8.3_14manyVersions

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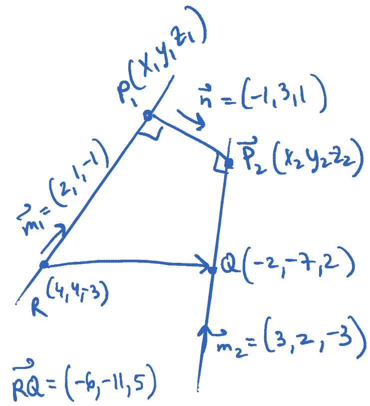
Using a Projection

14. You are given the two lines $L_1: x = 4 + 2t, y = 4 + t, z = -3 - t, t \in \mathbf{R}$, and $L_2: x = -2 + 3s, y = -7 + 2s, z = 2 - 3s, s \in \mathbf{R}$. If the point P_1 lies on L_1 and the point P_2 lies on L_2, determine the coordinates of these two points if $\overrightarrow{P_1P_2}$ is perpendicular to each of the two lines. (*Hint*: The vector $\overrightarrow{P_1P_2}$ is perpendicular to the direction vector of each of the two lines.)



$$\overrightarrow{P_1P_2} = \text{vector proj}(\vec{RQ} \text{ on } \vec{n})$$
$$= \frac{(\vec{RQ} \cdot \vec{n})}{|\vec{n}|^2}\vec{n}$$
$$= \frac{6 - 33 + 5}{\left(\sqrt{1^2 + 3^2 + 1^2}\right)^2}(-1, 3, 1)$$
$$= \frac{-22}{11}(-1, 3, 1)$$
$$\overrightarrow{P_1P_2} = (2, -6, -2)$$

$$(x_2 - x_1, y_2 - y_1, z_2 - z_1) = (2, -6, -2)$$

$x_2 - x_1 = 2$ $\quad$ $y_2 - y_1 = -6$ $\quad$ $z_2 - z_1 = -2$

$-2 + 3s - (4 + 2t) = 2$ $\quad$ $-7 + 2s - (4 + t) = -6$ $\quad$ $2 - 3s - (-3 - t) = -2$

(1) $3s - 2t = 8$ $\quad$ (2) $2s - t = 5$ $\quad$ (3) $-3s + t = -7$

(2)+(3) $\quad -s = -2$

$s = 2$, sub in (2)

$2(2) - t = 5$

$4 - 5 = t$

$-1 = t$

check in (1)

$3(2) - 2(-1) \stackrel{?}{=} 8$

$6 + 2$ ✓

sub $s = 2$ into L_2 to find $P_2 = (4, -3, -4)$

sub $t = -1$ into L_1 to find $P_1 = (2, 3, -2)$

using Dot product/Hint given

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$\overrightarrow{P_1P_2} \cdot \vec{m}_1 = 0$ or $\overrightarrow{P_1P_2} \cdot \vec{m}_2 = 0$

$P_1 = (4+2t, 4+t, -3-t)$
$P_2 = (-2+3s, -7+2s, 2-3s)$
$\therefore \overrightarrow{P_1P_2} = (3s-2t-6, 2s-t-11, -3s+t+5)$

$\vec{m}_1 = (2, 1, -1)$
$m_2 = (3, 2, -3)$

$\overrightarrow{P_1P_2} \cdot \vec{m}_1 = 0$
$2(3s-2t-6)+1(2s-t-11)+-1(-3s+t+5)=0$
$6s-4t-12+2s-t-11+3s-t-5=0$
$11s-6t-28=0 \quad (1)$

$\overrightarrow{P_1P_2} \cdot \vec{m}_2 = 0$
$3(3s-2t-6)+2(2s-t-11)-3(-3s+t+5)=0$
$9s-6t-18+4s-2t-22+9s-3t-15=0$
$22s-11t-55=0 \quad (2)$

$2\times(1) \quad 22s-12t-56=0$
$(2) \quad 22s-11t-55=0$

subtract $\quad -t-1=0$
$t=-1$

$\therefore P_1 = (4+2(-1), 4+-1, -3-(-1))$
$P_1 = (2, 3, -2)$

sub in (1) $\quad 11s-6(-1)-28=0$
$s=2$

$P_2 = (-2+3(2), -7+2(2), 2-3(2))$
$P_2 = (4, -3, -4)$