P377_13_17

October-09-12
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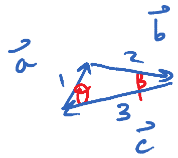
7.3

17. The vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ are such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Determine the value of $\vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{c}$ if $|\vec{a}| = 1$, $|\vec{b}| = 2$, and $|\vec{c}| = 3$.

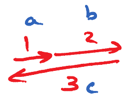


$$\cos\theta = \frac{1^2+3^2-2^2}{2(1)(3)} = \frac{6}{6} = 1$$

$$\cos^{-1}(1) = 0^\circ$$

$$\cos\beta = \frac{2^2+3^2-1^2}{2(2)(3)} = \frac{12}{12} = 1$$

wrong picture

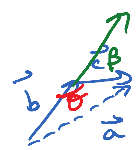


$$\vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{c}$$
$$= |\vec{a}||\vec{b}|\underbrace{\cos 0^\circ}_{1} + |\vec{a}||\vec{c}|\underbrace{\cos 180^\circ}_{-1} + |\vec{b}||\vec{c}|\underbrace{\cos 180^\circ}_{-1}$$
$$= (1)(2)(1) + (1)(3)(-1) + (2)(3)(-1)$$
$$= 2 - 3 - 6$$
$$= -7$$

7.3 13. The vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ satisfy the relationship $\vec{a} = \vec{b} + \vec{c}$.

a. Show that $|\vec{a}|^2 = |\vec{b}|^2 + 2\vec{b}\cdot\vec{c} + |\vec{c}|^2$.

b. If the vectors $\vec{b}$ and $\vec{c}$ are perpendicular, how does this prove the Pythagorean theorem?



ⓐ $|\vec{a}|^2 = |\vec{b}|^2 + |\vec{c}|^2 - 2|\vec{b}||\vec{c}|\cos\theta$ cosine law

Proving:

$$2\vec{b}\cdot\vec{c} = 2|\vec{b}||\vec{c}|\cos\beta$$
$$= 2|\vec{b}||\vec{c}|\cos(180^\circ - \theta)$$
$$= 2|\vec{b}||\vec{c}|(-1)(\cos\theta)$$

$\beta = 180 - \theta$

formula from advanced functions

$$\cos(180-\theta) = \underbrace{\cos 180}_{-1}\cos\theta + \cancel{\sin 180\sin\theta}$$ 0

$$\therefore |\vec{a}|^2 = |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b}\cdot\vec{c}$$

OR

$$(\vec{b}+\vec{c})\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{a}$$
$$= |\vec{a}|^2$$
$$\vec{b}\cdot\vec{b} + 2\vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{c}$$
$$|\vec{b}|^2 + \underbrace{2\vec{b}\cdot\vec{c}} + |\vec{c}|^2 = |\vec{a}|^2$$

ⓑ if $\vec{b} \perp \vec{c}$ then $\vec{b}\cdot\vec{c} = 0$

$$\therefore |\vec{b}|^2 + 2(0) + |\vec{c}|^2 = |\vec{a}|^2$$
$$|\vec{b}|^2 + |\vec{b}|^2 = |\vec{a}|^2$$ Pythagorean thrm.