# Calculus EXAM review

**Problem**

1. Explain the steps used to determine the slope to the curve $y = \sqrt{x-7}$ at the point with $x$-coordinate 11 by first principles.

2. Consider the function $f(x) = \dfrac{5}{2+x}$.
   a. Determine the slope at any point $x$ by first principles
   b. Determine the slope at the point with $x$-coordinate 1.
   c. Determine the equation of the tangent to the curve $f(x)$ at the point with $x$-coordinate 1.

3. Determine $\dfrac{dy}{dx}$ if $y = \dfrac{1}{6}x^6 + \dfrac{1}{4}x^4 - 2x$.

4. Determine the coordinates of the point(s) on the graph of $y = 3x - \dfrac{1}{x}$ at which the slope of the tangent is 7.

5. What is the slope of the tangent to $y = 3\sqrt{x} - x$ at (4, 2)?

6. Determine $f'(-1)$ for $f(x) = (x^3 + 3x^2 + 4)(x^2 - 2x + 1)$.

7. Determine the value of $b$ if the slope of the tangent to $f(x) = (3x^2 + 1)(2x^2 + b)$ at $x = -1$ is $-16$.

8. a) Determine the derivative of $f(x) = \dfrac{3x-1}{2x+5}$. b) find the $\lim\limits_{x \to \infty} f(x)$

9. For $g(x) = (x^3 - 2x + 3)^4$, determine $g'(x)$. Leave your answer in product form.

10. For what value(s) of $x$ does $g(x) = \dfrac{x+1}{\sqrt{x}}$ have a horizontal tangent?

11. The normal to $f(x) = \dfrac{1}{x-1}$ at (2, 1) intersects the graph of $f(x)$ at another point. What are the coordinates of the other point?

12. For what value(s) of $x$ is the tangent to $h(x) = \dfrac{x^2}{x+1}$ parallel to the line with equation $x - 2y - 8 = 0$?

13. a) Determine the value(s) of $k$ such that $g'(-1) = -\dfrac{1}{2}$ if $g(x) = \dfrac{x-k}{1+x^2}$. b) Find $\lim\limits_{x \to \infty} g(x)$

14. The function $E(x) = \dfrac{50x^2 + 40x + 33}{x^2 - 1}$ models a company's expenses in thousands of dollars per 1000 units to produce touch sensitive screens for small electronics. $x$ represents thousands of screens. At what rate is the expense per screen changing when the company produces 3000 screens?

15. If $f(x) = \dfrac{1}{1-x}$ and $g(x) = \dfrac{x}{x-1}$, what is $f(g(x))$?

16. Determine $g(f(2))$ if $f(x) = \sqrt{x^2 + 5}$ and $g(x) = x^2 + x - 2$.

17. Let $y = f(x^2 - 4x + 5)$. Determine $\dfrac{dy}{dx}$ when $x = 1$, given that $f'(2) = \dfrac{1}{2}$.

18. If $y = u^3 + u^2 - 1$, where $u = \dfrac{1}{1-x}$, determine $\dfrac{dy}{dx}$ at $x = 2$.

19. If $f(x)$ is a differentiable function, determine an expression for the derivative of $g(x) = 3x^2 f(x^2 - 3)$.

20. Determine $y'$ for $y = (1 + x^2)^{\frac{1}{4}}$.

21. Determine the derivative of $f(x) = [(3x^2 + 1)^3 + 1]^2$. Express your answer in a simplified factored form.

22. What is the slope of the tangent to the function $f(x) = 4xe^x$ at the point with $x$-coordinate $x = 0$?

23. At what point on the graph of $f(x) = 2xe^x$ is the tangent parallel to the line with equation $y = 2x - 12$?

24. Determine the derivative of the function $f(x) = \dfrac{e^{3x}}{3 + e^{3x}}$.

25. Determine the derivative of the function $f(x) = x(5^{-x})$.

26. Determine the equation of the tangent to the curve $f(x) = 3^{5x}$ at the point with $x$-coordinate $x = 0$.

27. Determine the derivative of the function $f(x) = (5^x)(x^5)$

28. A boy is selling lemonade throughout the hot summer. Suppose the number of cups sold is given by the function $n(x) = x2^{-x} + 2$ where the price, $x$, in dollars determines the number of cups sold per day, $n$, in hundreds. What is the price that maximizes the number of cups sold? How many cups are sold at this price?

29. Determine the maximum and minimum value of the function $f(x) = 3x3^x - 1$.

30. a. For the function $f(x) = x^4 e^x$, determine the intervals of increase and decrease.
b. Determine the absolute minimum value of $f(x)$ and the local maximum value of $f(x)$.

31. Differentiate the function $f(x) = \sin(\cos(x^2))$.

32. Differentiate the function $f(x) = \sqrt{3\cos x - e^x}$.

33. Differentiate the function $f(x) = \dfrac{e^{\cos x}}{x}$.

34. Determine the equation for the tangent to the curve $y = \sin(4x)$ at the point with $x$-coordinate $3\pi$.

35. Differentiate the function $f(x) = \cos^3 x \sin x$.

36. Determine $\dfrac{d^2y}{dx^2}$ for the function $f(x) = \tan^3 x$.

37. Determine the local extrema on the curve $y = 4x - \tan x$ in the interval $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$.

38. Differentiate the function $f(x) = \tan(4^x)$.

39. Differentiate the function $f(x) = \tan(\cos(e^x))$.

40. Differentiate the function $f(x) = 5^{\tan\sqrt{x}}$.

41. Find $\dfrac{dy}{dx}$ for

(1) $-3xy - 4y^2 = 2$

(2) $8x^2 = 2y^3 + 3xy^2$

(3) $\dfrac{3}{2x} + \dfrac{1}{y} = y$

(4) $3x^2 = \dfrac{2-y}{2+y}$

42. Differentiate the following:
a) $y = x\ln x$
b) $y = \ln 2x^2$
c) $y = \log_4 3x$
d) $y = \dfrac{\ln x^2}{e^{2x}}$
e) $\ln x^2 - \ln y = 2$

43. The growth in the population of a group of rabbits is given by $P(t) = 800e^{0.08t}$, where $P$ is the population at time $t$ measured in weeks.
a. What is the initial population of rabbits?
b. How many rabbits are there after 14 days?
c. What is the rate of change of rabbits after 14 days?

44. A certain population of squirrels is represented by the function $P(t) = 3t(e^{-\frac{t}{3}})$, where $P$ is the number of squirrels measured in hundreds after $t$ weeks.
a. Determine the function, $P'$, which represents the rate of change of squirrels.
b. After how many weeks are there a maximum number of squirrels present?
c. Determine the maximum number of squirrels present in the population.
d. Does the squirrel population ever die out?

45. A particle moves along a line so that, at time $t$, its position is $s(t) = 11\cos(3t),\ t \geq 0$.
a. What is the first time $t$ that the particle changes direction?
b. For what values of $t$ does the particle change direction?
c. What is the particle's maximum velocity?

46. Consider the function $f(x) = \sin^4(4x)$.
a. Determine $f'(x)$.
b. Determine $f''(x)$.
c. What is the $x$-coordinate of all points in which $f'(x) = 0$?

47. a. Graph the function $f(x) = \sqrt{16 - x^2}$.
b. Determine $\lim\limits_{x \to -4^+} f(x)$. Explain.
c. Determine $\lim\limits_{x \to -4^-} f(x)$. Explain.
d. From b. and c., determine $\lim\limits_{x \to -4} f(x)$.

48. Determine, using the properties of limits, $\lim\limits_{x \to 0} \dfrac{f(x) - 3x}{f(x) + x^2}$ given $\lim\limits_{x \to 0} f(x) = 1$.

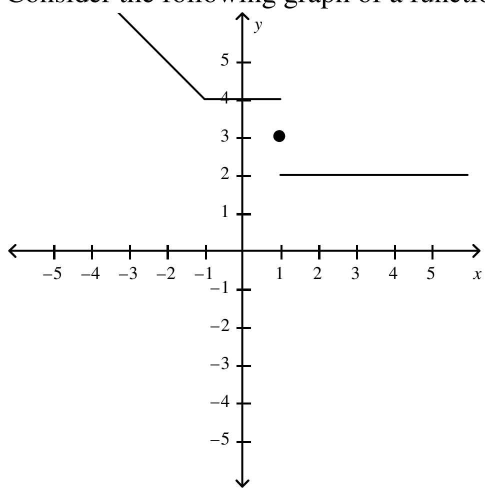
49. Determine $\lim\limits_{x \to -9} \dfrac{5x^3 + 40x^2 - 45x}{x + 9}$.

50. Determine $\lim\limits_{x \to \frac{1}{4}} \sqrt{14x + \sqrt{x}}$.

51. Determine $\lim\limits_{x \to 1} \dfrac{3 - \sqrt{8 + x}}{1 - x}$. Explain each step.

52. Determine $\lim_{x \to 125} \frac{125 - x}{x^{\frac{1}{3}} - 5}$. Explain each step.

53. Consider the following graph of a function $f(x)$.




Determine the following and explain each answer.

a. $\lim_{x \to 0} f(x)$

b. $\lim_{x \to 1^-} f(x)$

c. $\lim_{x \to 1^+} f(x)$

d. $\lim_{x \to 1} f(x)$

e. $f(1)$

54. Determine the real values of *a*, *b*, and *c* for the quadratic function $f(x) = ax^2 + bx + c$, $a \neq 0$ that satisfy the conditions $f(0) = 0$, $\lim_{x \to -1} f(x) = 3$, and $\lim_{x \to 2} f(x) = 6$. Explain all steps of the solution.

55. Let $f(x) = ax^3 + bx^2 + cx + 1$ . Determine the values of $a$, $b$, and $c$ so that $f(x)$ has a point of inflection at $x = 2$, a local minimum at $x = -2$, and $f(1) = 2$.

56. Let $f(x) = 2x^3 - 9x^2 - 60x + 1$ . Use the second derivative test to determine all local minima of $f(x)$.

57. Let $f'(x) = x^4 + 4x^3 + 2x^2 + 12x + 1$ .

a. For what values of $x$ is $f(x)$ concave up?

b. For what values of $x$ is $f(x)$ concave down?

c. For what values of $x$ does $f(x)$ have a point of inflection?

58. Let $f(x) = \frac{3x^2 + 9x - 54}{x^2 + 7x + 10}$ . Determine the equations of all of the asymptotes of $f(x)$.

59. Let $f(x) = (x-1)^{\frac{2}{3}}(x+3)$.

a. What are the critical numbers?

b. Classify the critical points using the first derivative test.

60. Sketch a possible graph for a function that has the following characteristics.

$f(0) = 2.5$

The horizontal asymptote is $y = 3$.

The vertical asymptote is $x = 4$.

$f'(x) < 0$ and $f''(x) > 0$ for $x > 4$.

$f'(x) < 0$ and $f''(x) < 0$ for $x < 4$.

61. Sketch the graph of $f(x) = (x-8)^{\frac{2}{3}}$. Justify your work.

62. Determine the right circular cylinder of greatest volume that can be inscribed upright in a right circular cone of radius 4 and height 8.

63. Express 18 as a sum of two positive numbers whose product of the first and square of the second is as large as possible. Explain why this is the largest possible product.

64. An isosceles triangle has a vertex at the origin and the base parallel to the x-axis. Determine the area of the largest such triangle that is bound by the function $x^2 + 6y = 48$ and the x-axis. Explain why this is in fact the maximum area.

65. A fireman has to reach a burning building. Determine the length of the shortest ladder that will reach over a 2 metre high fence to the burning building which is 1 metre behind the fence.

66. A woman wants to construct a box whose base length is twice the base width. The material to build the top and bottom is $\$9/m^2$ and the material to build the sides is $\$6/m^2$. If the woman wants the box to have a volume of 70 $m^3$, determine the dimensions of the box (in metres) that will minimize the cost of production. What is the minimum cost?

67. RELATED rates

a)

A spherical snowball is melting. Its radius is decreasing at 0.2 centimeters per hour when the radius is 15cm. How fast is the volume decreasing at this time?

b)

A water tank is in the shape of an inverted cone with depth 10m and top radius 8m. Water is flowing into the tank at $0.1\text{m}^3/\text{min}$ but leaking out at a rate of of $0.001h^2\text{m}^3/\text{min}$ where $h$ is the depth of the water in the tank. Can the tank ever overflow? Explain.

c)

When a rocket is 2 km high, it is moving vertically upward at a speed of 300 km per hour. At that instant, how fast is the angle of elevation of the rocket increasing, as seen by an observer on the ground 5 km from the launching pad?

d)

Water leaking onto a floor creates a circular pool with an area that increases at the rate of 3 $cm^2$ per minute. How fast is the radius of the pool increasing when the radius is 10 cm?

## Calculus EXAM review
## Answer Section

**PROBLEM**

1. ANS:

   Determine $\lim_{h \to 0} \frac{f(11+h) - f(11)}{h}$.

   So, $\lim_{h \to 0} \frac{f(11+h) - f(11)}{h} = \lim_{h \to 0} \frac{\sqrt{4+h} - 2}{h}$.

   Then, rationalize the numerator.

   $$\lim_{h \to 0} \frac{\sqrt{4+h} - 2}{h}$$

   $$= \lim_{h \to 0} \frac{\sqrt{4+h} - 2}{h} \times \frac{\sqrt{4+h} + 2}{\sqrt{4+h} + 2}$$

   $$= \lim_{h \to 0} \frac{4+h-4}{h(\sqrt{4+h} + 2)}$$

   $$= \lim_{h \to 0} \frac{1}{\sqrt{4+h} + 2}$$

   Finally, evaluate the limit.

   $$\lim_{h \to 0} \frac{1}{\sqrt{4+h} + 2}$$

   $$= \frac{1}{\sqrt{4+0} + 2}$$

   $$= \frac{1}{4}$$

   PTS: 1 REF: Communication OBJ: 1.2 - The Slope of a Tangent

2. ANS:

   a. Let *m* be the slope. Then,

$$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{x+h-x}$$

$$= \lim_{h \to 0} \left( \frac{5}{(2+x+h)} - \frac{5}{2+x} \right) \frac{1}{h}$$

$$= \lim_{h \to 0} \left( \frac{5(2+x) - 5(2+x+h)}{(2+x+h)(2+x)} \right) \frac{1}{h}$$

$$= \lim_{h \to 0} \left( \frac{-5h}{(2+x+h)(2+x)} \right) \frac{1}{h}$$

$$= \lim_{h \to 0} \left( \frac{-5}{(2+x+h)(2+x)} \right)$$

$$= \frac{-5}{(2+x)^2}$$

b. At the point with *x*-coordinate 1, the slope is

$$m = \frac{-5}{3^2}$$

$$= -\frac{5}{9}$$

c. $$y - \frac{5}{3} = -\frac{5}{9}(x-1)$$

$$9y - 15 = -5(x-1)$$

$$9y - 15 = -5x + 5$$

$$5x + 9y - 20 = 0$$

PTS: 1 REF: Application OBJ: 1.2 - The Slope of a Tangent

3. ANS:

$$\frac{dy}{dx} = x^5 + x^3 - 2$$

PTS: 1 REF: Knowledge and Understanding
OBJ: 2.2 - The Derivatives of Polynomial Functions

4. ANS:

$\left(-\frac{1}{2}, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, -\frac{1}{2}\right)$

PTS: 1 REF: Application OBJ: 2.2 - The Derivatives of Polynomial Functions

5. ANS:

$$-\frac{1}{4}$$

PTS: 1 REF: Knowledge and Understanding
OBJ: 2.2 - The Derivatives of Polynomial Functions

6. ANS:
$-36$

PTS: 1 REF: Knowledge and Understanding OBJ: 2.3 - The Product Rule

7. ANS:
$b = -2$

PTS: 1 REF: Thinking OBJ: 2.3 - The Product Rule

8. ANS:
a) $f'(x) = \dfrac{17}{(2x+5)^2}$ b) HA at y=3/2

PTS: 1 REF: Knowledge and Understanding OBJ: 2.3 - The Product Rule

9. ANS:
$g'(x) = 4(x^3 - 2x + 3)^3(3x^2 - 2)$

PTS: 1 REF: Knowledge and Understanding OBJ: 2.3 - The Product Rule

10. ANS:
$x = 1$

PTS: 1 REF: Knowledge and Understanding OBJ: 2.4 - The Quotient Rule

11. ANS:
$f'(x) = \dfrac{-1}{(x-1)^2}$, so $f'(2) = \dfrac{-1}{1} = -1$.

The slope of the tangent line at (2, 1) is $-1$, so the slope of the normal at (2, 1) is 1.
The equation of the normal at (2, 1) is $y - 1 = 1(x - 2)$

$$y = x - 1$$

The normal intersects the graph of $f(x)$ when $x - 1 = \dfrac{1}{x-1}$

$$(x-1)^2 = 1$$
$$x - 1 = \pm 1$$
$$x = 1 \pm 1$$
$$x = 2, 0$$

The normal intersects the graph of $f(x)$ at $x = 0$ and $x = 2$.
$f'(0) = -1$, so the other point where the normal intersects the graph of $f(x)$ is $(0, -1)$.

PTS: 1 REF: Thinking OBJ: 2.4 - The Quotient Rule

12. ANS:
$x = -1 + \sqrt{2}, -1 - \sqrt{2}$

PTS: 1 REF: Application OBJ: 2.4 - The Quotient Rule

13. ANS:
a) $k = +1$, b) HA at y=0

PTS: 1 REF: Thinking OBJ: 2.4 - The Quotient Rule

14. ANS:
−$14.03 per screen

PTS: 1 REF: Application OBJ: 2.4 - The Quotient Rule

15. ANS:
$f(g(x)) = 1 - x$

PTS: 1 REF: Knowledge and Understanding
OBJ: 2.5 - The Derivatives of Composite Function

16. ANS:
10

PTS: 1 REF: Knowledge and Understanding
OBJ: 2.5 - The Derivatives of Composite Function

17. ANS:
−1

PTS: 1 REF: Knowledge and Understanding
OBJ: 2.5 - The Derivatives of Composite Function

18. ANS:
1

PTS: 1 REF: Knowledge and Understanding
OBJ: 2.5 - The Derivatives of Composite Function

19. ANS:

$$\begin{aligned} g'(x) &= 3x^2 f'(x^2-3)(2x) + f(x^2-3)(6x) \\ &= 6x^3 f'(x^2-3) + 6xf(x^2-3) \\ &= 6x\left[x^2 f'(x^2-3) + f(x^2-3)\right] \end{aligned}$$

PTS: 1 REF: Thinking OBJ: 2.5 - The Derivatives of Composite Function

20. ANS:

$$y' = \frac{x}{2(1+x^2)^{\frac{3}{4}}}$$

PTS: 1 REF: Knowledge and Understanding
OBJ: 2.5 - The Derivatives of Composite Function

21. ANS:
$f'(x) = 36x(3x^2+1)^2[(3x^2+1)^3+1]$

PTS: 1 REF: Knowledge and Understanding
OBJ: 2.5 - The Derivatives of Composite Function

22. ANS:
4

PTS: 1 REF: Knowledge and Understanding
OBJ: 5.1 - Derivatives of Exponential Functions, y = e^x

23. ANS:
(0, 0)

PTS: 1 REF: Thinking OBJ: 5.1 - Derivatives of Exponential Functions, y = e^x

24. ANS:
$f'(x) = \dfrac{9e^{3x}}{(3+e^{3x})^2}$

PTS: 1 REF: Knowledge and Understanding
OBJ: 5.1 - Derivatives of Exponential Functions, y = e^x

25. ANS:
$f'(x) = 5^{-x}(-x\ln 5 + 1)$

PTS: 1 REF: Knowledge and Understanding
OBJ: 5.2 - The Derivative of the General Exponential Function, y = b^x

26. ANS:
$-5.49x + y - 1 = 0$

PTS: 1 REF: Application
OBJ: 5.2 - The Derivative of the General Exponential Function, y = b^x

27. ANS:
$f'(x) = x^4 5^x(5 + x\ln 5)$

PTS: 1 REF: Knowledge and Understanding
OBJ: 5.2 - The Derivative of the General Exponential Function, y = b^x

28. ANS:
Maximum value of 2.53 at $x = \dfrac{1}{\ln 2} \doteq$ 1.44.
So, a price of $1.44 will maximize the number of cups sold.
About 253 cups are sold per day at this price.

PTS: 1 REF: Application
OBJ: 5.3 - Optimization Problems Involving Exponential Functions

29. ANS:
No maximum value.
Minimum value of $-2.00$ at $x = -\dfrac{1}{\ln 3}$.

PTS: 1 REF: Knowledge and Understanding
OBJ: 5.3 - Optimization Problems Involving Exponential Functions

30. ANS:
a. increasing: $(-\infty, -4)$ and $(0, \infty)$; decreasing: $(-4, 0)$
b. absolute minimum of 0 at $x = 0$; local maximum value of 4.69 at $x = -4$

PTS: 1 REF: Thinking
OBJ: 5.3 - Optimization Problems Involving Exponential Functions

31. ANS:
$$f'(x) = -2x\cos\left(\cos\left(x^2\right)\right)\sin\left(x^2\right)$$

PTS: 1 REF: Knowledge and Understanding
OBJ: 5.4 - The Derivatives of y = sin x and y = cos x

32. ANS:
$$f'(x) = \frac{-3\sin x - e^x}{2\sqrt{3\cos x - e^x}}$$

PTS: 1 REF: Knowledge and Understanding
OBJ: 5.4 - The Derivatives of y = sin x and y = cos x

33. ANS:
$$f'(x) = \frac{e^{\cos x}(-x\sin x - 1)}{x^2}$$

PTS: 1 REF: Knowledge and Understanding
OBJ: 5.4 - The Derivatives of y = sin x and y = cos x

34. ANS:
$$4x - y - 12\pi = 0$$

PTS: 1 REF: Application OBJ: 5.4 - The Derivatives of y = sin x and y = cos x

35. ANS:
$$f'(x) = \cos^4 x - 3\sin^2 x\cos^2 x$$

PTS: 1 REF: Knowledge and Understanding
OBJ: 5.4 - The Derivatives of y = sin x and y = cos x

36. ANS:
$$\frac{d^2y}{dx^2} = 6\tan^3 x\sec^2 x + 6\tan x\sec^4 x$$

PTS: 1 REF: Knowledge and Understanding OBJ: 5.5 - The Derivative of y = tan x

37. ANS:
local maximum: $\frac{4\pi}{3} - \sqrt{3} \doteq 2.46$ at $x = \frac{\pi}{3}$
local minimum: $-\frac{4\pi}{3} + \sqrt{3} \doteq -2.46$ at $x = -\frac{\pi}{3}$

PTS: 1 REF: Thinking OBJ: 5.5 - The Derivative of y = tan x

38. ANS:
$$f'(x) = \sec^2(4^x)\cdot 4^x\ln 4$$

PTS: 1 REF: Knowledge and Understanding OBJ: 5.5 - The Derivative of y = tan x

39. ANS:
$$f'(x) = \sec^2(\cos(e^x))\cdot(-\sin(e^x))\cdot e^x$$

PTS: 1 REF: Knowledge and Understanding OBJ: 5.5 - The Derivative of y = tan x

40. ANS:

$$f'(x) = 5^{\tan\sqrt{x}} \cdot \ln 5 \cdot \sec^2(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$$

PTS: 1 REF: Knowledge and Understanding OBJ: 5.5 - The Derivative of y = tan x

41. ANS:

(1) $-3xy - 4y^2 = 2$

$-3y - 3xy' - 8yy' = 0$

$y' = \frac{3y}{-3x-8y}$

(2) $8x^2 = 2y^3 + 3xy^2$

$16x = 6y^2y' + 3y^2 + 3x(2yy')$

$y' = \frac{16x - 3y^2}{6y^2 + 6xy}$

(3) $\frac{3}{2x} + \frac{1}{y} = y$

$\frac{-3}{2x^2} - \frac{1}{y^2}y' = y'$

$y' = \frac{-3}{2x^2} \div \left(1 + \frac{1}{y^2}\right)$

$= \frac{-3y^2}{2x^2(y^2+1)}$

(4) $3x^2 = \frac{2-y}{2+y}$

$3x^2(2+y) = 2 - y$

$6x(2+y) + 3x^2(y') = -y'$

$y' = -\frac{6x(2+y)}{3x^2+1}$

PTS: 1

42. ANS:

$y = x\ln x$

$y' = 1\ln x + x\left(\frac{1}{x}\right)$

$= \ln x + 1$

$y = \ln 2x^2$

$y' = \frac{1}{2x^2}(4x)$

$y' = \frac{2}{x}$

$y = \log_4 3x$

$y' = \frac{1}{3x\ln 4}(3)$

$y' = \frac{1}{x\ln 4}$

$y = \frac{\ln x^2}{e^{2x}} = (\ln x^2)e^{-2x}$

$y' = \frac{1}{x^2}(2x)e^{-2x} + (\ln x^2)e^{-2x}(-2)$

$y' = \frac{2}{x}e^{-2x}\left[1 - x\ln x^2\right] = \frac{2\left[1 - x\ln x^2\right]}{xe^{2x}}$

$\ln x^2 - \ln y = 2$

$\frac{1}{x^2}(2x) - \frac{1}{y}y' = 0$

$\frac{2}{x} - \frac{1}{y}y' = 0$

$\frac{2y}{x} = y'$

PTS: 1

43. ANS:

a. The initial population is when $t = 0$. $P(0) = 800$ rabbits.

b. 14 days is equal to 2 weeks. So, the question asks for $P(2)$. $P(2) = 939$ rabbits.
c. 14 days is equal to 2 weeks. So, the question asks for $P'(2)$. $P'(2) = 75$ rabbits/week.

PTS: 1 REF: Application OBJ: 5.1 - Derivatives of Exponential Functions, y = e^x

44. ANS:

a. $P'(t) = e^{-\frac{t}{3}}(3 - t)$

b. Determine when $P'(t) = 0$.

$0 = e^{-\frac{t}{3}}(3 - t)$

$t = 3$

When $t < 3$, $P'(t) > 0$ and when $t > 3$, $P'(t) < 0$. Therefore, $t = 3$ weeks is when there is a maximum.

c. Determine $P(3)$.

$P(3) = 3(3)(e^{-1}) = \frac{9}{e} \doteq 3.31$

So, the maximum number of squirrels in the population is about 331.

d. Technically the function never reaches zero. However, there is certainly a point when only a fraction of a squirrel is left, which is not possible. So, most likely the squirrel population will actually die out.

PTS: 1 REF: Application
OBJ: 5.3 - Optimization Problems Involving Exponential Functions

45. ANS:

a. The particle will change direction at the points $t$ in which $s'(t) = 0$ and the slope changes from positive to negative or negative to positive.

$s'(t) = -33\sin(3t)$
$0 = -33\sin(3t)$
$0 = \sin(3t)$
$3t = 0, \pi, 2\pi, \ldots$

$t = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \ldots$

$t = \frac{\pi k}{3}$ where $k$ is a non-negative integer.

One can check that the slope switches sign around these points.

So, the first time that the particle changes direction is at $\frac{\pi}{3}$.

b. The particle changes direction for all $t = \frac{\pi k}{3}$ where $k$ is a non-negative integer.

c.

$s(t) = 11\cos(3t)$
$v(t) = s' = 11(-\sin(3t))3 = -33\sin(3t)$
$a(t) = s'' = -\cancel{66}\,99\cos(3t) = 0$
let $\theta = 3t$
$\cos\theta = 0$
$\theta_1 = \pi/2$
$\theta_2 = \frac{3\pi}{2}$
$\therefore$ $\left.\begin{array}{l} t_1 = \pi/6 \\ t_2 = \pi/2 \end{array}\right\}$ critical pts of velocity

compare: $v(0) = 0$
$v\left(\frac{\pi}{6}\right) = -33\sin\pi/2 = -33$
$v\left(\pi/2\right) = -33\sin\left(\frac{3\pi}{2}\right) = +33$

$\therefore$ 1st max velocity at $t = \pi/2 \doteq 1.57$ sec and velocity MAX is 33 units/time

46. ANS:

a. Using the power rule and the chain rule,

$f'(x) = 4(\sin^3(4x)) \cdot 4\cos(4x)$.
$\quad = 16\cos(4x) \cdot \sin^3(4x)$

b. Using the product rule, power rule, and chain rule,

$$f'''(x) = (16\cos(4x))(3\sin^2(4x)) \cdot 4\cos(4x)) + (\sin^3(4x))(-64\sin(4x))$$
$$= 192\cos^2(4x)\sin^2(4x) - 64\sin^4(4x)$$

c. $0 = 16\cos(4x) \cdot \sin^3(4x)$

After simplifying, $\cos(4x) = 0$ and $\sin(4x) = 0$

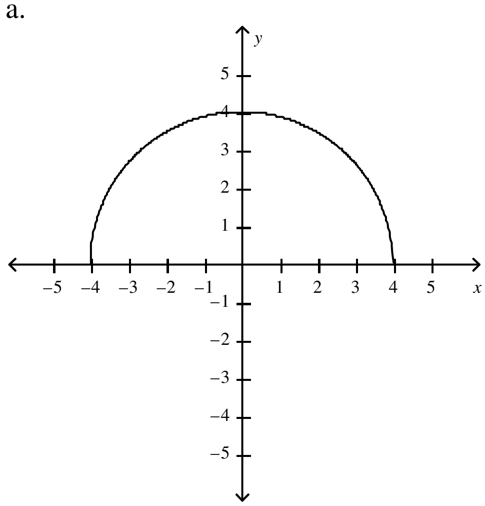
So, $4x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$.

$x = 0, \frac{\pi}{8}, \frac{2\pi}{8}, \frac{3\pi}{8}, \ldots$

So, the *x*-coordinates of all points in which $f''(x) = 0$ are given by $x = \frac{\pi k}{8}$ where *k* is an integer.

PTS: 1 REF: Knowledge and Understanding
OBJ: 5.4 - The Derivatives of y = sin x and y = cos x

47. ANS:
a.




b. From the graph, the right-hand limit at $x = -4$ is 0. So, $\lim_{x \to -4^+} f(x) = 0$.

c. The domain of the function is $-4 \le x \le 4$. The function is not defined for $x < -4$. So, there is no limit on the left of $-4$. Therefore, $\lim_{x \to -4^-} f(x)$ does not exist.

d. $\lim_{x \to -4} f(x)$ does not exist. This is because the right-hand limit at $x = -4$ and the left-hand limit at $x = -4$ does not exist.

PTS: 1 REF: Thinking OBJ: 1.5 - Properties of Limits

48. ANS:
1

PTS: 1 REF: Thinking OBJ: 1.5 - Properties of Limits

49. ANS:
450

PTS: 1 REF: Knowledge and Understanding OBJ: 1.5 - Properties of Limits

50. ANS:

2

PTS: 1 REF: Knowledge and Understanding OBJ: 1.5 - Properties of Limits

51. ANS:
First, try direction substitution. The denominator will be $1 - 1 = 0$, so direct substitution fails.
Next, notice that this appears to be a rationalizing limit problem. So, multiply numerator and denominator by $3 + \sqrt{8+x}$. Then, the limit is $\lim_{x \to 1} \left( \frac{3 - \sqrt{8+x}}{1-x} \times \frac{3+\sqrt{8+x}}{3+\sqrt{8+x}} \right)$. Now, simplify and see if anything cancels.

$$\lim_{x \to 1} \left( \frac{3 - \sqrt{8+x}}{1-x} \times \frac{3+\sqrt{8+x}}{3+\sqrt{8+x}} \right)$$

$$= \lim_{x \to 1} \frac{9 - (8+x)}{(1-x)(3+\sqrt{8+x})}$$

$$= \lim_{x \to 1} \frac{1-x}{(1-x)(3+\sqrt{8+x})}$$

$$= \lim_{x \to 1} \frac{1}{3+\sqrt{8+x}}$$

Finally, use direct substitution to finish the problem.

$$\lim_{x \to 1} \frac{1}{3+\sqrt{8+x}} = \frac{1}{3+3} \doteq 0.16$$

PTS: 1 REF: Knowledge and Understanding OBJ: 1.5 - Properties of Limits

52. ANS:
First, try direction substitution. The denominator will be $5 - 5 = 0$, so direct substitution fails.
Next, notice that this appears to be a change of variable limit problem. So, let $u = x^{\frac{1}{3}}$. Then, $x = u^3$. As $x$ approaches 125, $u$ approaches 5. Now, the limit can be rewritten as $\lim_{u \to 5} \frac{125 - u^3}{u-5}$. Factor the numerator to get $\lim_{u \to 5} \frac{(5-u)(25+5u+u^2)}{u-5}$. Take a negative out of the top expression to get $\lim_{u \to 5} \frac{-(u-5)(25+5u+u^2)}{u-5}$.
Finally, cancel the $u - 5$ and then use direct substitution to evaluate the limit. So,
$\lim_{u \to 5} -(25+5u+u^2) = -(25+25+25) = -75$.

PTS: 1 REF: Knowledge and Understanding OBJ: 1.5 - Properties of Limits

53. ANS:
a. $\lim_{x \to 0} f(x) = 4$. There is no discontinuity at (0, 4). It is a straight line. The limit can easily be seen to be 4.
b. $\lim_{x \to 1^-} f(x) = 4$. This limit is the value $x$ is approaching from the left of 1. This is on the line at the value
4. The limit is 4.
c. $\lim_{x \to 1^+} f(x) = 2$ This limit is the value $x$ is approaching from the right of 1. This is on the line at the value
2. The limit is 2.

d. $\lim_{x \to 1^-} f(x) = 4$ does not equal $\lim_{x \to 1^+} f(x) = 2$. Therefore, $\lim_{x \to 1} f(x)$ does not exist.

e. $f(1) = 3$. This is a point on the graph.

PTS: 1 REF: Communication OBJ: 1.4 - Limit of a Function

54. ANS:

Well, $f(0) = 0$. So, first substitute these values in the given function.

$$f(0) = a(0)^2 + b(0) + c$$

$$0 = 0 + 0 + c$$

$$0 = c$$

Now, work with the other 2 given expressions.

$\lim_{x \to -1} f(x) = 3$. So,

$$a(-1)^2 + b(-1) + 0 = 3$$

$$a - b = 3$$

$\lim_{x \to 2} f(x) = 6$. So,

$$a(2)^2 + b(2) + 0 = 6$$

$$4a + 2b = 6$$

Now, solve for *a* and *b*.

Using substitution,

$$4(3 + b) + 2b = 6$$

$$12 + 4b + 2b = 6$$

$$6b = -6$$

$$b = -1$$

Therefore,

$$a - b = 3$$

$$a - (-1) = 3$$

$$a + 1 = 3$$

$$a = 2$$

So, $a = 2$, $b = -1$, and $c = 0$.

PTS: 1 REF: Thinking OBJ: 1.4 - Limit of a Function

55. ANS:

$$f(x) = ax^3 + bx^2 + cx + 1$$
$$f(1) = a + b + c + 1$$
$$2 = a + b + c + 1$$
$$a + b + c = 1$$
$$f'(x) = 3ax^2 + 2bx + c$$
$$f'(x) = 3a(4) + 2b(-2) + c$$
$$0 = 12a - 4b + c$$
$$f''(x) = 6ax + 2b$$
$$f''(2) = 6a(2) + 2b$$
$$0 = 12a + 2b$$
$$12a = -2b$$
$$0 = (-2b) - 4b + c$$
$$c = 6b$$
$$\left(-\frac{1}{6}b\right) + b + (6b) = 1$$
$$-b + 6b + 36b = 6$$
$$41b = 6$$
$$b = \frac{6}{41}$$
$$c = \frac{36}{41}$$
$$a = -\frac{b}{6}$$
$$a = -\frac{1}{41}$$
$$f(x) = -\frac{1}{41}x^3 + \frac{6}{41}x^2 + \frac{36}{41}x + 1$$

PTS: 1 REF: Application OBJ: 4.4 - Concavity and Points of Inflection

56. ANS:

$$f(x) = 2x^3 - 9x^2 - 60x + 1$$

$$f'(x) = 6x^2 - 18x - 60$$

$$= 6(x^2 - 3x - 10)$$

$$f'(x) = 6(x-5)(x+2)$$

$$f''(x) = 12x - 18$$

$$f''(x) = 6(2x - 3)$$

$$f''(5) = 6(2(5) - 3)$$

$$= 6(7)$$

$$= 42$$

$$f''(5) > 0$$

$$f''(-2) = 6(2(-2) - 3)$$

$$= 6(-7)$$

$$= -42$$

$$f''(-2) < 0$$

$f(x)$ has a relative minimum at $x = 5$.

PTS: 1 REF: Knowledge and Understanding
OBJ: 4.4 - Concavity and Points of Inflection

57. ANS:

$$f'(x) = x^4 + 4x^3 + 2x^2 + 12x + 1$$

$$f''(x) = 4x^3 + 12x^2 + 4x + 12$$

$$= 4(x^3 + 3x^2 + x + 3)$$

$$= 4((x^3 + x) + (3x^2 + 3))$$

$$= 4(x(x^2 + 1) + 3(x^2 + 1))$$

$$f''(x) = 4(x+3)(x^2 + 1)$$

| Value of $x$ | $x < -3$ | $x > -3$ |
|---|---|---|
| Sign of $f''(x)$ | negative | positive |

a. $f(x)$ is concave up for $x > -3$.
b. $f(x)$ is concave down for $x < -3$.
c. $f(x)$ has a point of inflection at $x = -3$.

PTS: 1 REF: Knowledge and Understanding
OBJ: 4.4 - Concavity and Points of Inflection

58. ANS:

$$f(x) = \frac{3x^2 + 9x - 54}{x^2 + 7x + 10}$$

$$= \frac{3(x^2 + 3x - 18)}{x^2 + 7x + 10}$$

$$f(x) = \frac{3(x+6)(x-3)}{(x+5)(x+2)}$$

$f(x)$ has vertical asymptotes of $x = -5$ and $x = -2$.

$$f(x) = \frac{3x^2 + 9x - 54}{x^2 + 7x + 10}$$

$$= \frac{3x^2\left(1 + \frac{3}{x} - \frac{18}{x^2}\right)}{x^2\left(1 + \frac{7}{x} + \frac{10}{x^2}\right)}$$

$$f(x) = \frac{3\left(1 + \frac{3}{x} - \frac{18}{x^2}\right)}{\left(1 + \frac{7}{x} + \frac{10}{x^2}\right)}$$

As $x$ tends towards $+\infty$ and $-\infty$ $f(x)$ tends towards $y = 3$.
$f(x)$ has a horizontal asymptote of $y = 3$.

PTS: 1 REF: Thinking OBJ: 4.3 - Vertical and Horizontal Asymptotes

59. ANS:

a. $f(x) = (x-1)^{\frac{2}{3}}(x+3)$

$$f'(x) = \frac{2}{3}(x-1)^{\frac{-1}{3}}(x+3) + (x-1)^{\frac{2}{3}}$$

$$= (x-1)^{\frac{-1}{3}}\left(\left(\frac{2}{3}x + 2\right) + (x-1)\right)$$

$$f'(x) = (x-1)^{\frac{-1}{3}}\left(\frac{5}{3}x + 1\right)$$

$$0 = f'(x)$$

$$0 = (x-1)^{\frac{-1}{3}}\left(\frac{5}{3}x + 1\right)$$

$$\frac{5}{3}x = -1$$

$$x = -\frac{3}{5}$$

$x = 1$ is in the domain of $f(x)$ so $x = 1$ is also a critical number.

The critical numbers are $x = 1$ and $x = -\frac{3}{5}$.

b. $f'(x) = (x-1)^{\frac{-1}{3}}\left(\frac{5}{3}x + 1\right)$

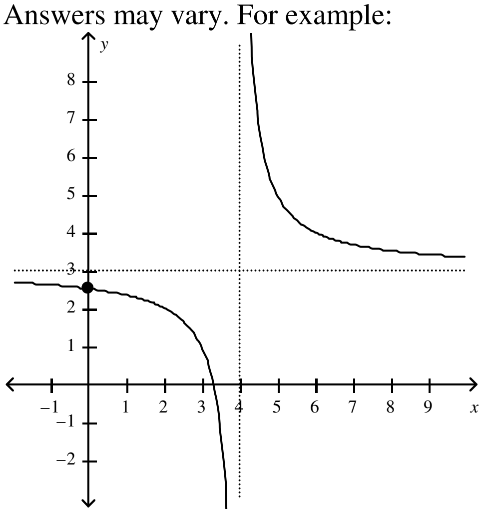
| Value of $x$ | $x < -\frac{3}{5}$ | $-\frac{3}{5} < x < 1$ | $x > 1$ |
|---|---|---|---|
| Sign of $f'(x)$ | positive | negative | positive |

$f(x)$ has a local maximum at $x = -\frac{3}{5}$.

$f(x)$ has a local minimum at $x = 1$.

PTS: 1 REF: Knowledge and Understanding
OBJ: 4.2 - Critical Points, Local Maxima, and Local Minima

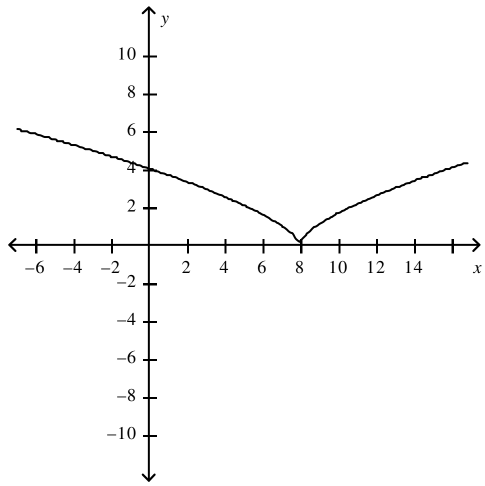
60. ANS:
Answers may vary. For example:




$f(x) = \frac{3x - 10}{x - 4}$

PTS: 1 REF: Application OBJ: 4.5 - An Algorithm for Curve Sketching

61. ANS:




PTS: 1 REF: Application OBJ: 4.5 - An Algorithm for Curve Sketching

62. ANS:

The volume of the inscribed cylinder, as a function of height $h$, is $V(h) = \pi \cdot h \cdot (4 - \frac{h}{2})^2$.

To determine the maximum volume, we take the derivative of $V(h)$ and determine when it equals zero.

$V'(h) = 0 = \pi \cdot [16 - 8h + \frac{3h^2}{4}]$

$h = 8, 2.67$

Because the height of the cone is 8, $0 < h < 8$, therefore $h \neq 8$.

Hence the solution is when $h \doteq 2.67$ and $r \doteq 2.67$.

PTS: 1 REF: Thinking OBJ: 3.3 - Optimization Problems

63. ANS:

From the given information, we can write two equations.

$x + y = 18$

$f(x) = x \cdot y^2 = \max$

Since we want to maximize $f(x)$, we must determine where $f'(x) = 0$.

Using the first equation, we can solve for $y$. Hence $y = 18 - x$. Substituting this into $f(x)$ we determine that $f(x) = x(18 - x)^2$.

$f'(x) = (18 - x)^2 - 2x(18 - x) = 0$

$18 - x = 2x$

$3x = 18$

$x = 6$

$y = 18 - 6 = 12$

To determine if (6, 12) is an absolute maximum we can examine $f'(x)$.

We know $0 < x < 18$ and $x = 6$ is a critical value.

$f'(x) > 0$ when $0 < x < 6$ and $f'(x) < 0$ when $6 < x < 18$, $x = 6$ is a maximum function value. Therefore when $x = 6$ and $y = 12$, the largest possible product occurs.

PTS: 1 REF: Communication OBJ: 3.3 - Optimization Problems

64. ANS:

The area of the triangle is $A(x) = \frac{1}{2}bh$. In this case $b = 2x$ and $h = y$. By using the given equation, we can solve for $y$ in terms of $x$.

$6y = 48 - x^2$

$y = 8 - \frac{x^2}{6}$

Then we can substitute this into $A(x)$ to get $A(x) = \frac{1}{2}(2x)(8 - \frac{x^2}{6})$ which simplifies to $A(x) = 8x - \frac{x^3}{6}$.

Since we want to maximize $A(x)$, we must determine where $A'(x) = 0$.

$A'(x) = 0 = 8 - \frac{x^2}{2}$

$\frac{x^2}{2} = 8$

$x^2 = 16$

$x = 4$

Since $0 < x < 4\sqrt{3}$ and $x = 4$ is a critical point, we must show it is where the maximum area occurs. Hence $f'(x) > 0$ when $0 < x < 4$ and $f'(x) < 0$ when $4 < x < 4\sqrt{3}$, therefore at $x = 4$ the largest possible area occurs.

By evaluating $A(4)$ we determine the maximum area to be $\frac{64}{3}$ square units.

PTS: 1 REF: Communication OBJ: 3.3 - Optimization Problems

65. ANS:

Let *L* represent the length of the ladder, *x* represent the distance the ladder is from the building, and *y* be the height of the building.

$\frac{y}{x} = \frac{2}{x-1}$

$y = \frac{2x}{x-1}$

Using the distance formula we can write the length of the ladder as:

$L = \sqrt{x^2 + y^2}$

Hence by substituting for *y* we get

$L = \sqrt{x^2 + \left(\frac{2x}{x-1}\right)^2}$

$L = \sqrt{x^2 + \frac{4x^2}{(x-1)^2}}$

Since we want to minimize *L*, we must differentiate.

$$L' = \frac{1}{2}\left(x^2 + \frac{4x^2}{(x-1)^2}\right)^{\frac{-1}{2}}\left(2x + \frac{8x(x-1)^2 - 8x^2(x-1)}{(x-1)^4}\right)$$

$$L' = \frac{2x - \frac{8x}{(x-1)^3}}{2\sqrt{x^2 + \frac{4x}{(x-1)^2}}}$$

$$L' = 0 = 2x\left(1 - \frac{4}{(x-1)^3}\right)$$

$$\frac{4}{(x-1)^3} = 1$$

$$(x-1)^3 = 4$$

$x \doteq 2.587$ m
$y \doteq 3.260$ m
$L \doteq 4.16$ m

PTS: 1 REF: Application OBJ: 3.3 - Optimization Problems

66. ANS:
Using the given information, we can write two equations.
$70 = lwh$
$C(x) = 9(2lw) + 6(2lh + 2wh)$
Since the base length is twice the base width, $l = 2w$, we can rewrite $C(x)$.
$C(x) = 36w^2 + 36wh$
Using the first equation, we can solve for *h* in terms of *w*.
$h = \frac{70}{lw} = \frac{70}{2w^2}$

Then using that fact, $C(x) = 36w^2 + \frac{1260}{w}$.
To determine the critical values, we will calculate $C'(x)$ and solve for $C'(x) = 0$.
$C'(x) = 72w - \frac{1260}{w^2}$
$0 = 72w - \frac{1260}{w^2}$
There are critical values when $w \doteq 2.596$. To show this is the minimum we will show that $C''(w) > 0$ then the cost function is always concave up, which means $w \doteq 2.596$ is an absolute minimum.
$C''(w) = 72 + \frac{2520}{w^3}$
$C''(2.596) = 72 + 72 = 144 > 0$
Hence $w \doteq 2.596$ m, $l \doteq 5.192$ m, $h \doteq 5.194$ m.
The minimum cost is $728.02.

PTS: 1 REF: Application OBJ: 3.4 - Optimization Problems in Economics and Science

67. ANS:

a)

- Variables Volume: $V$, radius: $r$ and time: $t$
- We want $dV/dt$ - rate of change of volume with respect to time.
- We know $dr/dt = -0.2$ - the rate of change of radius with respect to time (it is negative since the radius is decreasing).
- Equations relating variables: $V = 4\pi r^3/3$ (volume of a sphere in terms of radius).
- Solving the problem: We want $dV/dt$, so we need to differentiate both sides with respect to $t$. Differentiating, we get

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}.$$

Plugging in $r = 15$ and $dr/dt = -0.2$, we get $dv/dt = 4\pi(15)^2(-0.2) \sim -565.5 cm^3/hr$.

b)

This looks like a related rates problem, but can in fact be solved without using related rates. Observe that $dV/dt$ measures the rate of change of volume. Since the rate of change of volume will be equal to the difference between what is flowing into the tank and what is flowing out of the tank, we have

$$\frac{dV}{dt} = 0.1 - 0.001h^2$$

where $h$ is the height of the water in the tank. Notice that when $h = 10$ (so it is at the top of the tanks), we have $dV/dt = 0$, meaning there is no change in the height of the water. This means that when the water reaches the top of the tank, the amount of water will neither increase or decrease, but stay steady, so it follows that the tank will never overflow.

c)

$\frac{1500}{29} \frac{rad}{hr}$ (51.72)

d)

$\frac{3}{20\pi} \frac{cm}{min}$ (.048)

PTS: 1