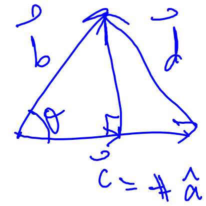
HW p377 15, 16, 18

18. The vector $\vec{a}$ is a unit vector, and the vector $\vec{b}$ is any other nonzero vector. If $\vec{c} = (\vec{b} \cdot \vec{a})\vec{a}$ and $\vec{d} = \vec{b} - \vec{c}$, prove that $\vec{d} \cdot \vec{a} = 0$.

$\vec{d}\cdot\vec{a} = |\vec{d}||\vec{a}|\cos\theta$ i.e. $\vec{d} \perp \vec{a}$



$\vec{c} = (\vec{b}\cdot\vec{a})\hat{a}$ dir. of $\hat{a}$ multiple of it

$= |\vec{b}||\vec{a}|\cos\theta$

$\underbrace{(|\vec{b}|\cos\theta)}_{\text{length of adjacent side to } \theta \text{ if made into right } \Delta}\hat{a}$

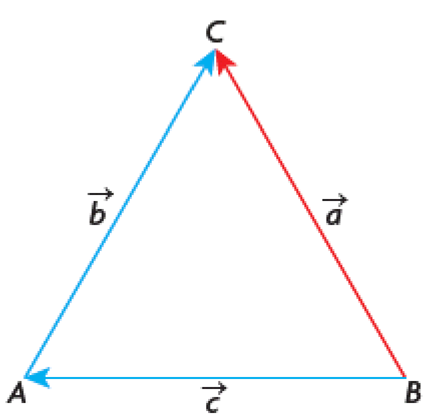
$\therefore \vec{c}$ stops at $90°$ to vector $\vec{d}$

$\vec{c} \perp \vec{d}$

$\vec{a} \perp \vec{d}$

$\therefore \vec{a}\cdot\vec{d} = 0$

16. The three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ are of unit length and form the sides of equilateral triangle ABC such that $\vec{a} - \vec{b} - \vec{c} = \vec{0}$ (as shown). Determine the numerical value of $(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b} + \vec{c})$.



$\vec{a}\cdot\vec{a} + \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} + \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{b} + \vec{b}\cdot\vec{c}$

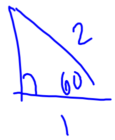
$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{c}$

$1 + 1 + 2\vec{a}\cdot\vec{b} + (\vec{a}+\vec{b})\cdot\vec{c}$

$2 + 2\vec{a}\cdot\vec{b} + (\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})$

$2 + 2|\vec{a}||\vec{b}|\cos 60° + \vec{a}\cdot\vec{a} - \vec{b}\cdot\vec{b}$

$2 + 2\cos 60° + 1 - 1$



$$2+2\left(\tfrac{1}{2}\right)=2+1=3$$

15. Prove the identity $|\vec{u}+\vec{v}|^2 + |\vec{u}-\vec{v}|^2 = 2|\vec{u}|^2 + 2|\vec{v}|^2$.

$$(\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v}) + (\vec{u}-\vec{v})\cdot(\vec{u}-\vec{v})$$

$$= \vec{u}\cdot\vec{u} + 2\vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{v} + \vec{u}\cdot\vec{u} - 2\vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{v}$$

$$= 2|\vec{u}|^2 + 2|\vec{v}|^2$$