6.3_22

January-22-13
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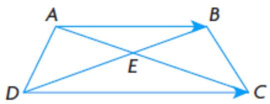
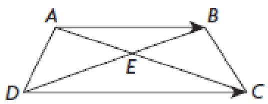
22. $ABCD$ is a trapezoid whose diagonals AC and BD intersect at the point E. If $\overrightarrow{AB} = \frac{2}{3}\overrightarrow{DC}$, prove that $\overrightarrow{AE} = \frac{3}{5}\overrightarrow{AB} + \frac{2}{5}\overrightarrow{AD}$.



22.



Applying the triangle law for adding vectors shows that

$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$

The given information states that

$\overrightarrow{AB} = \frac{2}{3}\overrightarrow{DC}$

$\frac{3}{2}\overrightarrow{AB} = \overrightarrow{DC}$

By the properties of trapezoids, this gives

$\frac{3}{2}\overrightarrow{AE} = \overrightarrow{EC}$, and since

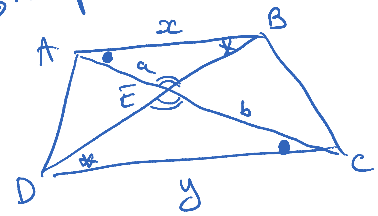
$\overrightarrow{AC} = \overrightarrow{AE} + \overrightarrow{EC}$, the original equation gives

$$\overrightarrow{AE} + \frac{3}{2}\overrightarrow{AE} = \overrightarrow{AD} + \frac{3}{2}\overrightarrow{AB}$$

$$\frac{5}{2}\overrightarrow{AE} = \overrightarrow{AD} + \frac{3}{2}\overrightarrow{AB}$$

$$\overrightarrow{AE} = \frac{2}{5}\overrightarrow{AD} + \frac{3}{5}\overrightarrow{AB}$$

not proven.



if $\frac{3}{2}x = y$ then $\frac{3}{2}a = b$

$\overrightarrow{AB}$ is parallel to $\overrightarrow{DC}$

$\therefore \angle ABD = \angle CDB$ * by Z pattern

$\angle BEA = \angle CED$ by X-pattern

$\therefore \triangle ABE \sim \triangle CDE$ (similar but different size $\triangle$'s since same shape)

similar $\triangle$'s means there's a scale factor that keeps sides proportional

$\therefore$ if $\frac{3}{2}x = y$ then $\frac{3}{2}a = b$