Vectors EXAM review

Problem

1. $\left|\overrightarrow{A}\right| = 8$ and $\left|\overrightarrow{B}\right| = 1$.

   a) Find the net force, assume that $\overrightarrow{A}$ points North, and $\overrightarrow{B}$ points East.
   b) Find the equilibrant force

2. $\left|\overrightarrow{A}\right| = 15$, $\left|\overrightarrow{B}\right| = 7$, and the angle between $\overrightarrow{A}$ and $\overrightarrow{B}$ is 60°. What is the magnitude of $\overrightarrow{B} - \overrightarrow{A}$?

3. $\left|\overrightarrow{A}\right| = 1$, $\left|\overrightarrow{B}\right| = 7$ and the angle formed by $\overrightarrow{A}$ and $\overrightarrow{B}$ is 120°. Determine the unit vector in the same direction of $\overrightarrow{A} + \overrightarrow{B}$.

4. $\vec{x} = 5\vec{i} + 2\vec{j} + \vec{z}$ and $\vec{y} = 3\vec{i} + \vec{j} + \vec{z}$, determine $\vec{x} - \vec{y}$.

5. A triangle has sides represented by the vectors (1, 2) and (5, 6). Determine the vector representing the third side.

6. If $\overrightarrow{A} = 2\vec{i} + \vec{j} - \vec{k}$ and $\overrightarrow{A} + \overrightarrow{B} = \vec{i} + 4\vec{j} + \vec{k}$, what is $\overrightarrow{B}$?

7. A parallelogram is defined in R^3 by the vectors $\overrightarrow{OA} = 3\vec{i} + 2\vec{j}$ and $\overrightarrow{OB} = \vec{i} + 2\vec{j} + 4\vec{k}$. Determine the coordinates of the vertices.

8. A triangle in R^3 has two sides represented by the vectors $\overrightarrow{OA} = (1, 3, -1)$ and $\overrightarrow{OB} = (2, 4, 1)$. Determine the measures of the angles of the triangle. Explain.

9. In the set: {(1, 0), {0, a)}, what must a be for this to span R^2? Why?

10. Does the set {($\frac{1}{2}$, 1), (4, 8)} span R^2?

11. Do the vectors (1, 0, 0), (0, 1, 0). and (2, 3, 0) lie on the same plane?

12. Do the vectors (2, 0, 1), (–2, 0, 0), and (2, 3, 0) lie on the same plane? Explain your reasoning.

13. $\vec{x} = \vec{i} - 2\vec{j} + 4\vec{k}$, $\vec{y} = \vec{i} + \vec{k}$, and $\vec{z} = 3\vec{i} + 2\vec{j} - 3\vec{k}$. Simplify the linear combination $(3\vec{x} + \vec{y}) + \frac{1}{2}(\vec{y} - \vec{z})$.

14. What do the set of vectors {(1, 3, 2), ($\frac{1}{2}$, $\frac{3}{2}$, 1), (–2, – 6, – 4)} span?

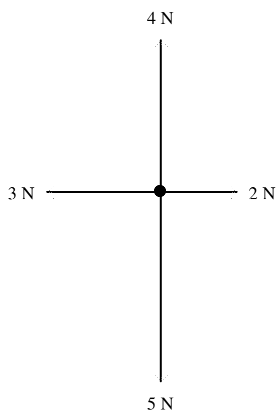
15. $\{(2, 3, 0), (0, 5, 0)\}$ spans a set in R^3.
a. Determine and describe the set spanned by the given set.
b. Can the vector (1, 1, 1) can be written as a linear combination of the given vectors? Explain.
c. Write vector (6, 8, 0) as a linear combination of the given two.

16. Two forces of 10 N and 30 N act at an angle of 40° to each other. Determine the resultant of these forces.

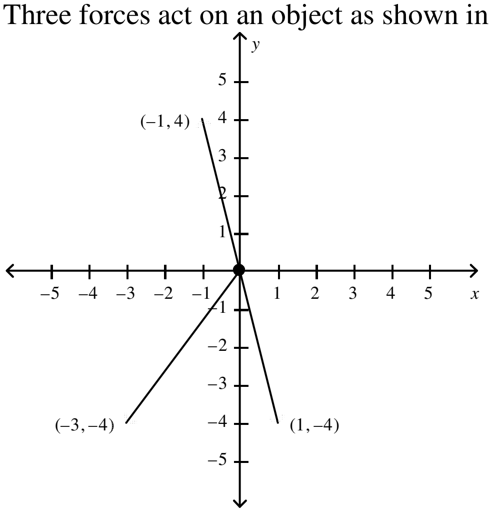
17. Two forces of 20 N and 25 N act on an object at angle of 30° to each other. Determine the equilibrant of these forces.

18. Paul pulls on a rope attached to his sleigh with a force of 60 N. If the rope makes an angle of 15° with the horizontal, determine the force that pulls the sleigh forward.

19. Four forces of magnitude 2 N, 3 N, 4 N, and 5 N act on an object. The forces are arranged as shown in the diagram. Determine the resultant of these forces



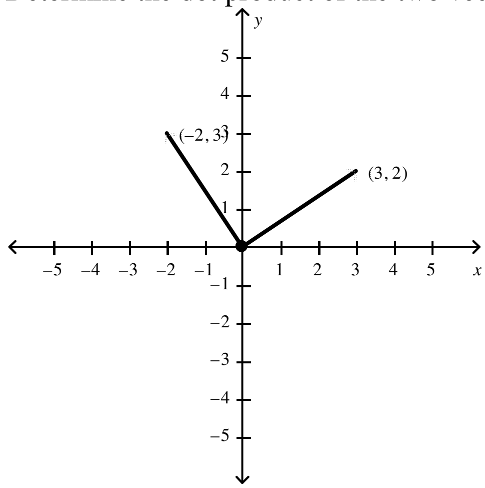
20. Three forces act on an object as shown in the diagram. Determine the equilibrant of these three vectors.



21. Determine the angle between $\vec{x}$ and $\vec{y}$, given that $\vec{x}$ and $\vec{y}$ are unit vectors and $\vec{x} \cdot \vec{y} = \frac{\sqrt{3}}{2}$.

22. Suppose $\vec{a} \cdot \vec{b} = 2$ and $\vec{a} \cdot \vec{c} = 5$. Determine the value of $\vec{a} \cdot \left(2\vec{b} - 3\vec{c}\right)$.

23. Determine the dot product of the two vectors shown in the diagram.



24. Determine the value of k such that the vectors (1, 2, 1) and (k, $2k + 1$, 8) are perpendicular.

25. Determine the vector projection of the vector (0, 8) on the vector (−1, −1).

26. The angle between the vectors $\vec{x}$ and $\vec{y}$ is 154°. The length of $\vec{x}$ is 7 and the length of $\vec{y}$ is 4. Determine the scalar projection of $\vec{x}$ on $\vec{y}$.

27. Suppose $(2,-3,4)\times(1,7,a)=(-16,12,17)$. Determine the value of a.

28. The vector $\vec{a}$ is twice as long as the vector $\vec{b}$. The angle between the vectors is 25°, and the magnitude of their cross product is 16.0. Determine $\left|\vec{b}\right|$.

29. The vectors $\vec{a}$ and $\vec{b}$ have magnitudes 3 and 5, respectively. The angle between the vectors is 130°. Determine the magnitude of the vector $-\vec{a}\times 4\vec{b}$.

30. Suppose $\vec{a}$, $\vec{b}$, and $\vec{c}$ are vectors such that $\vec{a}\times\vec{b}=(2,-1,7)$ and $\vec{a}\times\vec{c}=(10,8,-3)$. Determine $\left(3\vec{b}-\vec{c}\right)\times\left(2\vec{a}\right)$.

31. Calculate the amount of work done when a sleigh is pulled 50 m by a force of 30 N applied at an angle of 30° with the ground.

32. A 50 N force is applied at the end of a 30 cm wrench. If the force makes an angle of 67° with the wrench, what is the magnitude of the torque about the point of rotation?

33. The points $R(0, 2, 4)$, $S(1, 3, 2)$, and $T(-1, 2, 6)$ form a triangle. What is the area of ABC?

34. An object with a weight of 60 N is suspended by two lengths of rope from the ceiling. The angles that both lengths make with the ceiling are the same. The tension in each length is 40 N. Determine the angle that the lengths of ropes make with the ceiling.

35. Beth leaves a dock in a motorboat travelling at 14 m/s. She heads downstream at an angle of 40° to the current, which flows at 6 m/s. Determine how long it takes for Beth to travel 400 m.

36. An airplane is travelling S70°W with a resultant ground speed of 414 km/h. The nose of the plane is pointing west with an airspeed of 425 km/h. Determine is the speed and direction of the wind.

37. The vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ are unit vectors such that $\overrightarrow{a} \cdot \overrightarrow{b} = \frac{1}{2}$. Determine the value of j such that the vectors $\overrightarrow{a} + \overrightarrow{b}$ and $2\overrightarrow{a} + j\overrightarrow{b}$ are perpendicular.

38. Determine the value(s) of a such that the angle between the vectors $\overrightarrow{r} = (1, 2, a)$ and $\overrightarrow{s} = (1, 1, 1)$ is 60°.

39. The vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ are unit vectors such that $\left|\overrightarrow{a} + \overrightarrow{b}\right| = \sqrt{\frac{5}{2}}$. Determine the value of $\left(3\overrightarrow{a} - \overrightarrow{b}\right) \cdot \left(\overrightarrow{a} + \overrightarrow{b}\right)$.

40. The scalar projection of the vector $\overrightarrow{a} = (-2, 3, 1)$ on $\overrightarrow{b} = (2m, -1, 3)$ is -1. Determine the value of m.

41. The vector $\overrightarrow{r}$ is twice as long as the vector $\overrightarrow{s}$. The angle between the vectors is 110°. The vector projection of $\overrightarrow{s}$ on $\overrightarrow{r}$ is $(2, -1, 7)$. Determine $\overrightarrow{r}$.

42. A block slides 42 m down a ramp that makes an angle of 60° with the ground. Determine the mass of the block if 10 400 J of work was done. (Assume 1 kg exerts a force of 9.8 N.)

43. Write the vector equation for a line parallel to the line $L: \overrightarrow{r} = (1, 5) + s(-1, 3),\ s \in \mathbf{R}$ containing the point $P(2, 17)$.

44. Determine the parametric equations for the line perpendicular to $L: \overrightarrow{r} = (5, 6) + s(1, -5),\ s \in \mathbf{R}$ containing the point $P(2, 4)$.

45. A line passes through the points $P(7, 2)$ and $Q(1, 7)$. Find vector and parametric equations for this line.

46. Determine the Cartesian equation for a line passing through the point $P(1, 3)$ with normal vector $\overrightarrow{n} = (3, 5)$.

47. Determine the Cartesian equation of the line with parametric equations $x = 2t - 1,\ y = 4t + 2,\ t \in \mathbf{R}$.

48. Is the line with Cartesian equation $4x - 6y + 5 = 0$ parallel to the line with vector equation $\overrightarrow{r} = (1, 2) + s(3, 2)$, $s \in \mathbf{R}$?

49. Determine the symmetric equation of the line going through the points $P(-2, 0, 3)$ and $Q(1, 3, 7)$.

50. Does the line $\dfrac{x-7}{3} = \dfrac{y+8}{6} = \dfrac{z+4}{5}$ and the point $P(10, -2, 1)$ determine a plane?

51. Do the lines L_1: $\overrightarrow{r} = (1, 7, -5) + s(2, -2, 5)$, $s \in \mathbf{R}$, and the line L_2: $\overrightarrow{r} = (-2, 3, -6) + s(3, 2, 6)$, $s \in \mathbf{R}$, determine a plane?

52. Determine a set of parametric equations for the xz-plane.

53. Determine the Cartesian form of the plane whose equation in vector form is $\overrightarrow{r} = (-2, 2, 5) + s(2, -3, 1) + t(-1, 4, 2)$, $s, t \in \mathbf{R}$.

54. Determine the unit vector that is normal to the plane $\overrightarrow{r} = (1, -4, 3) + s(2, 0, -4) + t(0, 3, 1)$, $s, t \in \mathbf{R}$.

55. Determine the angle between the planes $\sqrt{3}x + z - 5 = 0$ and $2x + 2\sqrt{3}z + 10 = 0$.

56. a. Determine the vector and parametric equations of the line going through the points $P(1, 2, 3)$ and $Q(-1, 2, 6)$.
    b. Does this line have a system of symmetric equations? If it does have a system of symmetric equations, determine the system. If not, explain why.

57. Two lines have equations L_1: $x = 3t + 1$, $y = 3$, $z = -t + 2$, $t \in \mathbf{R}$ and L_2: $x = 2s + 3$, $y = -s + 1$, $z = 3s - 2$, $s \in \mathbf{R}$. Determine points P and Q if P is a point on L_1 and Q is a point on L_2 and $\overrightarrow{PQ}$ is perpendicular to both lines.

58. Determine if the lines with symmetric equations $\dfrac{x-3}{4} = \dfrac{y+2}{7} = \dfrac{z-1}{-3}$ and $\dfrac{x+1}{-8} = \dfrac{y+9}{-14} = \dfrac{z-4}{6}$ are the same. Explain your answer.

59. a. Determine the vector and parametric equations of the plane that passes through the points $Q\left(-\dfrac{3}{2}, 0, 0\right)$, $R(0, -1, 0)$, and $S(0, 0, 3)$.
    b. Determine if the point $P(1, 5, 6)$ is a point on this plane.

60. a. Explain why the vector equation $\overrightarrow{r} = (1, 3, -2) + s(2, 6, -4) + t(1, 3, -2)$, $s, t \in \mathbf{R}$ does not determine a plane.
    b. Determine the vector equation of a plane that contains line and the point $P(1, 1, 1)$.

61. At what point does the line $L\colon \overrightarrow{r} = (10, 7, 5,) + s(-4, -3, 2),\ s \in \mathbf{R}$ intersect the plane $P\colon 6x + 7y + 10z - 9 = 0$?

62. At what point does the line $L_1\colon (3, 0, 1) + s(5, 10, -15),\ s \in \mathbf{R}$ intersect the line $L_2\colon (2, 8, 12) + t(1, -3, -7),\ t \in \mathbf{R}$?

63. Solve the following system of equations.

$$x - y + 2z = 10$$
$$7x + 8y - z = 10$$
$$-6x - 5y + 2z = 12$$

64. Determine the solution to the following system of equations.

$$5x - 2y - z = -6$$
$$-x + y + 2z = 0$$
$$2x - y - z = -2$$

65. Describe the nature of intersection between the following planes.

$$2x - y + z = 3$$
$$4x - 2y + 2z = 6$$
$$-2x + y - z = -3$$

66. Show that the following system has no solutions.

$$x - y + 9z = 10$$
$$-4x + y - z = 0$$
$$-2x + 2y - 18z = -2$$

67. Determine the distance between the point $(6, 3, 4)$ and the line $\vec{r} = (0, 1, 3) + s(2, -1, -2),\ s \in \mathbf{R}$.

68. What is the distance between the origin and the line $\vec{r} = (1, 4) + s(-1, 2),\ s \in \mathbf{R}$?

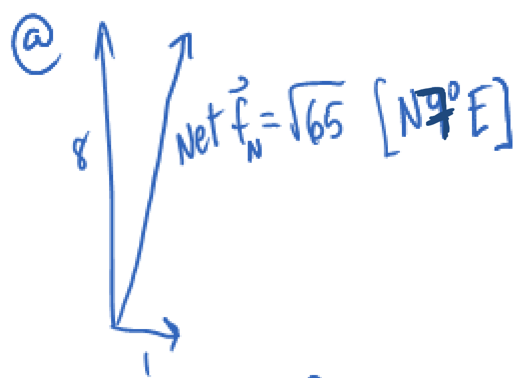
69. Calculate the distance between the point $(0, 2, -1)$ and the plane $2x - y + 2z + 13 = 0$.

70. Determine the minimal distance between the skew lines $L_1\colon (1, 1, 3) + s(2, -1, 1),\ s \in \mathbf{R}$ and $L_2\colon (0, 0, 2) + t(3, 1, 2),\ t \in \mathbf{R}$.

Vectors EXAM review
Answer Section

PROBLEM

1.



(b) equilibrant $\vec{f}_e = \sqrt{65}$ [S7°W]

2. 13
3. $\frac{1}{\sqrt{43}}(\vec{A} + \vec{B})$
4. $2\vec{i} + \vec{j}$
5. (4, 4) or (–4, –4)
6. $-\vec{i} + 3\vec{j} + 2\vec{k}$
7. (0, 0, 0), (3, 2, 0), (1, 2, 4), (4, 4, 4)
8. We will need the magnitudes of the vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OA} - \overrightarrow{OB}$.
   $\left|\overrightarrow{OA}\right| = \sqrt{1^2 + 3^2 + (-1)^2} = \sqrt{11}$
   $\left|\overrightarrow{OB}\right| = \sqrt{2^2 + 4^2 + 1^2} = \sqrt{21}$
   $\overrightarrow{OA} - \overrightarrow{OB} = (1-2, 3-4, -1-1) = (-1, -1, -2)$ $\left|\overrightarrow{OA} - \overrightarrow{OB}\right| = \sqrt{(-1)^2 + (-1)^2 + (-2)^2} = \sqrt{6}$
   θ = 31.20°, 44.54°, 104.26°
9. Any real number except 0, since in order to span R^2 you must have two noncollinear, nonzero vectors.
10. No, because the two vectors are collinear.
11. Yes, they are both on the plane $z = 0$.
12. No, there is no way to write the third as a linear combination of the first two. The first two are on the *xz*-plane, where the third is on the *xy*-plane.
13. $3\vec{i} - 7\vec{j} + 15\vec{k}$
14. They are all collinear, so they only span a line in R^3.

15. a. The set spanned by the given vectors is the xy-plane or the plane equal to $z = 0$.
b. The vector (1, 1, 1) cannot be written as a linear combination of the given vectors because it does not lie on $z = 0$ (or the xy-plane)
c. (6, 8, 0) = 3(2, 3, 0) -1/5(0, 5, 0)
16. The resultant has a magnitude of 38.21 N. It makes an angle of 30.31° with the 10 N force and of 9.69° with the 30 N force.
17. The equilibrant has a magnitude of 43.49 N. It makes an angle of 167° with the 25 N force and of 163° with the 20 N force.
18. 57.96 N
19. The resultant has a magnitude of 1.41 N. It makes an angle of 45° with the 5 N force and an angle of 45° with the 3 N force.
20. The equilibrant is the vector from the origin to (3, 4). Equivalently the resultant is a force of magnitude 5 N and makes an angle of 53.1° with the x-axis and of 36.9° with the y-axis.
21. 30°
22. –11
23. 0
24. $k = -2$
25. (4, 4)
26. -6.3
27. $a = -4$
28. 4.35
29. 46.0
30. (8, 22, –48)
31. 1299 J
32. 13.8 J
33. $\frac{\sqrt{5}}{2} \doteq 1.12$
34. $\theta \doteq 48.6°$.
35. 19.0 m/s is the resultant speed of the motorboat. So Beth travels 400 m in $\frac{400}{19.0} \doteq 21.0$ seconds.
36. Therefore the wind speed is 146 km/h.
So the wind blows from a direction 75.9° north from west. So the wind direction is S14.1°E.
37. $j = -2$.
38. $-12 + \sqrt{123}$
39. $\frac{5}{2}$.
40. m is $\sqrt{\frac{5}{6}}$.
41. (–11.7, 5.8, –40.9).
42. m = 29.2 kg.
43. The vector equation is $\overrightarrow{w} = (2, 17) + s(-1, 3)$, $s \in$ **R**.
44. The parametric equations are $x = 2 + 5t$ and $y = 4 + t$ for $t \in$ **R**.
45. The vector form is $\overrightarrow{r} = (7, 2) + s(-6, 5)$, $s \in$ **R**. The parametric form is $x = 7 - 6t$, $y = 2 + 5t$, $t \in$ **R**.
46. $3x + 5y - 18 = 0$
47. $2x - y + 4 = 0$.
48. Yes. The normal vector of the first line is $\overrightarrow{n_1} = (4, -6)$. The normal vector of the first line is $\overrightarrow{n_2} = (2, -3)$, and $2\overrightarrow{n_1} = \overrightarrow{n_2}$.
49. The equation is $\frac{x+2}{3} = \frac{y}{3} = \frac{z-3}{4}$.

50. No. The point P lies on the line, so there is only one direction vector.
51. No, the two lines do not intersect, so they cannot lie in the same plane.
52. One such set is $x = s$, $y = 0$, $z = t$, $s, t \in \mathbf{R}$.
53. $2x + y - z + 7 = 0$.
54. $\overrightarrow{n} = \left(\frac{6}{\sqrt{46}}, -\frac{1}{\sqrt{46}}, \frac{3}{\sqrt{46}}\right)$.
55. The angle between the two planes is $30°$.
56. a. The vector equation is determined by an initial point and the slope vector. The slope vector is $\overrightarrow{m} = (-2, 0, 3)$. Use P as the initial point, so the vector equation is $\overrightarrow{r} = (1, 2, 3) + s(-2, 0, 3)$, $s \in \mathbf{R}$. From the vector equation, it is easy to determine the parametric equation. The parametric equation is $x = -2t + 1$, $y = 2$, $z = 3t + 3$, $t \in \mathbf{R}$.

    b. This line does not have a system of symmetric equations. This is because $y = 2$ for every value of t.
57. $\left(\frac{605}{131}, 3, \frac{104}{131}\right), \left(\frac{573}{131}, \frac{41}{131}, \frac{8}{131}\right)$
58. They are the same line. A line is uniquely determined by two points, so if both lines contain two of the same points, then the lines are the same. It is clear that the point $P(3, -2, 1)$ is on the first line. Substituting these values into the symmetric equation for the second line gives $\frac{4}{-8} = \frac{7}{-14} = \frac{-3}{6}$, so P is on both lines. Doing the same calculation with the point $Q(-1, -9, 4)$ and the first line shows that these lines coincide.
59. a. The plane intersects the x-axis at $Q\left(-\frac{3}{2}, 0, 0\right)$. The plane intersects the y-axis at $R(0, -1, 0)$. The plane intersects the z-axis at $S(0, 0, 3)$. This gives two direction vectors: $\overrightarrow{QS} = \left(\frac{3}{2}, 0, 3\right)$ and $\overrightarrow{RS} = (0, 1, 3)$, so the vector equation is $\overrightarrow{r} = (0, 0, 3) + s\left(\frac{3}{2}, 0, 3\right) + t(0, 1, 3)$, $s, t \in \mathbf{R}$. Also, the parametric equations are $x = \frac{3}{2}s$, $y = t$, and $z = 3 + 3s + 3t$, $s, t \in \mathbf{R}$.

    b. Begin by using the parametric equations for the point P. $x = 1$ and $y = 5$, so $s = \frac{2}{3}$ and $t = 5$. $z = 3 + 3\left(\frac{2}{3}\right) + 3(5) = 20 \neq 6$, so the point P is not a point on this plane.
60. a. This vector equation is not a plane because the direction vectors are a scalar multiple of the other, i.e. $(2, 6, -4) = 2(1, 3, -2)$.

    b. The line from part a contains the origin, so $\overrightarrow{v} = (1, 1, 1)$ will be one of the two direction vectors. The other direction vector is given in the equation for the line in part a. The vector equation is $\overrightarrow{r} = s(1, 1, 1) + t(1, 3, -2)$, $s, t \in \mathbf{R}$.

    OTHER SOLUTIONS ALSO POSSIBLE
61. $(-14, -11, 17)$
62. $(4, 2, -2)$

63. $\left(\frac{-10}{3}, \frac{16}{3}, \frac{28}{3}\right)$

64. Line of intersection $x = -s - 2, y = -3s - 2, z = s$

65. The three planes are parallel to each other and equal.
Intersect in a plane. Many answers of vector equation for it,
one is: (x, y, z) =(0, 0, 3) +t(1, 0, -2) +r(0, 1, 1)

66.
Multiplying the first equation by 2 and adding it to the third equation we get $0x + 0y + 0z = 18$. This equation has no solution, therefore the system has no solution.

67. $\frac{\sqrt{305}}{3}$

68.
$\frac{6\sqrt{5}}{5}$

69. 3 units

70. $\frac{\sqrt{35}}{35}$