HW 415 9,10

Hw: p.415 # 3,4,6,8,9,10

10. For the vectors $\vec{p} = (1, -2, 3)$, $\vec{q} = (2, 1, 3)$, and $\vec{r} = (1, 1, 0)$, show the following to be true.
    a. The vector $(\vec{p} \times \vec{q}) \times \vec{r}$ can be written as a linear combination of $\vec{p}$ and $\vec{q}$.
    b. $(\vec{p} \times \vec{q}) \times \vec{r} = (\vec{p} \cdot \vec{r})\vec{q} - (\vec{q} \cdot \vec{r})\vec{p}$

(a) $\vec{p}$: 1 −2 3 1 −2 3
$\vec{q}$: 2 1 3 2 1 3

$(-6-3, 6-3, 1+4)$
$(-9, 3, 5)$

$\vec{p}\times\vec{q}$: −9 3 5 −9 3 5
$\vec{r}$: 1 1 0 1 1 0

$(0-5, 5-0, -9-3)$
$(-5, 5, -12)$

$(-5, 5, -12) = a(1, -2, 3) + b(2, 1, 3)$

$-5 = a + 2b$
$5 = -2a + b$ (×3)
$-12 = 3a + 3b$

$27 = -9a$
$-3 = a$

$-12 = 3(-3) + 3b$
$-12 + 9 = 3b$
$-3 = 3b$
$-1 = b$

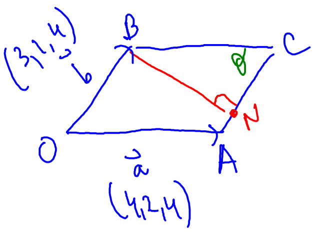
check
$-5 \mid -3 + 2(-1)$
-5 ✓

$\therefore (\vec{p} \times \vec{q}) \times \vec{r} = -3\vec{p} - 1\vec{q}$

(b) show $\vec{p}\cdot\vec{r} = -1$
$\vec{q}\cdot\vec{r} = +3$

$\vec{p}\cdot\vec{r} = 1 - 2 + 0 = -1$
$\vec{q}\cdot\vec{r} = 2 + 1 + 0 = 3$

9. Parallelogram $OBCA$ has its sides determined by $\overrightarrow{OA} = \vec{a} = (4, 2, 4)$ and $\overrightarrow{OB} = \vec{b} = (3, 1, 4)$. Its fourth vertex is point C. A line is drawn from B perpendicular to side AC of the parallelogram to intersect AC at N. Determine the length of BN.



$\sin\theta = \dfrac{|\overrightarrow{BN}|}{|\overrightarrow{BC}|}$

$\therefore |\overrightarrow{BN}| = |\overrightarrow{BC}| \sin\theta$

$= 6 \sin 11.6^\circ = 1.18$ units

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

$\dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \cos\theta$

$\dfrac{12+2+16}{\sqrt{9+1+16}\sqrt{16+4+16}} = \cos\theta$

$\dfrac{30}{\sqrt{26}\sqrt{36}} = \cos\theta$

$11.3^\circ \approx \theta$

$\overrightarrow{BC} = \overrightarrow{OA} = \vec{a}$

$\therefore |\overrightarrow{BC}| = \sqrt{16+4+16}$

$= \sqrt{36}$

$= 6$