p1NOTES

April-22-13
6:26 PM

☐ reprint blanks - unit 8 now

☐ insert note + question about extreme pts or open intervals

☐ p. 8 # 2(b) separate page

☐ p. 11 # 1(b) separate page

☐ p. 12 # 2(a) remove – no space

AppDerivNotesNEW

Inserted from: <file://C:\Users\MrsK\Desktop\LacieOct9\2 Math\Math 12\MCB 4U Calc Vect\2013\7 App of Deriv\AppDerivNotesNEW.doc>

Application of Derivatives Unit - Notes

Tentative TEST date TUE May 28

Big idea/Learning Goals

We live in a world that is always in flux. Sir Isaac Newton's name for calculus was "the method of fluxions." He recognized in the seventeenth century, as you probably recognize today, that understanding change is important. Newton was what we might call a "mathematical physicist." He developed calculus to gain a better understanding of the natural world, including motion and gravity. But change is not limited to the natural world, and, since Newton's time, the use of calculus has spread to include applications in the social sciences. Psychology, business, and economics are just a few of the areas in which calculus continues to be an effective problem-solving tool. As we shall see in this unit, anywhere functions can be used as models, the derivative is certain to be meaningful and useful.

Corrections for the textbook answers:

Success Criteria

☐ I understand the new topics for this unit if I can do the practice questions in the textbook/handouts

Date	pg	Topics	# of quest. done? You may be asked to show them
16th assemb May 15	2-4	Velocity & Acceleration 3.1	
May 17	5-6	Extreme Values 3.2	
21st TEST → May 22-23	7-13	Optimization Problems – 2 days 3.3 & 3.4	
May 24		Review + Journals DUE	

Mon. May 27 start new unit

Reflect – previous TEST mark __________, Overall mark now__________.

TUE May 28 TEST part 1

WED May 29 TEST part 2 (TIPS part)

1

Velocity & Acceleration

Higher Order Derivatives

1st derivative notation $f'(x)=\frac{df}{dx}$ 2nd deriv: $f''(x)=\frac{d}{dx}\frac{d}{dx}f = \frac{d^2f}{dx^2}$

Find the second derivative

1. $y=\frac{1}{3}x^3-x^2-3x+4$

$y'=x^2-2x-3$ ← factor to see zeros

$(x-3)(x+1)$

x-int at 3 and -1

vertex $(1,-4)$

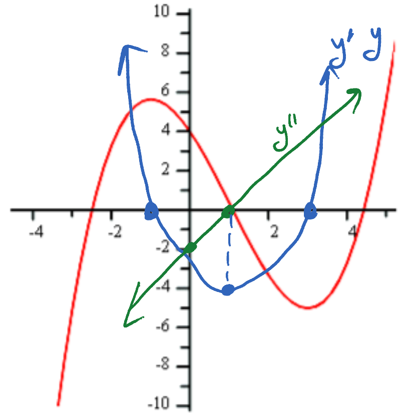
$\frac{3+-1}{2}$

$y''=2x-2$

y-int -2

x-int 1

2. Graph the first and second derivatives



(x-values) inflection pt. of f → is t.p. of f'

turning pt. of f → is the zero of f'

(y) - y-value represents position relative to origin
- slope tells you speed
- concavity tells accelation

(y') - y-value tells you speed
- slope tells you accelation

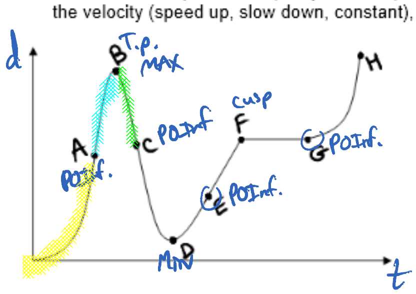
(y'') - y-value tells you accelation

3. Explain what each graph tells you if the original function represented displacement. Also talk about slopes and concavity

distance with direction

- at X=-3 start at negative side of origin and slowing down from a fast speed past the origin
- at x=-1 turn around and speeding up until the origin (p.o.Inf) the slowing down, still in the same direction
- at x=3 turn around ...

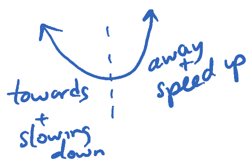
4. The graph shows the position function of a motorcycle. d
Describe the displacement (away, towards) for each interval, the slope of the graph (pos, neg, zero, inc, dec), the velocity (speed up, slow down, constant), and acceleration (pos, neg, zero)



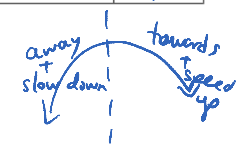
Interval	Displ.	Slope	Velocity	Accel
0 to A	away	+ inc	speed up	+ CU
A to B	away	+ dec	slow down	− CD
B to C	toward	− inc in (mag.)	speed up	− CD
C to D	toward	−		+ CU
D to E	away	+		+ CU
E to F	away	+		∅
F to G	still	∅		∅
G to H	away	+		+ CU

5. Summarize key information:

pos. acceleration CU



neg. accel CD



★ direction is positive is when speed is pos.

6. A construction worker accidently drops a hammer from a height of 90m while working on the roof of a new apartment building. The height of the hammer, s, in meters, after t seconds is modelled by the function $s(t)=90-4.9t^2, t\geq 0$

a. Determine the average velocity of the hammer between 1s and 4s.
b. Determine the velocity of the hammer at 1s and 4s.
c. When will the hammer hit the ground?
d. Determine the impact velocity of the hammer.
e. Determine the acceleration function. What do you notice?

$s'(t) = -9.8t$

ⓐ aroc of $s(t) = \frac{s(4)-s(1)}{4-1} = \frac{11.6-85.1}{3} \doteq -24.5$ m/s

going down

ⓑ iroc or deriv. $s'(1) = -9.8$ m/s
$s'(4) = -39.2$ m/s

ⓒ $s=0$ $\quad 0 = 90-4.9t^2$
$4.9t^2 = 90$
$\sqrt{t^2} = \sqrt{\frac{90}{4.9}}$
$t \doteq 4.29$ sec

impact is when it touched ground

ⓓ $s'(4.29) \doteq -42$ m/s
$s' =$ velocity.

ⓔ accel $= s'' = -9.8$ m/s²
this is acceleration due to gravity.

7. The position of a particle moving along a straight line is represented by the function $s(t) = t^3 - 12t^2 + 36t$, where distance, s, is in metres, time, t, is in seconds, and $t \geq 0$

a. At what time(s) is the object at rest?
b. When is the object moving in a positive direction?
c. When does the object return to its original postion?
d. Graph separately the position-time graph, the velocity-time graph, and acceleration-time graph
e. Describe the position of the particle using information from above.

(a) at rest vel = 0 $\quad s'(t) = 3t^2 - 24t + 36$

$0 = 3(t^2 - 8t + 12)$

$0 = 3(t-6)(t-3)$

$\therefore$ at $t=3$ and $t=6$ sec object is at rest

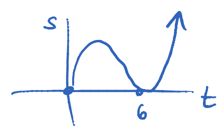
start at zero!! domain

(b) pos. dir. is when speed is pos

	$-\infty$ / 0	3	6	∞
3	+	+	+	
$t-6$	−	−	+	
$t-3$	−	+	+	
v'	+	−	+	

$\therefore$ moving in pos. direction for $t \in (0,3)$ and $(6,\infty)$

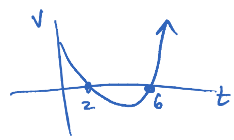
(d)

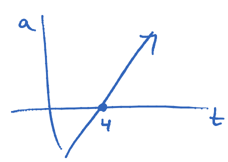


(c) origin at $s=0$ $\quad 0 = t(t^2 - 12t + 36)$

$0 = t(t-6)(t-6)$

$\therefore$ $t=0$ or $t=6$ sec it's at the origin.





(e) $t=0$ going away and slowing down

$t=2$ turning back + speeding up till $t=4$

continue going same direction but slowing down

$t=6$ at the origin + turn back around and speed away

Extreme Values on closed interval

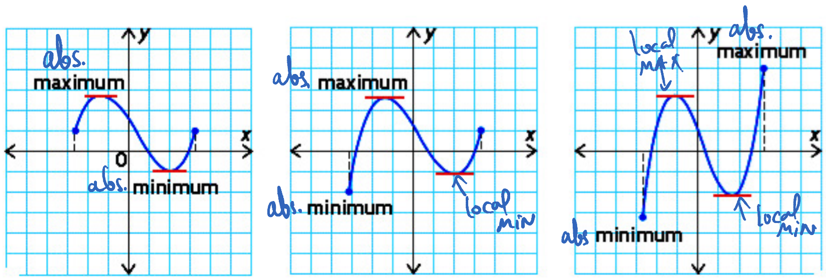
MAX/MIN

critical numbers when $f'(x) = 0$ or $f'(x) = \text{undefined}$

MAX/MIN is this one

local max & min
relative
→ at turning pts only

(absolute) max & min → at turning pts OR at endpts of interval.



1. Find the absolute maximum and minimum of the function $f(x) = x^3 - 12x - 3$ on the interval $-3 \le t \le 4$

Eg.

$f'(x) = 3x^2 - 12$

$0 = 3(x^2 - 4)$

$0 = 3(x+2)(x-2)$

$\therefore$ crit. pts. are $x = -2, x = 2$

are they in interval?

check y-values of endpts + crit. pts.

$f(-3) = 6$

$f(-2) = 13$

$f(2) = -19$

$f(4) = 13$

compare

$\therefore$ at $(2, -19)$ abs. MIN

at $(-2, 13)$ and $(4, 13)$ abs MAX.

2. A section of roller coaster is in the shape of $f(x) = -x^3 - 2x^2 + x + 15$ where $-2 \le x \le 2$
   a. Find all local extreme values.
   b. Is the highest point of this section of the ride at the beginning, the end, or neither?

abs. MAX

a) $f'(x) = -3x^2 - 4x + 1$

$0 = -3x^2 - 4x + 1$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{\cancel{4}^{\,2} \pm \sqrt{28}}{-\cancel{6}_{\,3}}$ $\quad \sqrt{4}\sqrt{7}$

$x = \dfrac{2 + \sqrt{7}}{-3}$ or $x = \dfrac{2 - \sqrt{7}}{-3}$

$x \doteq -1.55$ $\quad x \doteq 0.22$

b) $f(-2) = 13$

$f\left(\dfrac{2+\sqrt{7}}{-3}\right) \doteq 12.4$

$f\left(\dfrac{2-\sqrt{7}}{-3}\right) \doteq 14.8$

$f(2) = 1$

compare

$\therefore$ abs. MAX or highest point is at $x \doteq 0.22$

(neither beginning nor end)

3. The surface area of a cylindrical container is to be 100 cm^2. Its volume is given by the function $V(r) = 50r - \pi r^3$, where r represents the radius, in centimetres, of the cylinder. Find the maximum volume of the cylinder in each case.
   a. The radius cannot exceed 3 cm. $r \in [0, 3]$
   b. The radius cannot exceed 2 cm. $r \in [0, 2]$

$V'(r) = 50 - 3\pi r^2$

$0 = 50 - 3\pi r^2$

$0 = (\sqrt{50} - \sqrt{3\pi}r)(\sqrt{50} + \sqrt{3\pi}x)$

$3\pi r^2 = 50$

$r^2 = \frac{50}{3\pi}$

$r = \pm\sqrt{\frac{50}{3\pi}} \sim 2.3$

Better to keep on one side (may lose some crit. pts otherwise!)

ignore the neg. since not in domain/interval

@ $V(0) \sim \lim_{r \to 0} 50r - \pi r^3 = 0$

$V\left(\sqrt{\frac{50}{3\pi}}\right) \doteq 76.78$

$V(3) = 65.18$

$\therefore$ Max volume is 76.78 cm^3

b) $V(0) \sim 0$

$V(2) \doteq 74.87$

$\therefore$ Max volume is 74.87 cm^3 at $r = 2$

4. In a certain manufacturing process the unit cost is $U(x) = \dfrac{6000 + 9x + 0.05x^2}{x}$, where x is the number of units sold. What level of production, x, minimizes the unit cost, if $1 \le x \le 400$?

$U(x) = 6000x^{-1} + 9 + 0.05x$

$U'(x) = -6000x^{-2} + 0.05$

$0 = -\frac{6000}{x^2} + 0.05$

$0 = \frac{-6000 + 0.05x^2}{x^2}$

$0 = \frac{0.05(x^2 - 120\,000)}{x^2}$

$0 = \frac{0.05(x - \sqrt{120\,000})(x + \sqrt{120\,000})}{x^2}$

crit. pts. $x = 0$ and $x = \pm\sqrt{120\,000} \sim 346.4$

$x = 0$ and $x = -\sqrt{\ }$ is not in the interval

$U(1) = 6009.05$

$U(\sqrt{120\,000}) \doteq 43.64$

$U(400) \doteq 44$

$\therefore$ absolute min at $x = 346$ units of production

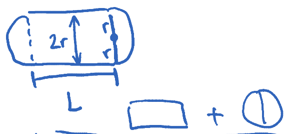
Optimization Problems

Eg.

1. A 400m track is to be constructed of two straight-ways and two semicircular ends. The straight-ways can be no less than 100m long. What radius would produce the maximum area?

Steps
1. Find equation to optimize MAX/MIN
   - only 2 variables
   - use diagrams to help you
2. Find domain (in terms of independent variable)
3. Find crit. pts. in domain
4. Show it is a MAX/MIN
   - use f on closed/open interval
   - use f' 1st deriv. test
   - use f'' 2nd deriv test
5. Answer question.

$L \geq 100$



Maximize $\boxed{A = 2rL + \pi r^2}$

need 2 variables

$400 = 2L + 2\pi r$

$400 - 2\pi r = 2L$

$200 - \pi r = L$

sub in

$A = 2r(200 - \pi r) + \pi r^2$

$A = 400r - 2\pi r^2 + \pi r^2$

eqtn: $A(r) = A = 400r - \pi r^2$

domain: $L \geq 100$

$200 - \pi r \geq 100$

$100 \geq \pi r$

$\frac{100}{\pi} \geq r$

$\therefore$ domain $r \in \left(0, \frac{100}{\pi}\right]$

crit. pts: $A'(r) = \frac{dA}{dr} = A' = 400 - 2\pi r$

those that create MAX

$f' = 0$

$0 = 400 - 2\pi r$

$2\pi r = 400$

$r = \frac{200}{\pi} \doteq 64$ not in domain.

will not use it.

So maximum must happen at the endpt.

$r = \frac{100}{\pi}$ not ϕ

show MAX:

$A(0) \to \lim_{r \to 0} 400r - \pi r^2 = 0$

$A\left(\frac{100}{\pi}\right) \doteq 9554.1 \text{ m}^2$

compare to choose abs. MAX

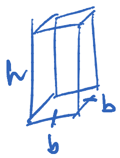
answer question:

The radius will be approx. 31.8 m and length of straight edge

$L = 200 - \pi r$

$L = 100 \text{ m}$

2. A cardboard box with a square base is to have a volume of 8 L. (1 L = 1000 cm^3)
   a. Find the dimensions of the box that will minimize the amount of cardboard used. top + bottom
   b. The cardboard for the box costs 0.1¢/cm^2, but the cardboard for the bottom is thicker, so it costs three times as much. Find the dimensions of the box that will minimize the cost.



Minimize Surface Area = $\boxed{A = \underbrace{2b^2}_{\text{top + bottom}} + \underbrace{4bh}_{\text{sides}}}$

use $V = b^2h$ to eliminate one variable

$8000 = b^2h$

$\frac{8000}{b^2} = h$

sub in

$A = 2b^2 + 4b\left(\frac{8000}{b^2}\right)$

equation: $A = 2b^2 + \frac{32000}{b}$

domain: $b > 0$, $h > 0$

$\frac{8000}{b^2} > 0$

can't solve
always true
no new restrictions

$\therefore$ domain $b \in (0, \infty)$

crit. pts. $A' = 4b - \frac{32000}{b^2}$

$0 = \frac{4b^3 - 32000}{b^2}$

$0 = \frac{4(b^3 - 8000)}{b^2}$

$0 = \frac{4(b-20)(b^2+20b+400)}{b^2}$

never factor

* don't isolate for b (don't move terms) just LCD, factor & lose any so you don't crit. pts.

crit pts $b = 20$ and $\cancel{b = 0}$

not in domain
also will be a cusp or vert. tangent.

show it is MIN:
use 1st deriv. test.

	$-\infty$ … 0	0 … 20	20 … ∞
4	+	+	+
$b-20$	−	−	+
$b^2+20b+400$	+	+	+
b^2	+	+	+
A'	−	−	+
A	↘	↘	↗

at $b = 20$ is a local MIN. AND will be abs. MIN since it's the only crit. pt.

answer question:
$\therefore$ dimensions of box base = 20 cm
height $= \frac{8000}{b^2} = 20$ cm

(b) • Minimize Cost $C = \overset{4}{0.3}(\underbrace{2b^2}_{\text{base}}) + 0.1(\underbrace{4bh}_{\text{sides}})$

$$C = 0.6b^2 + 0.4b\left(\frac{8000}{b^2}\right)$$

• domain $b \in (0, \infty)$ $\quad C = 0.6b^2 + \frac{3200}{b}$

• crit. pts.

$$C' = 1.2b - \frac{3200}{b^2}$$

$$0 = \frac{1.2b^3 - 3200}{b^2}$$

$$0 \doteq \frac{1.2(b^3 - 2666.\bar{6})}{b^2}$$

$$0 \doteq \frac{1.2(b - 13.87)(b^2 + 13.87b + 192.3)}{b^2}$$

$\therefore$ crit. pt $b = 13.87$ and ~~$b = 0$~~ not in domain

• show a min:
use 2nd deriv. test

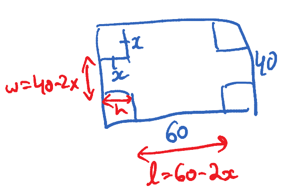
$$C'' = 1.2 + \frac{6400}{b^3}$$

$$C''(13.87) = 1.2 + \frac{6400}{13.87^3}$$

$= \text{pos}$ $\therefore$ CU
$\therefore$ ~~local~~ abs min. since the only crit. pt.

• answer question
$\therefore$ base should be about 13.87 cm
and height $= \frac{8000}{13.87^2}$
$\doteq 41.6$ cm

3. A 60cm by 40cm piece of tin has squares cut out of each corner, then the sides are folded up to form an open-top box. Find the dimensions that will maximize the volume of this box.



Maximize: $\boxed{V = lwh}$

$V = (60-2x)(40-2x)(x)$

$V = (60-2x)(40x-2x^2)$

$V = 2400x - 120x^2 - 80x^2 + 4x^3$

$V = 4x^3 - 200x^2 + 2400x$

domain: $l > 0$ $w > 0$ $h > 0$

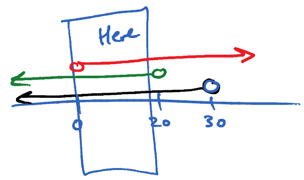
$60-2x > 0$ $40-2x>0$ $x>0$

$-2x > -60$ $40 > 2x$

$x < 30$ $20 > x$

where are all true?

$x \in (0, 20)$



crit. pt: $V' = \frac{dV}{dx} = 12x^2 - 400x + 2400$

$0 = 4(3x^2 - 100x + 600)$

quad formula $x \doteq \cancel{25.5}$ not in domain $x \doteq 7.85$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

this can be saddle pt MIN MAX

show it's a MAX
do 2nd deriv. test. $V'' = 24x - 400$

$V''(7.85) = 24(7.85) - 400$

neg

$\therefore$ CD

$\therefore \frown$ MAX.

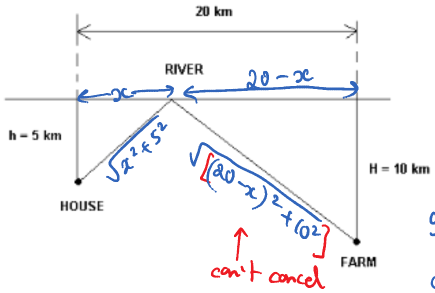
answer the question: the dimensions for MAX volume are $h = x = 7.85$ cm

$l = 60-2x = 44.3$ cm

$w = 40-2x = 24.3$ cm

eg.

4. The diagram below shows the path that Wilson follows every morning to take water from the river to his farm. Help Wilson minimize the total distance travelled from his house to the farm.



$$\boxed{\text{Minimize distance } = \sqrt{x^2+25} + \sqrt{400-40x+x^2+100}}$$

$$D = \sqrt{x^2+25} + \sqrt{x^2-40x+500}$$

domain: $x \in [0, 20]$

crit. pts: $D' = \frac{1}{\cancel{2}}(x^2+25)^{-1/2}(\cancel{2}x) + \frac{1}{\cancel{2}}(x^2-40x+500)^{-1/2}(\cancel{2}x-\cancel{40})$ $(1x-20)$

usually, best not to move terms around (the result is not going to be a derivative but an equivalent equation)

$$0 = \frac{x}{\sqrt{x^2+25}} + \frac{x-20}{\sqrt{x^2-40x+500}}$$

but since it's a complex equation we'll do it remembering that some critical pts (the undefined ones) will be lost AND the equation at the end is NOT same as the derivative so you can't use it for 1st or 2nd deriv test !!

$$\left(\frac{-x}{\sqrt{x^2+25}}\right)^2 = \left(\frac{x-20}{\sqrt{x^2-40x+500}}\right)^2$$

$$\frac{x^2}{x^2+25} = \frac{x^2-40x+400}{x^2-40x+500}$$

$$\cancel{x^4}-\cancel{40x^3}+500x^2 = \cancel{x^4}-\cancel{40x^3}+400x^2+25x^2-1000x+10\,000$$

$$0 = -75x^2-1000x+10\,000$$

$$0 = -25(3x^2+40x-400)$$

$$0 = -25(3x-20)(x+20)$$

$\therefore$ $x = \cancel{-20}$ or $x = \frac{20}{3}$

not in domain

show it is a MIN:

use original function and check endpts + crit. pts outputs and compare

} don't use 1st deriv. test – since we don't have D' factored + simplified for +/– chart

don't use 2nd deriv. test – since D'' too complex

$D(0) \doteq 27.4$

$D\left(\frac{20}{3}\right) \doteq 25$ ← abs. MIN

$D(20) \doteq 30.6$

answer question: the shortest distance will be achieved if Wilson walks towards a point that's 6.6 km horizontally to the right of his house OR at an angle of 52.8°

More Optimization

Eg.

1. A company sells 1500 movie DVDs per month at $10 each. Market research has shown that sales will decrease by 125 DVDs per month for each $0.25 increase in price.
   a. Determine the price for maximum revenue. Be sure to state what your variables represent or next question may be done wrong!
   b. The actual cost of producing x number of DVDs is $C(x) = -0.004x^2 + 9.2x + 5000$. Determine the price that would maximize the profit.

n

sub in

ⓐ Rev = (price)(quantity)

maximize $\boxed{R = (10 + 0.25x)(1500 - 125x)}$

$R = 15000 - 1250x + 375x - 31.25x^2$

$R = -31.25x^2 - 875x + 15000$

let x be # of times inc. price by 25¢

domain: price ≥ 0 quantity ≥ 0

$10 + 0.25x \geq 0$ $1500 - 125x \geq 0$

$10 \geq -0.25x$ $1500 \geq 125x$

$-40 \leq x$ $12 \geq x$

$\therefore$ domain: $x \in [-40, 12]$

crit pt. $R' = -62.5x - 875$

$0 = -62.5x - 875$

$x = -14$

show it's MAX

$R(-40) = 0$

$R(-14) = 21\,125$

$R(12) = 0$

$\therefore$ max at $x = -14$

$\therefore$ price = $6.50

ⓑ Profit = Rev − Cost

$= [-31.25x^2 - 875x + 15000]$

$- [-0.004(1500 - 125x)^2 + 9.2(1500 - 125x) + 5000]$

$P = -31.25x^2 - 875x + 15000$

$- [-0.004(2250000 - 375000x + 15625x^2)$

$+ 13800 - 1150x + 5000]$

$P = -31.25x^2 - 875x + 15000$

$+ 9000 + 1500x - 62.5x^2$

$- 13800 + 1150x - 5000$

Minimize $\boxed{P = 31.25x^2 + 1775x + 5200}$

domain quantity ≥ 0 Cost ≥ 0

$1500 - 125x \geq 0$ $-0.004n^2 + 9.2n + 5000 \geq 0$

$12 \geq x$ $-0.004(n + 453.9)(n - 2753.9) \geq 0$

not linear so can't do it seperately use +/− chart

⊖ −453.9 ⊕ 2753.9 ⊖

n = quantity

$\therefore 0 \leq n \leq 2753.9$

$1500 - 125x \leq 2753.9$

$-10 \leq x$

$\therefore$ domain $x \in [-10, 12]$

crit pt: $P' = 62.50x + 1775$

$0 = 62.50x + 1775$

$\cancel{-28.4} = x$

not in domain

Max. profit $P(-10) = -\$9425$

$P(12) = \$31\,000$

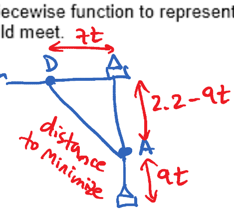
$\therefore$ MAX at $x = 12$

$\therefore$ price = $13

Eg.

2. Andrew and David both leave their houses at 7 A.M. for their Sunday run. Andrew's house is 2.2km south of David's house. Andrew runs north at 9km/h, while David runs west at 7km/h.
   a. Determine the rate of change of the distance between the two runners after 1 hour.
   b. Determine the minimum distance between the two joggers on the domain of 0 hrs to 2 hrs assuming Andrew continues to run north.
   c. If Andrew turned at David's house and ran west to catch up, write a piecewise function to represent the distance travelled and then find when and where the two runners would meet.

let t be time
let d be distance between



$D = VT$
$D = 9t$

$$d = \sqrt{(7t)^2 + (2.2-9t)^2}$$
$$= \sqrt{130t^2 - 39.6t + 4.84}$$

(a) Rate of change is $d'(t)$

$$d' = \frac{1}{2}\left(130t^2 - 39.6t + 4.84\right)^{-1/2}\left(260t - 39.6\right)$$

$$d'(1) = \frac{220.4}{2\sqrt{95.24}} \doteq 11.29 \text{ km/h}$$ is the rate of how distance is changing between them at $t = 1$ hr

(b) domain given $t \in [0, 2]$

min at crit. pt. $d' = 0 = \dfrac{260t - 39.6}{2\sqrt{130t^2 - 39.6t + 4.84}}$ ← doesn't factor

show min — only crit. pt. $t = 0.1523$

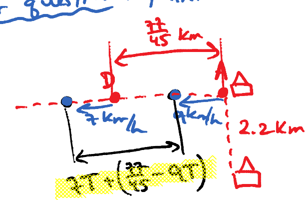
$d(0) = 2.2$
$d(0.1523) = 1.35$
$d(2) = 21.11$

$\therefore$ abs. min at $t = 0.152$ or 9.13 min

answer question: Minimum distance is 1.35km

(c) → turn corner at: $2.2 - 9t = 0$

$$\frac{2.2}{9} = t$$
$$\frac{11}{24} = 0.2\overline{4} = t$$



let T be the new time after Andrew turned corner

$$T = t - \frac{11}{45}$$

→ at this time distance between them is $d = \sqrt{49(0.2\overline{4})^2 - 0^2} = 1.7\overline{1} = \frac{77}{45}$

$\therefore$ piecewise function

$$d(t) = \begin{cases} \sqrt{130t^2 - 39.6t + 4.84} & \text{if } 0 \le t \le 0.2\overline{4} \text{ or } \frac{11}{45} \\ \underbrace{\frac{77}{45} - 2\left(t - \frac{11}{45}\right)}_{\text{this is } \frac{77}{45} - 2T} & \text{if } t > \frac{11}{45} \end{cases}$$

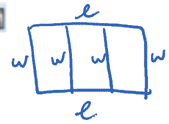
exact is best for equations

$\therefore$ meet if $d(t) = 0$
must happen on 2nd piece

$$\frac{77}{45} - 2t + \frac{22}{45} = 0$$
$$\frac{11}{5} = 2t$$
$$\frac{11}{10} = t$$

$\therefore$ meet 1.1hrs
9.9 km west of Dave's house

3. A rectangular pen is to be built with 1200m of fencing. The pen is to be divided into three parts using two parallel partitions.
   a. Find the maximum possible area of the pen.
   b. How does the maximum area change if each side of the pen had to be at least 200m long.



$1200 = 2l + 4w$ } $\frac{1200}{2} - \frac{4w}{2} = l$

Maximize $\boxed{A = lw}$

$A = (600 - 2w)w$

$A = -2w^2 + 600w$

domain: $l > 0$ $w > 0$

$600 - 2w > 0$

$300 > w$

$\therefore w \in (0, 300)$

crit pt: $A' = -4w + 600$

$0 = -4w + 600$

$150 = w$

show MAX: $A'' = -4$ neg $\therefore$ CD $\therefore$ MAX at crit. pt.

answer quest: max Area $A(150) = 450\,000$

(b) if domain $w \in [200, 300)$ then crit pt. not in $\therefore$ MAX at endpt.

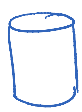
$A(200) = 40\,000$

$A(300) \to \lim_{w \to 300} -2w^2 + 600w = 0$

$\therefore$ max at $w = 200$

Area $= 40\,000$

4. A cylindrical drum with an open top is to be constructed using $1m^2$ of aluminum.
   a. Find the radius that gives maximum volume.
   b. What is the maximum volume if the radius can be a maximum of 0.2m?



Maximize $\boxed{V = \pi r^2 h}$ need only 2 variables use $SA = \underbrace{\pi r^2}_{\text{bottom}} + \underbrace{2\pi r h}_{\text{side}}$

sub in

$\bigcirc + h\,\boxed{}_{2\pi r}$

$1 = \pi r^2 + 2\pi r h$

$\frac{1 - \pi r^2}{2\pi r} = h$

$V = \cancel{\pi} r^{\cancel{2}} \left[\frac{1 - \pi r^2}{2 \cancel{\pi r}} \right]$

$V = \frac{1}{2}(r - \pi r^3)$

domain: $r > 0$ $h > 0$

$\frac{1 - \pi r^2}{2\pi r} > 0$

not linear
use +/− chart

$\frac{(1 + \sqrt{\pi} r)(1 - \sqrt{\pi} r)}{2\pi r} > 0$

$\oplus \quad \ominus \quad \oplus \quad \ominus$

$\frac{-1}{\sqrt{\pi}} \quad 0 \quad \frac{1}{\sqrt{\pi}}$

$\therefore$ domain $r \in (0, \frac{1}{\sqrt{\pi}})$

crit pt: $V' = \frac{1}{2}(1 - 3\pi r^2)$

$0 = \frac{1}{2}(1 + \sqrt{3\pi} r)(1 - \sqrt{3\pi} r)$

$\therefore$ ~~$r = -\frac{1}{\sqrt{3\pi}}$~~ or $r = \frac{1}{\sqrt{3\pi}}$ ~0.33

not in domain

show MAX $V(0) \to 0$

$V(\frac{1}{\sqrt{3\pi}}) = 0.109$

$V(\frac{1}{\sqrt{\pi}}) \to 0$

answer quest: radius of 0.33 m will give MAX volume.

(b) if domain is $r \in [0.2, \frac{1}{\sqrt{\pi}})$ then MAX will be at endpt. $r = 0.2m$ when compared

0.56

$V(0.2) = 0.0874$

$V(\frac{1}{\sqrt{\pi}}) \to 0$

abs. MAX/MIN

Extreme values on open interval

Extreme Values on Open Intervals

ex. Find extrema of $f(x) = \frac{1}{x} + 2x$ on $x \in (0, \infty)$

$f'(x) = \frac{-1}{x^2} + 2$

$0 = \frac{-1 + 2x^2}{x^2} = \frac{(2x^2 - 1)}{x^2}$

$0 = \frac{(\sqrt{2}x + 1)(\sqrt{2}x - 1)}{x^2}$

crit. pts. $x = 0$, $\frac{1}{\sqrt{2}}$, $\frac{-1}{\sqrt{2}}$

only this inside interval

$f(0) \rightsquigarrow$ do with a limit: $\lim_{x \to 0} \cancel{\frac{1}{x}}^{\infty} + \cancel{2x}^{0} =$ D.N.E. but do know it's $+\infty$

$f\left(\frac{1}{\sqrt{2}}\right) \doteq 2.8$

$f(\infty) \rightsquigarrow$ do with a limit: $\lim_{x \to \infty} \cancel{\frac{1}{x}}^{0} + \cancel{2x}^{\infty} =$ D.N.E. also $+\infty$

} compare

$\therefore$ no abs. MAX

but there is abs. MIN at $x = \frac{1}{\sqrt{2}}$

ex. Find extrema of $f(x) = \frac{1}{x - 2}$ on $x \in [5, \infty)$

$f'(x) = \frac{-1}{(x-2)^2}(1)$

$0 = \frac{-1}{(x-2)^2}$

cr. pt. $x = 2$

not inside interval

$f(5) = \frac{1}{3}$

$f(\infty) \rightarrow$ do with limit $\lim_{x \to \infty} \frac{1}{x - 2} = 0$

} compare

$\therefore$ abs. MAX at $x = 5$

no abs. MIN since it never reaches $y = 0$

ex. $f(x) = x^2 - 4x$ on $x \in [1, 4)$ Find the extrema

$f'(x) = 2x - 4$

$0 = 2(x - 2)$

crit. pt. $x = 2$

$f(1) = -3$

$f(2) = -4$

$f(4)$ do with limit $\lim_{x \to 4} x^2 - 4x = 0$

} $\therefore$ abs MIN at $x = 2$

no abs MAX since it doesn't reach zero

ex. Draw a function with no abs. MAX but with absolute min on $x \in (-1, 1)$