

HW: p.415 # 3,4,6,8,9,10

10. For the vectors $\vec{p} = (1, -2, 3)$, $\vec{q} = (2, 1, 3)$, and $\vec{r} = (1, 1, 0)$, show the following to be true.

- a. The vector $(\vec{p} \times \vec{q}) \times \vec{r}$ can be written as a linear combination of \vec{p} and \vec{q} .
b. $(\vec{p} \times \vec{q}) \times \vec{r} = (\vec{p} \cdot \vec{r})\vec{q} - (\vec{q} \cdot \vec{r})\vec{p}$

$$\textcircled{a} \quad \begin{array}{c} \vec{p} \\ \vec{q} \end{array} \quad \left| \begin{array}{ccc} -2 & 3 & 1 \\ 1 & 3 & 2 \\ 1 & 1 & 3 \end{array} \right| \quad \begin{array}{c} \vec{p} \times \vec{q} \\ \vec{r} \end{array} \quad \left| \begin{array}{ccc} -9 & 3 & -9 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{array} \right| \quad \begin{array}{c} (-6, 6, -3) \\ (-9, 3, 5) \end{array}$$

$$\begin{array}{c} \vec{p} \times \vec{q} \\ \vec{r} \end{array} \quad \left| \begin{array}{ccc} 3 & 5 & 3 \\ 1 & 0 & 1 \\ 0 & -5 & -3 \end{array} \right| \quad \begin{array}{c} (-5, 5, -12) \end{array}$$

$$(-5, 5, -12) = a(1, -2, 3) + b(2, 1, 3)$$

$$\begin{aligned} -5 &= a + 2b \\ 5 &= -2a + b \\ -12 &= 3a + 3b \\ 27 &= -9a \\ -3 &= a \end{aligned}$$

$$\begin{aligned} -12 &= 3(-3) + 3b \\ -12 + 9 &= 3b \\ -3 &= 3b \end{aligned}$$

$$\textcircled{-1 = b}$$

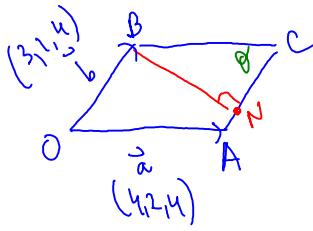
$$\therefore (\vec{p} \times \vec{q}) \times \vec{r} = -3\vec{p} - 1\vec{q}$$

$$\text{check} \quad \begin{array}{c} -5 \\ -5 \end{array} \quad \left| \begin{array}{c} -3 + 2(-1) \\ -5 \end{array} \right. \quad \checkmark$$

$$\textcircled{b} \quad \begin{array}{l} \text{show } \vec{p} \cdot \vec{r} = -1 \\ \vec{q} \cdot \vec{r} = +3 \end{array}$$

$$\begin{aligned} \vec{p} \cdot \vec{r} &= 1 - 2 + 0 = -1 \\ \vec{q} \cdot \vec{r} &= 2 + 1 + 0 = 3 \end{aligned}$$

9. Parallelogram $OBCA$ has its sides determined by $\overrightarrow{OA} = \vec{a} = (4, 2, 4)$ and $\overrightarrow{OB} = \vec{b} = (3, 1, 4)$. Its fourth vertex is point C . A line is drawn from B perpendicular to side AC of the parallelogram to intersect AC at N . Determine the length of BN .



$$\overrightarrow{OC} = \overrightarrow{OA} = \vec{a}$$

$$\therefore |\overrightarrow{BC}| = \sqrt{16+4+16} \\ = \sqrt{36} \\ = 6$$

$$\sin \theta = \frac{|\overrightarrow{BN}|}{|\overrightarrow{BC}|}$$

$$\therefore |\overrightarrow{BN}| = |\overrightarrow{BC}| \sin \theta$$

$$= 6 \sin 11.6^\circ = 1.18 \text{ units}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$\frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \cos \theta$$

$$\frac{\sqrt{12+2+16}}{\sqrt{9+1+16} \sqrt{16+4+16}} = \cos \theta$$

$$\frac{3\sqrt{6}}{\sqrt{36} \sqrt{36}} = \cos \theta$$

$$11.3^\circ \approx \theta$$