HW p497 13,15,17,18

Hw: p. 497 #4,5,7,13,15,17,18

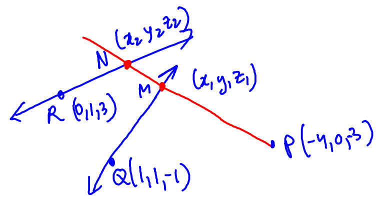
18. A line passing through point $P(-4, 0, -3)$ intersects the two lines with equations $L_1 : \vec{r} = (1, 1, -1) + s(1, 1, 0), s \in \mathbf{R}$, and $L_2 : \vec{r} = (0, 1, 3) + t(-2, 1, 3), t \in \mathbf{R}$. Determine a vector equation for this line.

needed line $L_3 \; \vec{r} = (-4, 0, -3) + u(a, b, c)$

show lines not parallel don't meet check!

L_2: $x = 0 - 2t$, $y = 1 + t$, $z = 3 + 3t$

L_1: $x = 1 + s$, $y = 1 + s$, $z = -1$



L_1 meets L_3:

pt. M
$x_1 = 1 + s$
$y_1 = 1 + s$ some $s \in \mathbb{R}$
$z_1 = -1$

L_2 meets L_3:

pt. N
$x_2 = -2t$
$y_2 = 1 + t$ some $t \in \mathbb{R}$
$z_2 = 3 + 3t$

$\vec{PM}$ and $\vec{PN}$ are parallel $P = (-4, 0, -3)$

ie. $\vec{PM} = k(\vec{PN})$

$(1+s+4, 1+s, -1+3) = k(-2t+4, 1+t, 3+3t+3)$

$(5+s, 1+s, 2) = k(4-2t, 1+t, 6+3t)$

① $5+s = 4k - 2kt$
② $1+s = k + kt$
③ $2 = 6k + 3kt$

$3\times$② $3 + 3s = 3k + 3kt$
③ $2 = 6k + 3kt$ subtract

④ $\boxed{1 + 3s = -3k}$

① $5 + s = 4k - 2kt$
$2\times$② $2 + 2s = 2k + 2kt$ add

⑤ $\boxed{7 + 3s = 6k}$

⑤ $7 + 3s = 6k$ subtract

$-6 = -9k$

$\frac{6}{9} = k$

$\frac{2}{3} = k$

$\therefore$ sub in ④
$1 + 3s = -3(\frac{2}{3})$
$1 + 3s = -2$
$3s = -3$
$s = -1$

sub in ③
$2 = 6(\frac{2}{3}) + 3(\frac{2}{3})t$
$2 = 4 + 2t$
$-2 = 2t$
$-1 = t$

$\therefore \vec{PN} = (6, 0, 3)$
$\vec{PM} = (4, 0, 2)$
simplest dir $= (2, 0, 1)$

$\therefore L_3 = (-4, 0, -3) + t(2, 0, 1)$

check in ①
$5 - 1 \stackrel{?}{=} 4(\frac{2}{3}) - 2(\frac{2}{3})(-1)$
$4 \quad \frac{8}{3} + \frac{4}{3}$
$\frac{12}{3}$
4 ✓

17. a. Show that the lines $\frac{x}{1} = \frac{y-7}{-8} = \frac{z-1}{2}$ and $\frac{x-4}{3} = \frac{z-1}{-2}$, $y = -1$, lie on the plane with equation $2x + y + 3z - 10 = 0$.

b. Determine the point of intersection of these two lines.

L_1 dir $\vec{m} = (1, -8, 2)$ pt. $(0, 7, 1)$

L_2 dir $\vec{n} = (3, 0, -2)$ pt $= (4, -1, 1)$

need to show $\underbrace{\text{vectors on the plane}}_{\text{if cross product gives normal } (2,1,3)}$ and $\underbrace{\text{pts on the plane}}_{\text{sub to check}}$

OR sub

L_1: $x = 0 + t$, $y = 7 - 8t$, $z = 1 + 2t$

to see if LS = RS

same with L_2

$$\begin{matrix} \cancel{1} & -8 & 2 & 1 & -8 & \cancel{2} \\ \cancel{3} & 0 & -2 & 3 & 0 & \cancel{-2} \end{matrix}$$

$(16 - 0, 6 + 2, 0 + 24)$

$(16, 8, 24)$

or dir. $(2, 1, 3)$ ✓ yes.

$2(0) + 7 + 3(1) - 10 \stackrel{?}{=} 0$ ✓

$2(4) + -1 + 3(1) - 10 \stackrel{?}{=} 0$

$8 - 1 + 3 - 10$

$7 + 3 - 10$ ✓

b) Since both lines are coplanar (on one plane) they $\underline{\text{must}}$ intersect, not skew.

L_1: $x = 0 + t$, $y = 7 - 8t$, $z = 1 + 2t$

L_2: $x = 4 + 3p$, $y = -1$, $z = 1 - 2p$

equate

(1) $t = 4 + 3p$

(2) $7 - 8t = -1 \longrightarrow 8 = 8t$, $1 = t$

(3) $1 + 2t = 1 - 2p$

sub in (1)

$1 = 4 + 3p$

$-3 = 3p$

$-1 = p$

check in (3)

$1 + 2(1) \stackrel{?}{=} 1 - 2(-1)$

$3 \quad 3$ ✓

$\therefore$ meet at $(1, -1, 3)$

15. The lines $\vec{r} = (-1, 3, 2) + s(5, -2, 10)$, $s \in \mathbf{R}$, and $\vec{r} = (4, -1, 1) + t(0, 2, 11)$, $t \in \mathbf{R}$, intersect at point A.

a. Determine the coordinates of point A.

b. Determine the vector equation for the line that is perpendicular to the two given lines and passes through point A.

ⓐ L_1: $x = -1 + 5s$, $y = 3 - 2s$, $z = 2 + 10s$

L_2: $x = 4$, $y = -1 + 2t$, $z = 1 + 11t$

equate

① $-1 + 5s = 4$

② $3 - 2s = -1 + 2t$

③ $2 + 10s = 1 + 11t$

$5s = 5$

$s = 1$

sub in ②

$3 - 2(1) = -1 + 2t$

$1 + 1 = 2t$

$1 = t$

check in ③

$2 + 10(1) \overset{?}{=} 1 + 11(1)$

$12 \qquad 12$ ✓

$\therefore$ pt $A = (4, 1, 12)$

ⓑ $\perp$ if do cross of 2 dir. vectors

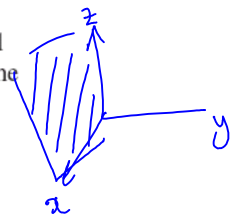
$$\begin{array}{cccccc} 0 & 2 & 11 & 0 & 2 & 11 \\ 5 & -2 & 10 & 5 & -2 & 10 \end{array}$$

$(20 + 22,\ 55 - 0,\ 0 - 10)$

$(44, 55, -10)$

$\vec{r} = (4, 1, 12) + t(44, 55, -10)$

13. The line $\vec{r} = (-8, -6, -1) + s(2, 2, 1)$, $s \in \mathbf{R}$, intersects the xz- and yz-coordinate planes at the points A and B, respectively. Determine the length of line segment AB.



pt. $A = (x, 0, z_1)$

$x = -8 + 2s$

$0 = -6 + 2s$

$z_1 = -1 + s$

$s = 3$

$x = -2$

$z_1 = 2$

$\therefore$ pt. $A(-2, 0, 2)$

pt. $B(0, y, z_2)$

$0 = -8 + 2s$

$y = -6 + 2s$

$z_2 = -1 + s$

$s = 4$

$y = 2$

$z_2 = 3$

$\therefore$ pt. $B(0, 2, 3)$

$|\vec{AB}| = \sqrt{(0+2)^2 + (2-0)^2 + (3-2)^2}$

$= \sqrt{4 + 4 + 1}$

$= \sqrt{9}$

$= 3$