5.3_8

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9:58 AM

a) $$\boxed{F(t) = \begin{cases} 2^{t/5} & \text{if } 0 < t \le 60 \\ 2^{t/5} - e^{\frac{t-60}{3}} & \text{if } t \ge 60 \end{cases}}$$ equation to maximize

domain: $t \in (0, \infty)$

crit pt: $$F'(t) = \begin{cases} 2^{t/5}\ln(2)\left(\frac{1}{5}\right) & \text{if } 0 < t < 60 \\ 2^{t/5}\ln(2)\left(\frac{1}{5}\right) - e^{\frac{t-60}{3}}\left(\frac{1}{3}\right) & \text{if } t \ge 60 \end{cases}$$

$$0 = 2^{t/5}\ln(2)\left(\frac{1}{5}\right) \quad \text{or} \quad 0 = 2^{t/5}\frac{\ln 2}{5} - \frac{e^{\frac{t-60}{3}}}{3}$$

N/A

$$\frac{e^{\frac{t-60}{3}}}{3} = 2^{t/5}\frac{\ln 2}{5}$$

$$e^{\frac{t-60}{3}} = \frac{3}{5}(\ln 2)\,2^{t/5}$$ take ln of both sides

pull exponent down $$\ln\left[e^{\frac{t-60}{3}}\right] = \ln\left[\frac{3}{5}(\ln 2)\,2^{t/5}\right]$$

separate then can pull exponent down

$$\left(\frac{1}{3}t - 20\right)\underbrace{\ln e}_{1} = \ln\left[\frac{3}{5}(\ln 2)\right] + \ln\left(2^{t/5}\right)$$

$$\frac{1}{3}t - 20 = \ln\left[\frac{3}{5}(\ln 2)\right] + \frac{1}{5}t\ln(2)$$

$$\frac{1}{3}t - \frac{1}{5}(\ln 2)t = \ln\left[\frac{3}{5}(\ln 2)\right] + 20$$

$$t\left(\frac{1}{3} - \frac{1}{5}\ln 2\right) = \ln\left[\frac{3}{5}(\ln 2)\right] + 20$$

$$t = \frac{\ln\left[\frac{3}{5}\ln 2\right] + 20}{\frac{1}{3} - \frac{1}{5}\ln 2}$$

$\frac{1}{3} - \frac{1}{5}\ln 2$

$t = 98.2$

show MAX:

$F''(t) = 2^{t/5}\left(\frac{\ln 2}{5}\right)^2 - \left(\frac{1}{3}\right)^2 e^{\frac{t-60}{3}}$

$F''(98.2) = 2^{19.64}(0.019) - \frac{1}{9}(338856.9) = \text{neg} \therefore CD \therefore t = 98.2$ is a MAX

answer question: $\therefore$ at 98.2 min (38.2 min after drug introduced)

$F(98.2) \doteq 478158.5$ is MAX # of bacteria

(b) obliterated if $F(t) = 0$

$2^{t/5} = 0$ or $2^{t/5} - e^{\frac{t-60}{3}} = 0$

N/A

$2^{t/5} = e^{\frac{t-60}{3}}$ take ln of both sides

$\ln 2^{t/5} = \ln e^{\frac{t-60}{3}}$

$\frac{t}{5}\ln 2 = \frac{t-60}{3}(\ln e)$ 1

$\frac{t}{5}\ln 2 - \frac{t}{3} = -20$

$t\left(\frac{1}{5}\ln 2 - \frac{1}{3}\right) = -20$

$t = \frac{-20}{\frac{1}{5}\ln 2 - \frac{1}{3}}$

$t \doteq 102.72$ min (42.72 min after drug introduced)

the bacteria is obliterated.