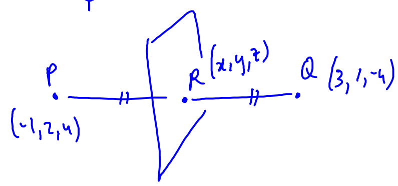
HW p468 13,14,15,16,17

Hw: p.468 #3,7,10,11,13-17

17. Determine the equation of the plane that lies between the points $(-1, 2, 4)$ P and $(3, 1, -4)$ Q and is equidistant from them.




now normal to the plane is $\vec{PQ}$ !!

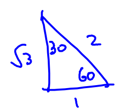
$\vec{n} = (3+1, 1-2, -4-4)$
$= (4, -1, -8)$

R midpoint
$= \left(\frac{-1+3}{2}, \frac{2+1}{2}, \frac{4+-4}{2}\right)$
$= \left(\frac{2}{2}, \frac{3}{2}, \frac{0}{2}\right)$
$= (1, \frac{3}{2}, 0)$

$Ax + By + Cz + D = 0$
$4x - y - 8z + D = 0$
$4(1) - \frac{3}{2} - 8(0) + D = 0$
$D = -4 + \frac{3}{2}$
$D = -\frac{5}{2}$

$\therefore 4x - y - 8z - \frac{5}{2} = 0$
or
$8x - 2y - 16z - 5 = 0$

16. Determine an equation of the plane that is perpendicular to the plane $x + 2y + 4 = 0$, contains the origin, and has a normal that makes an angle of 30° with the $z$-axis.




normal $(1,2,0)$ need ⊥ to it $\vec{n} \cdot (1,2,0) = 0$
$(A,B,C) \cdot (1,2,0) = 0$
$A + 2B = 0$
$A = -2B$

also need
$\cos 30 = \dfrac{\vec{n} \cdot (0,0,1)}{|\vec{n}| \sqrt{0^2+0^2+1^2}}$

$\frac{\sqrt{3}}{2}\sqrt{A^2+B^2+C^2} = 0A + 0B + C$

$3A^2 + 3B^2 + 3C^2 = 4C^2$
$3(-2B)^2 + 3B^2 = C^2$
$12B^2 + 3B^2 = C^2$
$15B^2 = C^2$

let $c^2 = 15 \rightsquigarrow c = \pm\sqrt{15}$
then $B^2 = 1$ $\quad B = \pm 1$
then $A = -2B$ $\quad A = -2B$
$= -2(1)$ $\quad = -2(-1)$
$= -2$ $\quad = 2$
if $B = 1$ $\quad$ if $B = -1$

many choices!

let $(A,B,C)$
$= (2, -1, \pm\sqrt{15})$

then $Ax + By + Cz + D = 0$
$2x - y \pm \sqrt{15}z + D = 0$ sub $(0,0,0)$
$D = 0$

$\therefore 2x - y \pm \sqrt{15}z = 0$ is the equation

Back of Book $\quad -\frac{2}{\sqrt{5}}x + \frac{1}{\sqrt{5}}y + \sqrt{3}z = 0 \quad \times\sqrt{5}$

$-2x + y + \sqrt{15}z = 0$ same!

15. Determine the Cartesian equation of the plane that passes through the points $(1, 4, 5)$ and $(3, 2, 1)$ and is perpendicular to the plane $2x - y + z - 1 = 0$.

A B

normal to this is $(2,-1,1)$

$\vec{AB} = (3-1, 2-4, 1-5)$
$= (2,-2,-4)$
or simpler dir. vector $(1,-1,-2)$

take this as another direction vector in our plane since need it $\perp$ to given one.

$\therefore \vec{r} = (1,4,5) + t(1,-1,-2) + s(2,-1,1)$ need Cartesian!

cross product: $\begin{matrix} 1 & -1 & -2 & 1 & -1 & -2 \\ 2 & -1 & 1 & 2 & -1 & 1 \end{matrix}$

$(-1-2, -4-1, -1+2)$
$(-3,-5,1)$
or $(3,5,-1)$ as a normal to our plane

$\therefore Ax + By + Cz + D = 0$
$3x + 5y - z + D = 0$ sub $(1,4,5)$
$3(1) + 5(4) - 5 + D = 0$
$D = -3 - 20 + 5$
$D = -18$

$\therefore 3x + 5y - z - 18 = 0$

14. a. What is the value of $k$ that makes the planes $4x + ky - 2z + 1 = 0$ and $2x + 4y - z + 4 = 0$ parallel?

b. What is the value of $k$ that makes these two planes perpendicular?

c. Can these two planes ever be coincident? Explain.

(a) $\parallel$ if $\vec{n}_1 = t\vec{n}_2$ for some $t \in \mathbb{R}$ $\quad \vec{n}_1 = (4, k, -2)$, $\vec{n}_2 = (2, 4, -1)$ $\times 2$ $\quad \therefore k = 8$

(b) $\perp$ if $\vec{n}_1 \cdot \vec{n}_2 = 0$ $\quad (4,k,-2)\cdot(2,4,-1) = 0$
$8 + 4k + 2 = 0$
$4k = -10$
$k = -\frac{5}{2}$

(c) coincident if equations can be multiples $\quad 4x + ky - 2z + 1 = 0$
$\times 2$ but $2x + 4y + z + 4 = 0$ $\div 4$ but $\therefore$ NOT possible

13. a. Determine the angle between the planes $x + 2y - 3z - 4 = 0$ and $x + 2y - 1 = 0$.

b. Determine the Cartesian equation of the plane that passes through the point $P(1, 2, 1)$ and is perpendicular to the line $\frac{x-3}{-2} = \frac{y+1}{3} = \frac{z+4}{1}$.

(a) $\cos\theta = \frac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}||\vec{n_2}|} = \frac{(1,2,-3)\cdot(1,2,0)}{\sqrt{1^2+2^2+3^2}\sqrt{1^2+2^2}}$

$= \frac{1+4}{\sqrt{14}\sqrt{5}}$

$\cos\theta = \frac{5}{\sqrt{70}}$

$\theta \doteq 53°$

(b) dir. vector of line

$(-2,3,1)$

need $\perp$ plane

$\therefore$ take this as normal

$Ax + By + Cz + D = 0$

$-2x + 3y + z + D = 0$ sub $(1,2,1)$

$-2(1) + 3(2) + 1 + D = 0$

$D = +2 - 6 - 1$

$D = -5$

$\therefore -2x + 3y + z - 5 = 0$