 12CV Date:____________ Name: ________________________

Application of Derivatives Unit - Notes

Tentative TEST date_______________

Big idea/Learning Goals

We live in a world that is always in flux. Sir Isaac Newton's name for calculus was "the method of fluxions." He recognized in the seventeenth century, as you probably recognize today, that understanding change is important. Newton was what we might call a "mathematical physicist." He developed calculus to gain a better understanding of the natural world, including motion and gravity. But change is not limited to the natural world, and, since Newton's time, the use of calculus has spread to include applications in the social sciences. Psychology, business, and economics are just a few of the areas in which calculus continues to be an effective problem-solving tool. As we shall see in this unit, anywhere functions can be used as models, the derivative is certain to be meaningful and useful.

Corrections for the textbook answers:

Success Criteria

- ☐ I understand the new topics for this unit if I can do the practice questions in the textbook/handouts

Date	pg	Topics	# of quest. done? You may be asked to show them
	2-4	Velocity & Acceleration 3.1	
	5-6	Extreme Values 3.2	
	7-13	Optimization Problems – 2 days 3.3 & 3.4	
		Review	

Reflect – previous TEST mark ___________, Overall mark now___________.

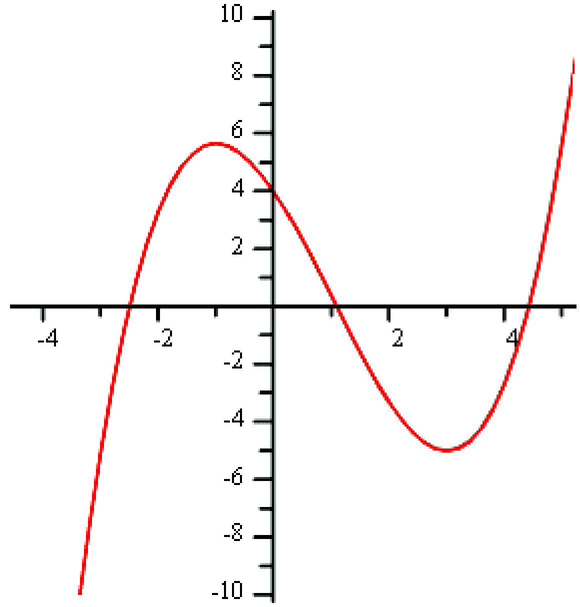
Velocity & Acceleration

Higher Order Derivatives

Find the second derivative

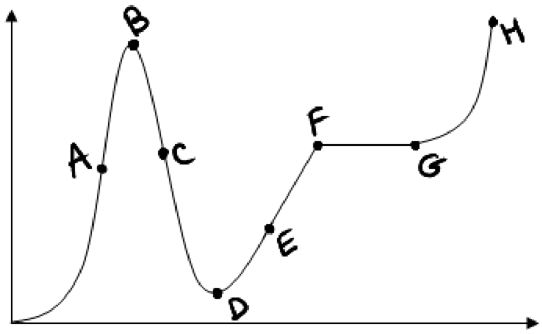
1. $y = \frac{1}{3}x^3 - x^2 - 3x + 4$

2. Graph the first and second derivatives



3. Explain what each graph tells you if the original function represented displacement. Also talk about slopes and concavity

4. The graph shows the position function of a motorcycle.
   Describe the displacement (away, towards) for each interval, the slope of the graph (pos, neg, zero, inc, dec), the velocity (speed up, slow down, constant), and acceleration (pos, neg, zero)



Interval	Displ.	Slope	Velocity	Accel
0 to A				
A to B				
B to C				
C to D				
D to E				
E to F				
F to G				
G to H				

5. Summarize key information:

6. A construction worker accidently drops a hammer from a height of 90m while working on the roof of a new apartment building. The height of the hammer, *s*, in meters, after *t* seconds is modelled by the function $s(t) = 90 - 4.9t^2, t \geq 0$
   a. Determine the average velocity of the hammer between 1s and 4s.
   b. Determine the velocity of the hammer at 1s and 4s.
   c. When will the hammer hit the ground?
   d. Determine the impact velocity of the hammer.
   e. Determine the acceleration function. What do you notice?

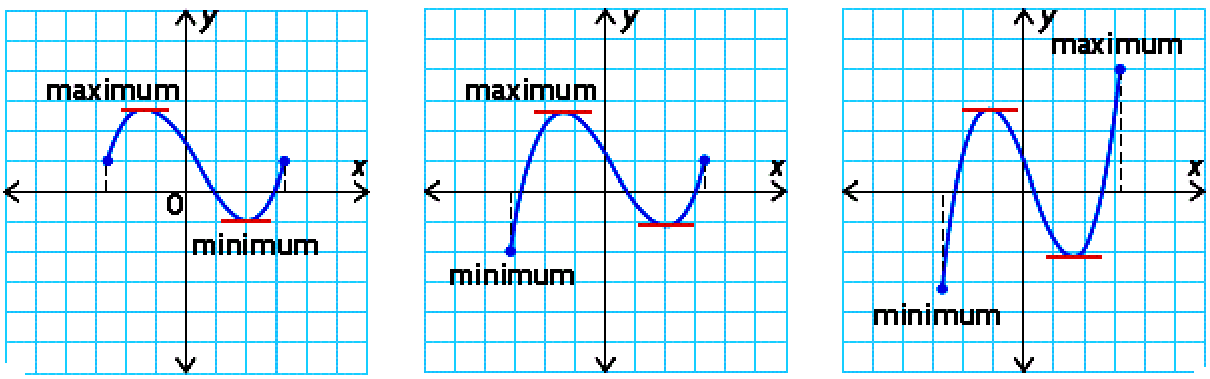
7. The position of a particle moving along a straight line is represented by the function $s(t) = t^3 - 12t^2 + 36t$, where distance, *s*, is in metres, time, *t*, is in seconds, and $t \geq 0$
   a. At what time(s) is the object at rest?
   b. When is the object moving in a positive direction?
   c. When does the object return to its original postion?
   d. Graph separately the position-time graph, the velocity-time graph, and acceleration-time graph
   e. Describe the position of the particle using information from above.

Extreme Values

critical numbers

local max & min

(absolute) max & min



1. Find the absolute maximum and minimum of the function $f(x) = x^3 - 12x - 3$ on the interval $-3 \le t \le 4$

2. A section of roller coaster is in the shape of $f(x) = -x^3 - 2x^2 + x + 15$ where $-2 \le x \le 2$
   a. Find all local extreme values.
   b. Is the highest point of this section of the ride at the beginning, the end, or neither?

3. The surface area of a cylindrical container is to be 100 cm^2. Its volume is given by the function $V(r) = 50r - \pi r^3$, where *r* represents the radius, in centimetres, of the cylinder. Find the maximum volume of the cylinder in each case.
   a. The radius cannot exceed 3 cm.
   b. The radius cannot exceed 2 cm.

4. In a certain manufacturing process the unit cost is $U(x) = \dfrac{6000 + 9x + 0.05x^2}{x}$, where *x* is the number of units sold. What level of production, *x*, minimizes the unit cost, if $1 \le x \le 400$?

Optimization Problems

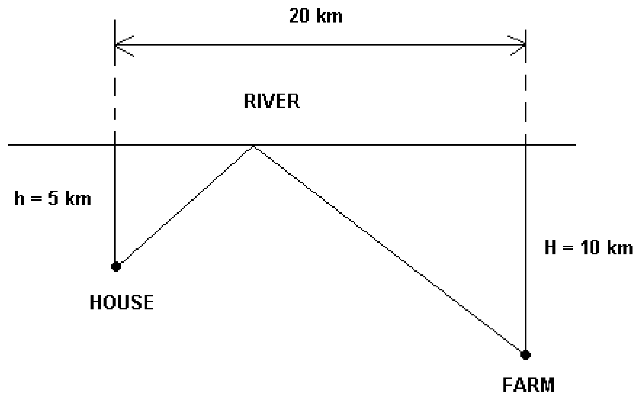
1. A 400m track is to be constructed of two straight-ways and two semicircular ends. The straight-ways can be no less than 100m long. What radius would produce the maximum area?

Steps

2. A cardboard box with a square base is to have a volume of 8 L. (1 L = 1000 cm^3)
    a. Find the dimensions of the box that will minimize the amount of cardboard used.
    b. The cardboard for the box costs 0.1¢/cm^2, but the cardboard for the bottom is thicker, so it costs three times as much. Find the dimensions of the box that will minimize the cost.

3. A 60cm by 40cm piece of tin has squares cut out of each corner, then the sides are folded up to form an open-top box. Find the dimensions that will maximize the volume of this box.

4. The diagram below shows the path that Wilson follows every morning to take water from the river to his farm. Help Wilson minimize the total distance travelled from his house to the farm.



More Optimization

1. A company sells 1500 movie DVDs per month at $10 each. Market research has shown that sales will decrease by 125 DVDs per month for each $0.25 increase in price.
   a. Determine the price for maximum revenue. Be sure to state what your variables represent or next question may be done wrong!
   b. The actual cost of producing *x* number of DVDs is $C(x) = -0.004x^2 + 9.2x + 5000$. Determine the price that would maximize the profit.

2. Andrew and David both leave their houses at 7 A.M. for their Sunday run. Andrew's house is 2.2km south of David's house. Andrew runs north at 9km/h, while David runs west at 7km/h.
   a. Determine the rate of change of the distance between the two runners after 1 hour.
   b. Determine the minimum distance between the two joggers on the domain of 0 hrs to 2 hrs assuming Andrew continues to run north.
   c. If Andrew turned at David's house and ran west to catch up, write a piecewise function to represent the distance travelled and then find when and where the two runners would meet.

3. A rectangular pen is to be built with 1200m of fencing. The pen is to be divided into three parts using two parallel partitions.
    a. Find the maximum possible area of the pen.
    b. How does the maximum area change if each side of the pen had to be at least 200m long.

4. A cylindrical drum with an open top is to be constructed using $1m^2$ of aluminum.
    a. Find the radius that gives maximum volume.
    b. What is the maximum volume if the radius can be a maximum of 0.2m?