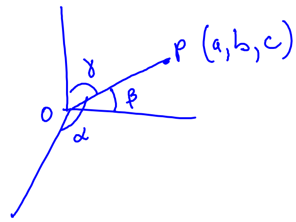
HW p399 15,17

HW: p. 399 #5, 6, 10, 11, 12, 14, 15, 17

17. If α, β, and γ represent the direction angles for vector $\overrightarrow{OP}$, prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.

$(a,b,c) = \overrightarrow{OP}$ unit vector $\frac{\overrightarrow{OP}}{|\overrightarrow{OP}|} = \left(\frac{a}{|\overrightarrow{OP}|}, \frac{b}{|\overrightarrow{OP}|}, \frac{c}{|\overrightarrow{OP}|}\right)$



$= (\cos\alpha, \cos\beta, \cos\gamma)$

since unit $\therefore \sqrt{\cos^2\alpha + \cos^2\beta + \cos^2\gamma} = 1$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ $\qquad \sin^2\theta + \cos^2\theta = 1$

$1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$

$3 - 1 \qquad = \sin^2\alpha + \sin^2\beta + \sin^2\gamma$

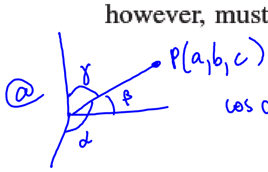
2

15. a. If α, β, and γ represent the direction angles for vector $\overrightarrow{OP}$, prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

b. Determine the coordinates of a vector $\overrightarrow{OP}$ that makes an angle of 30° with the y-axis, 60° with the z-axis, and 90° with the x-axis.

c. In Example 3, it was shown that, in general, the direction angles do not always add to 180°—that is, $\alpha + \beta + \gamma \neq 180°$. Under what conditions, however, must the direction angles always add to 180°?

(a)



$\cos\alpha = \frac{a}{|\overrightarrow{OP}|}$ $\quad \cos\beta = \frac{b}{|\overrightarrow{OP}|}$ $\quad \cos\gamma = \frac{c}{|\overrightarrow{OP}|}$

but $\left(\frac{a}{|\overrightarrow{OP}|}, \frac{b}{|\overrightarrow{OP}|}, \frac{c}{|\overrightarrow{OP}|}\right) = \frac{\overrightarrow{OP}}{|\overrightarrow{OP}|}$ = unit vector

$\therefore \left|(\cos\alpha, \cos\beta, \cos\gamma)\right| = 1$

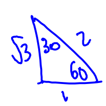
$\sqrt{\cos^2\alpha + \cos^2\beta + \cos^2\gamma} = 1$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

(b) $\alpha: \cos 90° = \frac{a}{\sqrt{a^2+b^2+c^2}} = 0$

$\beta: \cos 30° = \frac{b}{\sqrt{}} = \frac{\sqrt{3}}{2}$

$\gamma: \cos 60° = \frac{c}{\sqrt{}} = \frac{1}{2}$



$\therefore a = 0$

① $2b = \sqrt{3(b^2+c^2)}$ $\qquad$ ② $2c = \sqrt{b^2+c^2}$

$4b^2 = 3b^2 + 3c^2$ $\qquad 4c^2 = b^2 + c^2$

$b^2 = 3c^2 \longleftrightarrow$ SAME $\quad 3c^2 = b^2$

① − ② $\quad b^2 = 3c^2$

$0 = 0$

$\therefore$ choose $c = 1$

$b = \sqrt{3}$

$a = 0$

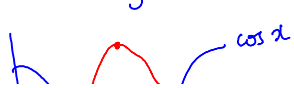
(c) $\alpha + \beta + \gamma = 180°$

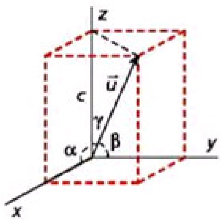
$\alpha + \beta = 180° - \gamma$

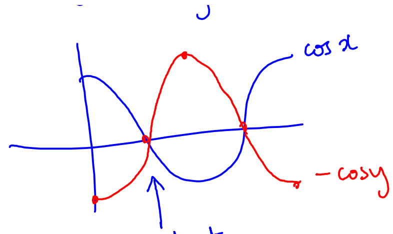
$\cos(\alpha+\beta) = \cos(180° - \gamma)$

$\cos(\alpha+\beta) = -\cos\gamma$

$\cos x = -\cos y$







meet at 90°

$\therefore$ $\alpha + \beta = 90°$ and $\gamma = 90°$ then all 3 will add to 180°