HW p450 12,13,14

HW: p.450 #6,8,9,12-15

14. You are given the two lines $L_1: x = 4 + 2t, y = 4 + t, z = -3 - t, t \in \mathbf{R}$, and $L_2: x = -2 + 3s, y = -7 + 2s, z = 2 - 3s, s \in \mathbf{R}$. If the point P_1 lies on L_1 and the point P_2 lies on L_2, determine the coordinates of these two points if $\overrightarrow{P_1P_2}$ is perpendicular to each of the two lines. (*Hint*: The vector $\overrightarrow{P_1P_2}$ is perpendicular to the direction vector of each of the two lines.)

L_1 direction vector $(2,1,-1)$

L_2 dir vector $(3,2,-3)$ > cross product will be ⊥ to both

$$\begin{matrix} \cancel{2} & 1 & -1 & 2 & 1 & \cancel{-1} \\ \cancel{3} & 2 & -3 & 3 & 2 & \cancel{-3} \end{matrix}$$

$(-3+2, -3+6, 4-3)$

let $\overrightarrow{P_1P_2} = K(-1,3,1)$

may be longer / shorter

let $P_1 = (x_1, y_1, z_1)$ on L_1

$P_2 = (x_2, y_2, z_2)$ on L_2

$x_1 - x_2 = -1K$

$y_1 - y_2 = 3K$

$z_1 - z_2 = 1K$

$4 + 2t - (-2 + 3s) = -1K$

$4 + t - (-7 + 2s) = 3K$

$-3 - t - (2 - 3s) = 1K$

$2t - 3s + K = -6$

$t - 2s - 3K = -11$

$-t + 3s - K = 5$

add

$t = -1$

$-2s - 3K = -10$

$3s - K = 4$ ×3 ×3 ×3

-10

-12

$-$

$-11s = -22$

$s = 2$

$-(-1) + 3(2) - K = 5$

$1 + 6 - 5 = K$

$7 - 5$

$2 = K$

check:

$2(-1) - 3(2) + 2 \stackrel{?}{=} -6$

$-2 - 6 + 2$ ✓

$\therefore P_1 = (4 + 2(-1), 4 - 1, -3 - (-1))$

$P_1 = (2, 3, -2)$

$P_2 = (-2 + 3(2), -7 + 2(2), 2 - 3(2))$

$P_2 = (4, -3, -4)$

13. A line with parametric equations $x = 10 + 2s, y = 5 + s, z = 2, s \in \mathbf{R}$, intersects a sphere with the equation $x^2 + y^2 + z^2 = 9$ at the points A and B. Determine the coordinates of these points.

$(10+2s)^2 + (5+s)^2 + 2^2 = 9$

$4s^2 + 40s + 100 + 25 + 10s + s^2 + 4 - 9 = 0$

$5s^2 + 50s + 120 = 0$

$5(s^2 + 10s + 24) = 0$

$5(s+6)(s+4) = 0$

$s = -6$ or $s = -4$

$(-2, -1, 2)$ or $(2, 1, 2)$

125
-5
24
$5\overline{)120}$

12. Determine the parametric equations of the line whose direction vector is perpendicular to the direction vectors of the two lines $\frac{x}{-4} = \frac{y + 10}{-7} = \frac{z + 2}{3}$ and $\frac{x - 5}{3} = \frac{y - 5}{2} = \frac{z + 5}{4}$ and passes through the point $(2, -5, 0)$

$(a_1, b_1, c_1) = (-4, -7, 3)$

$(a_2, b_2, c_2) = (3, 2, 4)$

$\perp$ to both
cross prod

$\begin{matrix} \cancel{-4} & -7 & 3 & -4 & -7 & \cancel{3} \\ \cancel{3} & 2 & 4 & 3 & 2 & \cancel{4} \end{matrix}$

$(-28-6, 9+16, -8+21)$

$(-34, 25, 13)$

$\vec{r} = (2, -5, 0) + t(-34, 25, 13)$

$x = 2 - 34t$

$y = -5 + 25t$ $\quad t \in \mathbb{R}$

$z = 13t$