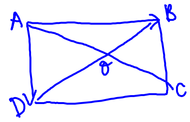
HW p385 14,16,17,18,19

19. The rectangle $ABCD$ has vertices at $A(-1, 2, 3)$, $B(2, 6, -9)$, and $D(3, q, 8)$.
    a. Determine the coordinates of the vertex C.
    b. Determine the angle between the two diagonals of this rectangle.

ⓐ $\vec{AB} = (2+1, 6-2, -9-3)$
$= (3, 4, -12)$

$\vec{AB} + \vec{AD}$
$= \vec{AC}$



$\vec{AD} = (3+1, q-2, 8-3)$
$= (4, q-2, 5)$

$\vec{AB} + \vec{AD} = (7, q+2, -7)$
$\vec{AC} = (7, q+2, -7)$

$\vec{AC} = \vec{AO} + \vec{OC}$
$= -\vec{OA} + \vec{OC}$

$(7, q+2, -7) = -(-1, 2, 3) + (x, y, z)$

$7+1 = x$
$8 = x$

$q+2-2 = y$
$q = y$

$-7-3 = z$
$-10 = z$

$\therefore$ pt. $C = (8, q, -10)$
$= (8, 14, -10)$

$\vec{AB} \cdot \vec{AD} = 0$ since at $90°$
$(3, 4, -12) \cdot (4, q-2, 5) = 0$
$12 + 4q - 8 - 60 = 0$
$4q = 56$
$q = 14$

ⓑ $\vec{AC} \cdot \vec{BD} = |\vec{AC}||\vec{BD}| \cos\theta$
$(7, 16, -7) \cdot (1, 8, 17)$

$\dfrac{7 + 128 - 119}{\sqrt{7^2+16^2+(-7)^2}\sqrt{1^2+8^2+17^2}} = \cos\theta$

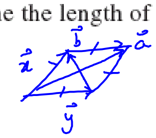
$\dfrac{16}{\sqrt{354}\sqrt{354}} = \cos\theta$

$\dfrac{16}{354} = \cos\theta$

$87.4° = \theta$

ANS wrong?
$(6, 18, -4)$
$87.4°$

18. The diagonals of a parallelogram are determined by the vectors $\vec{a} = (3, 3, 0)$ and $\vec{b} = (-1, 1, -2)$.
    a. Show that this parallelogram is a rhombus.
    b. Determine vectors representing its sides and then determine the length of these sides.
    c. Determine the angles in this rhombus.



ⓐ $\vec{a} \cdot \vec{b} = 0$ then $\vec{a} \perp \vec{b}$ and is a rhombus

$\vec{a} \cdot \vec{b} = (3, 3, 0) \cdot (-1, 1, -2)$
$= 3 - 3 + 0$
$= 0$

ⓑ let $\vec{x} + \vec{y} = \vec{a}$
$\vec{x} - \vec{y} = \vec{b}$

$|\vec{a}|^2 = |\vec{x}+\vec{y}|^2$ $\quad |\vec{x}| = |\vec{y}|$
$3^2 + 3^2 = x \cdot x + 2x \cdot y + y \cdot y$
$18 = 2|\vec{x}|^2 + 2|\vec{x}|^2 \cos\theta$

add

$x_1 + y_1 = 3$ $\quad x_1 - y_1 = -1$
$x_2 + y_2 = 3$ $\quad x_2 - y_2 = 1$
$x_3 + y_3 = 0$ $\quad x_3 - y_3 = -2$

$2x_1 = 2$ $\quad 2x_2 = 4$

$|\vec{b}|^2 = |\vec{x} - \vec{y}|^2$
$1^2 + 1^2 + 2^2 = x \cdot x - 2x \cdot y + y \cdot y$
$6 = 2|x|^2 - 2|\vec{x}|^2 \cos\theta$
$+ \quad 18 = 2|\vec{x}|^2 + 2|\vec{x}| \cos\theta$
$24 = 4|\vec{x}|^2$

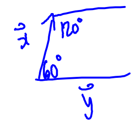
$\rightsquigarrow 6 = 2(6) - 2(6)\cos\theta$
$\dfrac{-6}{-12} = \cos\theta$
$60° = \theta$

$2x_1 = 2$ $\quad$ $2x_2 = 4$ $\quad$ $24 = 4|\vec{x}|^2$ $\quad$ $60° = \theta$

$x_1 = 1$ $\quad$ $x_2 = 2$ $\quad$ $2x_3 = -2$ $\quad$ $\sqrt{6} = |\vec{x}| = |\vec{y}|$

$y_1 = 2$ $\quad$ $y_2 = 1$ $\quad$ $x_3 = -1$

$y_3 = 1$



$\therefore \vec{x} = (1, 2, -1)$ $\quad$ $\vec{y} = (2, 1, 1)$

17. The vectors $\vec{x} = (-4, p, -2)$ and $\vec{y} = (-2, 3, 6)$ are such that $\cos^{-1}\left(\frac{4}{21}\right) = \theta$, where θ is the angle between $\vec{x}$ and $\vec{y}$. Determine the value(s) of p.

$\downarrow$

$\cos\theta = \frac{4}{21}$

$\vec{x} \cdot \vec{y} = |\vec{x}||\vec{y}|\cos\theta$

$8 + 3p - 12 = \sqrt{16 + p^2 + 4}\ \underbrace{\sqrt{4 + 9 + 36}}_{7}\left(\frac{4}{21}\right)$

$3p - 4 = \sqrt{20 + p^2}\left(\frac{28}{21}\right)$

$\frac{21(3p - 4)}{28} = \sqrt{20 + p^2}$

$\frac{441(3p - 4)^2}{784} = 20 + p^2$

$441(9p^2 - 24p + 16) = 15680 + 784p^2$

$3969p^2 - 10584p + 7056 = 15680 + 784p^2$

$3185p^2 - 10584p - 8624 = 0$

$p = \frac{10584 \pm 14846}{6370} = 4$ or $-0.67\ldots$

$-\frac{44}{65}$

16. Given the vectors $\vec{r} = (1, 2, -1)$ and $\vec{s} = (-2, -4, 2)$, determine the components of two vectors perpendicular to each of these vectors. Explain your answer.

$\vec{r} = -2\vec{s}$ $\quad$ $\therefore \vec{r} \parallel \vec{s}$

$(a, b, c) \cdot \vec{r} = 0$ $\quad$ to be $\perp$

$a + 2b - c = 0$

choose $a = 1$, $b = 1$ $\quad$ $1 + 2 - c = 0$ $\quad$ $1 + 2 = c$

then $c = 3$

$\therefore (1, 1, 3)$

or choose $a = 0$, $b = 1$ $\quad$ $0 + 2 = c$

then $c = 2$

$\therefore (0, 1, 2)$

14. Find the value of p if the vectors $\vec{r} = (p, p, 1)$ and $\vec{s} = (p, -2, -3)$ are perpendicular to each other.

$\vec{r} \cdot \vec{s} = 0$

$p^2 - 2p - 3 = 0$

$(p-3)(p+1) = 0 \quad p = 3$ or $p = -1$