6.4_12

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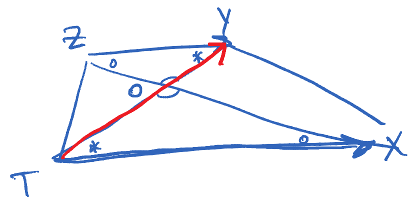
12. In the trapezoid $TXYZ$, $\overrightarrow{TX} = 2\overrightarrow{ZY}$. If the diagonals meet at O, find an expression for $\overrightarrow{TO}$ in terms of $\overrightarrow{TX}$ and $\overrightarrow{TZ}$.



Textbook solution in

12. Applying the triangle law for adding vectors shows that

$\overrightarrow{TY} = \overrightarrow{TZ} + \overrightarrow{ZY}$

The given information states that

$\overrightarrow{TX} = 2\overrightarrow{ZY}$

$\frac{1}{2}\overrightarrow{TX} = \overrightarrow{ZY}$

By the properties of trapezoids, this gives — not proven

$\frac{1}{2}\overrightarrow{TO} = \overrightarrow{OY}$, and since

$\overrightarrow{TY} = \overrightarrow{TO} + \overrightarrow{OY}$, the original equation gives

$\overrightarrow{TO} + \frac{1}{2}\overrightarrow{TO} = \overrightarrow{TZ} + \frac{1}{2}\overrightarrow{TX}$

$\frac{3}{2}\overrightarrow{TO} = \overrightarrow{TZ} + \frac{1}{2}\overrightarrow{TX}$

$\overrightarrow{TO} = \frac{2}{3}\overrightarrow{TZ} + \frac{1}{3}\overrightarrow{TX}$

Since this is a trapezoid two sides are parallel

$\circ$ $\angle YZO = \angle TXO$ by Z-pattern

$*$ $\angle ZYO = \angle XTO$ by Z-pattern

$\therefore \triangle YZO \sim \triangle TXO$ by Angle-Angle similarity

($\sim$ similar to)

The proportion of each side is twice

ie. $\left\{\begin{array}{l} \overrightarrow{TX} = 2\overrightarrow{ZY} \\ \overrightarrow{XO} = 2\overrightarrow{OZ} \\ \overrightarrow{TO} = 2\overrightarrow{OY} \end{array}\right.$

sub in.

Now $\overrightarrow{TY} = \overrightarrow{TO} + \overrightarrow{OY} = \overrightarrow{TO} + \frac{1}{2}\overrightarrow{TO}$ (1)

sub in

also $\overrightarrow{TY} = \overrightarrow{TZ} + \overrightarrow{ZY} = \overrightarrow{TZ} + \frac{1}{2}\overrightarrow{TX}$ (2)

make (1) = (2) since BOTH are $\overrightarrow{TY}$

$\overrightarrow{TO} + \frac{1}{2}\overrightarrow{TO} = \overrightarrow{TZ} + \frac{1}{2}\overrightarrow{TX}$

$\frac{3}{2}\overrightarrow{TO} = \overrightarrow{TZ} + \frac{1}{2}\overrightarrow{TX}$ isolate $\overrightarrow{TO}$ so it's in terms of other two

$\therefore \overrightarrow{TO} = \frac{2}{3}\overrightarrow{TZ} + \frac{1}{3}\overrightarrow{TX}$

This is not the only way to do this

Here → started with $\overrightarrow{TY}$ written in two ways using required vectors $\overrightarrow{TO}, \overrightarrow{TZ}, \overrightarrow{TX}$ then eliminated $\overrightarrow{TY}$

Can also do $\overrightarrow{TO}$ written in two ways then eliminate anything thats not $\overrightarrow{TO}, \overrightarrow{TZ}, \overrightarrow{TX}$ by using what was proven in similar $\triangle$'s

etc. ⋮