HW p459 11,14,15

HW: p.459 #5,6,7,11,14,15

15. The plane with equation $\vec{r} = (1, 2, 3) + m(1, 2, 5) + n(1, -1, 3)$ intersects the y- and z-axes at the points A and B, respectively. Determine the equation of the line that contains these two points.

$A = (0, y, 0)$
$B = (0, 0, z)$

$(0, y, 0) = (1,2,3) + m(1,2,5) + n(1,-1,3)$

$0 = 1 + m + n$
$y = 2 + 2m - n$
$0 = 3 + 5m + 3n$

elimination

$0 = 5 + 5m + 5n$
$0 = 3 + 5m + 3n$
$0 = 2 + 2n$
$-2 = 2n$
$-1 = n$

$0 = 3 + 5m + 3(-1)$
$0 = 5m$
$0 = m$

check
$0 \stackrel{?}{=} 1 + 0 - 1$ ✓

now sub $m = 0$ $n = -1$ to find $y = 2 + 2(0) - (-1)$
$y = 2 + 1 = 3$

$\therefore A = (0, 3, 0)$

Similarly find z:

$0 = 1 + m + n$
$0 = 2 + 2m - n$ add $\rightarrow$ $0 = 3 + 3m$
$z = 3 + 5m + 3n$

$-1 = m$

$0 = 1 - 1 + n$
$0 = n$

check $0 \stackrel{?}{=} 2 + 2(-1) - 0$ ✓

$\therefore z = 3 + 5(-1) + 3(0)$
$z = 3 - 5$
$z = -2$

$\therefore B = (0, 0, -2)$

$\vec{AB} = (0-0, 0-3, -2-0)$
$= (0, -3, -2)$ is direction vector of the line
or
$(0, 3, 2)$ also parallel but more simple

$\therefore$ line $\vec{r} = (0, 3, 0) + t(0, 3, 2)$

14. Show that the following equations represent the same plane:
$\vec{r} = u(-3, 2, 4) + v(-4, 7, 1),\ u, v \in \mathbf{R}$, and
$\vec{r} = s(-1, 5, -3) + t(-1, -5, 7),\ s, t \in \mathbf{R}$
(*Hint*: Express each direction vector in the first equation as a linear combination of the direction vectors in the second equation.)

OR find that have normals the same ie. parallel planes but both also share pt. $(0,0,0)$ $\therefore$ must be coincident

$\begin{matrix} \cancel{-3} & 2 & 4 & -3 & 2 & \cancel{4} \\ \cancel{-4} & 7 & 1 & -4 & 7 & \cancel{1} \end{matrix}$

$(2-28, 16+3, -21+8)$
$(-26, -13, -13)$
or $\vec{n}_1 = (2, 1, 1)$

$\begin{matrix} \cancel{-1} & 5 & -3 & -1 & 5 & \cancel{3} \\ \cancel{-1} & -5 & 7 & -1 & -5 & \cancel{7} \end{matrix}$

$(35-15, 3+7, 5+5)$
$(20, 10, 10)$
or $\vec{n}_2 = (2, 1, 1)$

OR using Hint:

$(-3,2,4) = a(-1,5,-3) + b(-1,-5,7)$ if can write as lin. comb $\therefore$ coplanar!

$-3 = -a - b$
$2 = 5a - 5b$
$4 = -3a + 7b$

$\rightarrow$ $-15 = -5a - 5b$
$2 = 5a - 5b$
$-13 = -10b$

$\frac{13}{10} = b$

$-3 = -a - \frac{13}{10}$
$a = 3 - \frac{13}{10}$
$a = \frac{17}{10}$

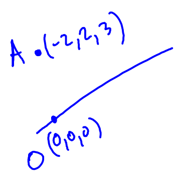
check $4 \stackrel{?}{=} -3\left(\frac{17}{10}\right) + 7\left(\frac{13}{10}\right)$
$\frac{-51}{10} + \frac{91}{10}$
4 ✓

do Similarly

$(-4,7,1) = a(-1,5,-3) + b(-1,-5,7)$

find a and b

11. Determine the equation of the plane that contains the point $A(-2, 2, 3)$ and the line $\vec{r} = m(2, -1, 7), m \in \mathbf{R}$.



$\vec{OA} = (-2,2,3)$ another vector in the plane

$\therefore \vec{r} = (0,0,0) + t(2,-1,7) + s(-2,2,3)$

or pt. A