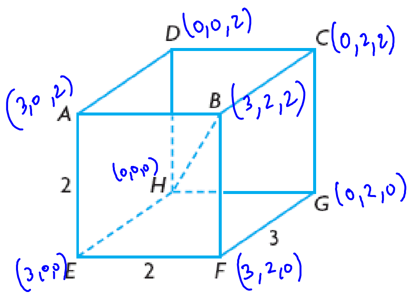
HW p540 9,10,11 p550 5,6

Hw: p. 540 # 5, 9-11 p. 550 #5, 6

11. A rectangular box with an open top, measuring 2 by 2 by 3, is constructed. Its vertices are labelled as shown.

suppose $H = (0,0,0)$ then $B = (3,2,2)$ and $A = (3,0,2)$



$\rightarrow$ in $\mathbb{R}^3$ $\quad \dfrac{|\vec{PQ} \times \vec{m}|}{|\vec{m}|}$

a. Determine the distance from A to the line segment HB.

b. What other vertices on the box will give the same distance to HB as the distance you found in part a.?

c. Determine the area of the $\triangle AHB$.

ⓐ $\vec{m} = \vec{HB} = (3,2,2)$

$\vec{PQ} = \vec{BA}$ or $\vec{HA} = (3,0,2)$

$$\begin{matrix} 3 & 0 & 2 & 3 & 0 & 2 \\ 3 & 2 & 2 & 3 & 2 & 2 \end{matrix}$$

$(0-4, 6-6, 6-0)$

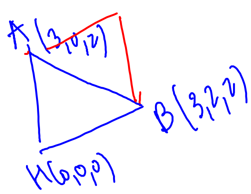
$(-4, 0, 6)$

$$\text{distance} = \frac{\sqrt{4^2+0^2+6^2}}{\sqrt{3^2+2^2+2^2}} = \sqrt{\frac{52}{17}} = \frac{\sqrt{884}}{17} = \frac{2\sqrt{221}}{17}$$

~ 1.75

ⓑ D and G

ⓒ



Area of $\Delta = \frac{1}{2}$ Area of $\parallel$gm $\vec{HB}$ and $\vec{HA}$

$= \frac{1}{2} |\vec{HB} \times \vec{HA}| \leftarrow$ did

$= \frac{1}{2}\sqrt{4^2+0^2+6^2}$

$= \frac{\sqrt{52}}{2} = \sqrt{13} \sim 3.6 \text{ units}^2$

10. The point $A(2, 4, -5)$ is reflected in the line with equation $\vec{r} = (0, 0, 1) + s(4, 2, 1)$, $s \in \mathbf{R}$, to give the point A'. Determine the coordinates of A'.

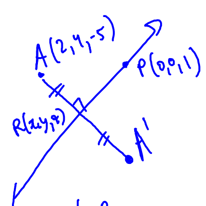


image on other side

and $\vec{AR} \perp$ to $(4,2,1)$

so $(4,2,1) \cdot (x-2, y-4, z+5) = 0$

$4x - 8 + 2y - 8 + z + 5 = 0$

$4x + 2y + z - 11 = 0$

but pt. R (x,y,z) on line:

pt. R

$x = 40/21$

$y = 20/21$

$y = \frac{20}{21}$

$z = \frac{31}{21}$

$\therefore$ dir $\overrightarrow{AR}$ $\left(\frac{-2}{21}, \frac{-64}{21}, \frac{136}{21}\right)$

$\overrightarrow{AR}$ same vector as $\overrightarrow{RA'}$

$\therefore$ pt. $A' =$ pt. $R + \overrightarrow{RA'}$

$= \left(\frac{40}{21} - \frac{2}{21}, \frac{20}{21} - \frac{64}{21}, \frac{31}{21} + \frac{136}{21}\right)$

$= \left(\frac{38}{21}, \frac{-44}{21}, \frac{167}{21}\right)$

but pt. $R(x,y,z)$ on line:

$$\begin{cases} x = 4s \\ y = 2s \\ z = 1+s \end{cases}$$

$4(4s) + 2(2s) + 1 + s - 11 = 0$

$16s + 4s + s - 10 = 0$

$21s - 10 = 0$

$s = \frac{10}{21}$

9. Two planes with equations $x - y + 2z = 2$ and $x + y - z = -2$ intersect along line L. Determine the distance from $P(-1, 2, -1)$ to L, and determine the coordinates of the point on L that gives this minimal distance.

① $x - y + 2z = 2$

② $x + y - z = -2$ subtract

$-2y + 3z = 4$

let $z = t$

then $-2y = 4 - 3t$

$y = -2 + \frac{3}{2}t$

and $x = -2 + \frac{3}{2}t - 2t + 2$

$x = -\frac{1}{2}t$

$\vec{r} = (0,-2,0) + t\left(\frac{-1}{2}, \frac{3}{2}, 1\right)$

or $(-1,3,2)$ direction

distance from $\underset{Q}{(-1,2,-1)}$ to $\underset{P}{(0,-2,0)} + t\underset{\vec{m}}{(-1,3,2)}$

$\text{distance} = \frac{|\overrightarrow{PQ} \times \vec{m}|}{|\vec{m}|}$

$\overrightarrow{PQ}$ $(-1, 2+2, -1)$

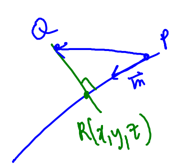
$$\begin{matrix} -1 & 4 & -1 & -1 & 4 & -1 \\ -1 & 3 & 2 & -1 & 3 & 2 \end{matrix}$$

$(8+3, 1+2, -3+4)$

$(11, 3, 1)$

$= \frac{\sqrt{11^2 + 3^2 + 1^2}}{\sqrt{1^2 + 3^2 + 2^2}}$

$= \sqrt{\frac{131}{14}} \sim 3.06$



vector $\overrightarrow{QR} \perp$ to $\vec{m}$

$(x+1, y-2, z+1) \cdot (-1,3,2) = 0$

$-x - 1 + 3y - 6 + 2z + 2 = 0$

$-x + 3y + 2z - 5 = 0$

also pt. R is on line!

$x = -t$

$y = -2 + 3t$

$z = 2t$

sub

$+t + 3(-2+3t) + 2(2t) - 5 = 0$

$t - 6 + 9t + 4t - 5 = 0$

$14t = 11$

$t = \frac{11}{14}$

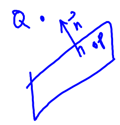
pt. R

$\left(\frac{-11}{14}, \frac{5}{14}, \frac{22}{14}\right)$

5. Points $A(1, 2, 3)$, $B(-3, -1, 2)$, and $C(13, 4, -1)$ lie on the same plane. Determine the distance from $P(1, -1, 1)$ to the plane containing these three points.

6. The distance from $R(3, -3, 1)$ to the plane with equation $Ax - 2y + 6z = 0$ is 3. Determine all possible value(s) of A for which this is true.

5. distance $= \dfrac{|\vec{PQ} \cdot \vec{n}|}{|\vec{n}|}$



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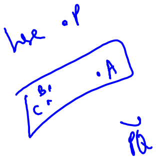


find the normal $\vec{n}$

$\vec{AB} = (-3-1, -1-2, 2-3)$
$= (-4, -3, -1)$

$\vec{AC} = (13-1, 4-2, -1-3)$
$= (12, 2, -4)$

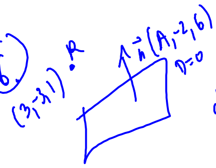
$$\begin{array}{cccccc} \cancel{-4} & -3 & -1 & -4 & -3 & \cancel{-1} \\ \cancel{12} & 2 & -4 & 12 & 2 & \cancel{-4} \end{array}$$

$(12+2, -12-16, -8+36)$
$(14, -28, 28)$
$\therefore \vec{n} = (1, -2, 2)$

$\vec{PQ}$ is like $\vec{AP} = (1-1, -1-2, 1-3)$
$= (0, -3, -2)$

$\therefore$ distance $= \dfrac{|(0,-3,-2) \cdot (1,-2,2)|}{\sqrt{1^2+2^2+2^2}} = \dfrac{|0+6-4|}{\sqrt{9}} = \dfrac{2}{3}$

6.



distance $= \dfrac{|\vec{PQ} \cdot \vec{n}|}{|\vec{n}|}$ or $= \dfrac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2+B^2+C^2}}$ where $R = (x_1, y_1, z_1)$

$3 = \dfrac{|A(3) - 2(-3) + 6(1) + 0|}{\sqrt{A^2+2^2+6^2}}$

$3\sqrt{A^2+40} = 3A + 12$

$A^2 + 40 = (A+4)^2$

$A^2 + 40 = A^2 + 8A + 16$

$24 = 8A$

$3 = A$