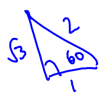
HW p443 8,12,13,14

HW: p.443 #7-14

14. The lines $x - y + 1 = 0$ and $x + ky - 3 = 0$ have an angle of $60°$ between them. For what values of k is this true?

$\vec{n}_1 = (1,-1)$ $\quad$ $\vec{n}_2 = (1,k)$

$\cos\theta = \dfrac{\vec{n}_1 \cdot \vec{n}}{|\vec{n}_1||\vec{n}_2|}$



$\cos 60° = \dfrac{1-k}{\sqrt{1^2+1^2}\sqrt{1+k^2}}$

$\dfrac{1}{2} = \dfrac{1-k}{\sqrt{2}\sqrt{1+k^2}}$

$\dfrac{\sqrt{2}\sqrt{1+k^2}}{2} = 1-k$

$\left(\sqrt{2+2k^2}\right)^2 = (2-2k)^2$

$2+2k^2 = 4-8k+4k^2$

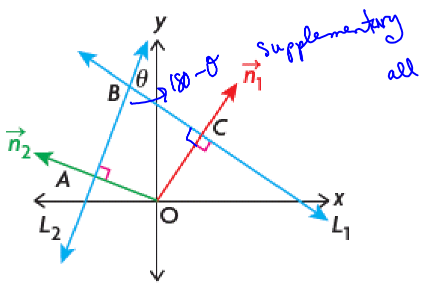
$0 = 2k^2 - 8k + 2$

$0 = 2(k^2-4k+1)$

$k = \dfrac{4 \pm \sqrt{12}}{2}$ $\quad 2\sqrt{3}$

$k = 2 \pm \sqrt{3}$

13. Lines L_1 and L_2 have $\vec{n_1}$ and $\vec{n_2}$ as their respective normals. Prove that the angle between the two lines is the same as the angle between the two normals.



all angles in quadrilateral add = 360°

$90° + 90° + 180 - \theta + \angle AOC = 360°$

$\angle AOC = \theta$

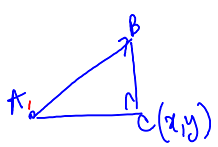
(*Hint:* Show that $\angle AOC = \theta$ by using the fact that the sum of the angles in a quadrilateral is 360°.)

12. The line segment joining $A(-3, 2)$ and $B(8, 4)$ is the hypotenuse of a right triangle. The third vertex, C, lies on the line with the vector equation $(x, y) = (-6, 6) + t(3, -4)$.

a. Determine the coordinates of C.

b. Illustrate with a diagram.

c. Use vectors to show that $\angle ACB = 90°$.



$\begin{cases} x = -6 + 3t \\ y = 6 - 4t \end{cases}$

(c) $\vec{AC} \cdot \vec{CB} \stackrel{?}{=} 0$

$(0+3, -2-2) \cdot (8-0, 4+2)$

$(3,-4) \cdot (8,6)$

$24 - 24 = 0$

$|\vec{AB}|^2 = |\vec{AC}|^2 + |\vec{CB}|^2$

$(8+3)^2 + (4-2)^2 = (\underbrace{-6+3t+3}_{3t-3})^2 + (\underbrace{6-4t-2}_{4-4t})^2 + (\underbrace{8+6-3t}_{14-3t})^2 + (\underbrace{4-6+4t}_{4t-2})^2$

$11^2 + 2^2 = 9t^2 - 18t + 9 + 16 - 32t + 16t^2 + 196 - 84t + 9t^2 + 16t^2 - 16t + 4$

$11^2 + 2^2 = 9t^2 - 18t + 9 + 16 - 32t + 16t^2 + 196 - 84t + 9t^2 + 16t^2 - 16t + 4$

$121 + 4 = 50t^2 - 150t + 225$

$125 =$

$0 = 50t^2 - 150t + 100$

$0 = 50(t^2 - 3t + 2)$

$0 = 50(t-2)(t-1)$

$t = 2$ or $t = 1$ $\quad C = (0,-2)$ or $\cancel{C = (-3, 2)}$ is A.

8. A line is perpendicular to the line $2x - 4y + 7 = 0$ and that passes through the point $P(7, 2)$. Determine the equation of this line in Cartesian form.

$\vec{n} = (2,-4)$ normal to given line can be direction vector of our line

$\therefore \vec{r} = (7,2) + t(2,-4)$ but need Cartesian line

$Ax + By + C = 0$ where (A,B) is normal

or in $\mathbb{R}^2$ can use slope $= \frac{b}{a} = m$ $\therefore y = mx + b$

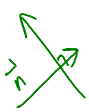
sub pt. to find y-int b

rearrange to standard form

need $(2,-4) \cdot (A,B) = 0$

$2A - 4B = 0$ $\perp$ slopes

$(A,B) = (4,2)$



unique in $\mathbb{R}^2$

~~can't choose like in $\mathbb{R}^3$~~

CAN

if $A = 1$

$2 - 4B = 0$

$2 = 4B$

$\frac{1}{2} = B$

$(1, \frac{1}{2})$

equal direction $= (2,1)$

$= (4,2)$

$4x + 2y + C = 0$ sub $(7,2)$

$4(7) + 2(2) + C = 0$

$C = -32$

$4x + 2y - 32 = 0$

or $2x + y - 16 = 0$