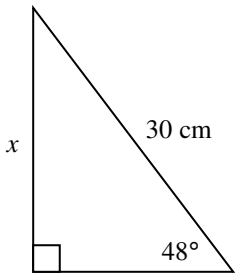
Strand Review: Trigonometry

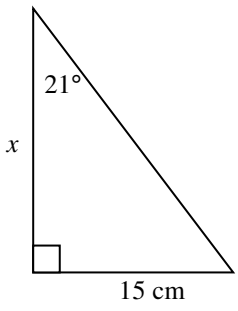
Answer each of the following questions in the space provided. Show all steps of work.

1. Find the missing angle or side specified for each of the following.

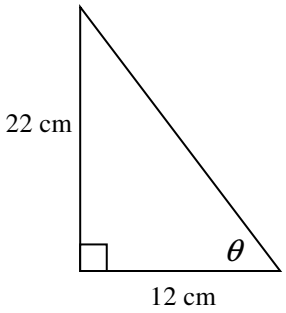
   a. Solve for x.



   b. Solve for x.

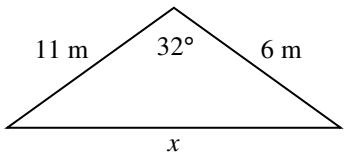


   c. Solve for θ.

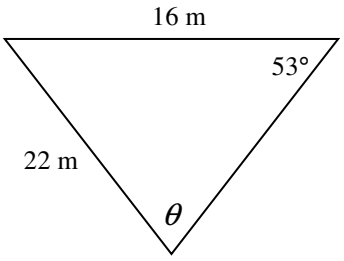


2. Find the missing angle or side specified for each of the following.

   a. Solve for x.



   b. Solve for θ.



3. The angle of depression from the top of castle to a boat is 25°. If the distance from the top of the castle to the boat is determined to be 100 m, how high is the castle?

4. A ski run reaches a height of 250 m along a hill 1600 m long. What interior angle is formed where the bottom of the hill meets the horizontal?

5. Two angles of a triangle are 30° and 65° and the longest side is 34 cm. How long is the shortest side?

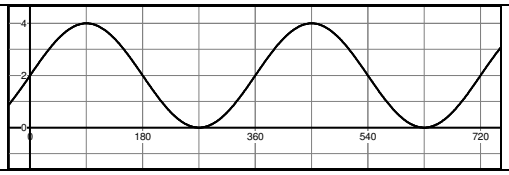
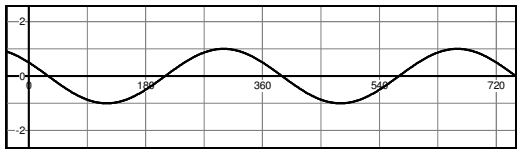
6. Find the measure, to the nearest degree, of the smallest angle in a triangle with sides 4 m, 7 m, and 8 m.

7. Two points, A and B, are on the same side of a tower. The angle of elevation of the top of the tower form point A is 28°. Point B is closer to the tower, 140 m from point A. The angle of elevation of the top of the tower from point B is 37°. Determine the height of the tower.

8. A flagpole stands on top of a 27 m building. From a point on the ground some distance away, the angle of elevation of the top of the flagpole is 43°. The angle of elevation of the bottom of the flagpole is 32°.
   a. How far is it from the point on the ground to the bottom of the building?

   b. How high is the flagpole?

9. Complete the chart for each graph.

		
max & min values		
axis of the curve		
amplitude		
Phase shift		
period		
equation of the function		

10. Complete the chart for each function.

	$-3\sin(2(\theta+90))-4$	$y=\frac{1}{3}\sin 3\theta-5$	$y=\sin 5(\theta-30^\circ)+7$
amplitude			
period			
phase shift			
vertical translation			

11. Write an equation to represent each of the transformations of the function $y=\sin\theta$.

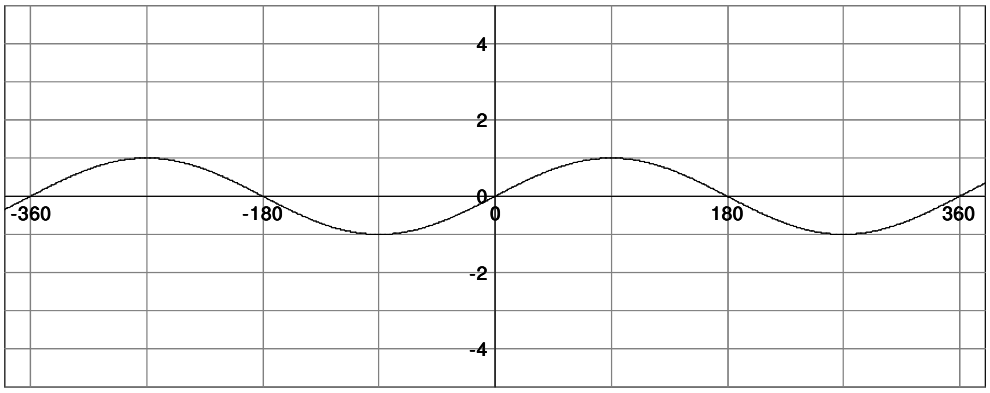
a. a reflection in the x-axis, a phase shift right 60° and a vertical translation down 2

b. an amplitude of 3 and a vertical translation up 6

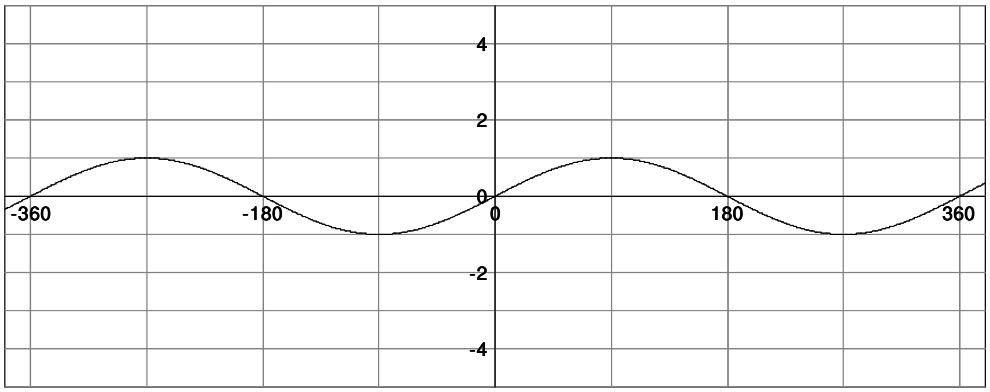
c. an amplitude of ½, a period of 180° and a vertical translation down 1

12. Sketch each of the following functions.

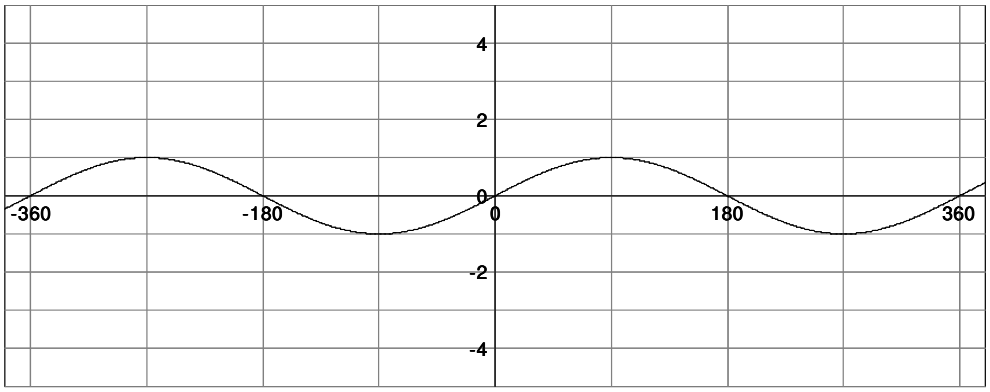
a. $y = 3\sin(\theta - 20^\circ)$



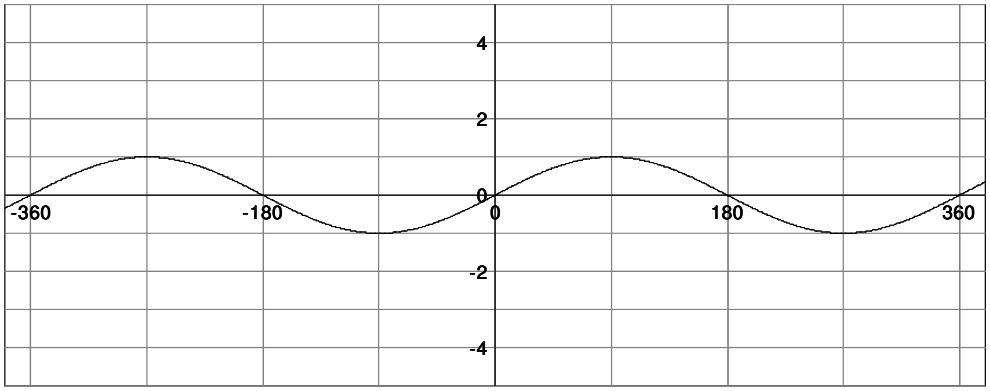
b. $y = -\sin(\theta - 45^\circ) + 2$



c. $y = 2\sin 4\theta - 3$



d. $y = -3\sin(\theta + 90^\circ) - 1$



13. The table below shows how the number of hours of daylight observed at Trois-Rivières, Québec varies over several days. Trois-Rivières is located at 72º 33' W and 46º 21' N. Day 0 represents January 1st.

Day Number	# of hrs of daylight
0	9.84
31	10.72
59	12.08
90	13.79
120	15.45
151	16.79
181	17.04

Day Number	# of hrs of daylight
212	16.02
243	14.39
273	12.74
304	11.15
334	10.03
365	9.82

a. Draw a scatter plot of the data and then sketch the curve of best fit.

b. What is the possible domain for this situation, if the data continued for many days?

c. What is the range of this function?

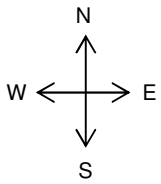
d. Write an equation for the axis of the curve.

e. On the next season of the Amazing Race, they want to run a detour in Trois-Rivières. The detour will require exactly 16 hours of daylight. When should they plan to have the teams arrive in Trois-Rivières? Express your answer(s) in terms of month, i.e. early January, mid February, late March, etc. Explain your reasoning.

14. A clock pendulum swings back and forth with the amplitude of 1 cm.
    a. In order to generate a sine curve, what measurements should be used for the horizontal and vertical axes?

    b. Explain why the sine function would be an appropriate model.

15. Two boats depart from the same dock in a marina. A sailboat sails at 12 km/h in a direction of 15° west of north. At the same time, a powerboat travels 10 km/h in a direction of 26° east of north. How far apart are the boats after 4 hours?

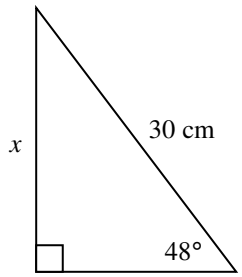


Strand Review: Trigonometry

Answer each of the following questions in the space provided. Show all steps of work.

1. Find the missing angle or side specified for each of the following.

a. Solve for x.

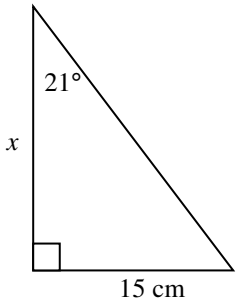


$\sin 48^\circ = \frac{x}{30}$

$30 \sin 48^\circ = x$

$22.3_{cm} \doteq x$

b. Solve for x.



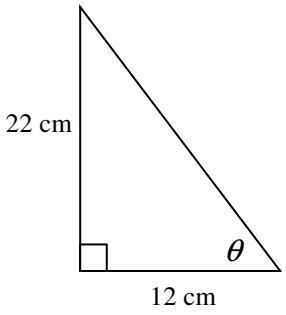
$\tan 21^\circ = \frac{15}{x}$

$x \tan 21^\circ = 15$

$x = \frac{15}{\tan 21^\circ}$

$x = 39.1\,cm$

c. Solve for θ.



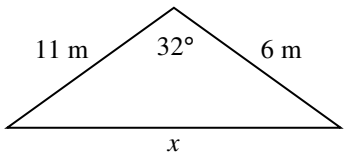
$\tan\theta = \frac{22}{12}$

$\theta = \tan^{-1}\left(\frac{22}{12}\right)$

$\theta = 61^\circ$

2. Find the missing angle or side specified for each of the following.

a. Solve for x.



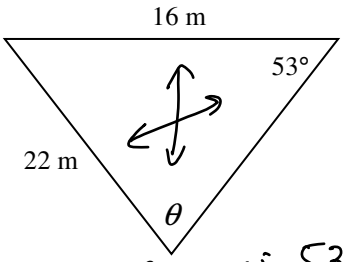
SAS

$x^2 = 11^2 + 6^2 - 2(11)(6)\cos 32^\circ$

$x^2 = 45.06$

$x \doteq 6.7\,m$

b. Solve for θ.



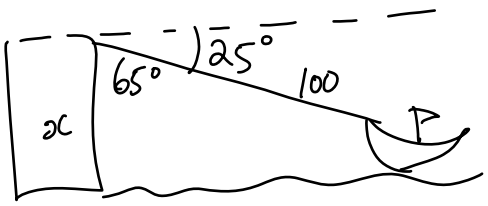
$\frac{\sin\theta}{16} = \frac{\sin 53}{22}$

$\sin\theta = \frac{16 \sin 53}{22}$

$\theta = \sin^{-1}(0.5808)$

$\theta = 36^\circ$

3. The angle of depression from the top of castle to a boat is 25°. If the distance from the top of the castle to the boat is determined to be 100 m, how high is the castle?



$\cos 65^\circ = \frac{x}{100}$

$100 \cos 65^\circ = x$

$42.3 \doteq x$

∴ castle is 42.3 m tall.

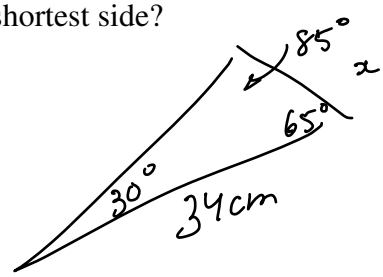
4. A ski run reaches a height of 250 m along a hill 1600 m long. What interior angle is formed where the bottom of the hill meets the horizontal?

$\sin\theta = \frac{250}{1600}$

$\theta = \sin^{-1}\left(\frac{250}{1600}\right)$

$\theta = 9^\circ$ is the interior angle

5. Two angles of a triangle are 30° and 65° and the longest side is 34 cm. How long is the shortest side?

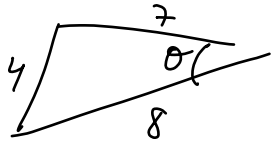


$\frac{x}{\sin 30^\circ} = \frac{34}{\sin 85}$

$x = \frac{34 \sin 30^\circ}{\sin 85^\circ}$

$x \doteq 17.1$ cm

6. Find the measure, to the nearest degree, of the smallest angle in a triangle with sides 4 m, 7 m, and 8 m.

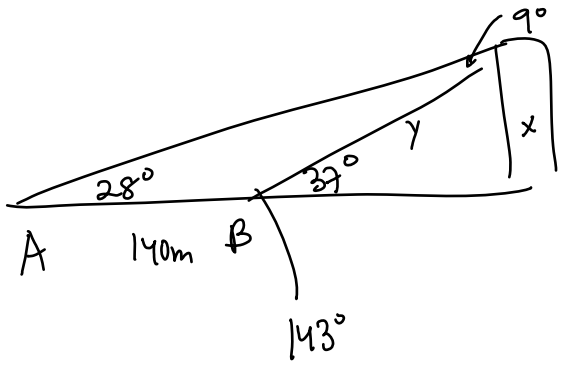


$\cos\theta = \frac{7^2 + 8^2 - 4^2}{2(7)(8)}$

$\cos\theta = \frac{97}{112}$

$\theta = 30^\circ$

7. Two points, A and B, are on the same side of a tower. The angle of elevation of the top of the tower form point A is 28°. Point B is closer to the tower, 140 m from point A. The angle of elevation of the top of the tower from point B is 37°. Determine the height of the tower.



find y

$\frac{y}{\sin 28^\circ} = \frac{140}{\sin 9^\circ}$

$y = \frac{140 \sin 28^\circ}{\sin 9^\circ}$

$y \doteq 420.2\text{m}$

find x

SOH $\sin 37^\circ = \frac{x}{420.2}$

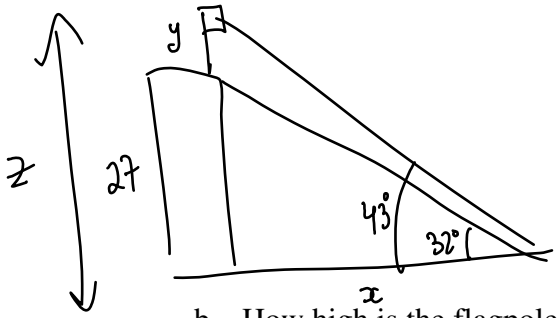
$420.2 \sin 37^\circ = x$

$252.9\text{m} \doteq x$

∴ tower is 252.9m tall

8. A flagpole stands on top of a 27 m building. From a point on the ground some distance away, the angle of elevation of the top of the flagpole is 43°. The angle of elevation of the bottom of the flagpole is 32°.

a. How far is it from the point on the ground to the bottom of the building?



$\tan 32^\circ = \frac{27}{x}$

$x \tan 32 = 27$

$x = \frac{27}{\tan 32}$

$x \doteq 43.2\text{m}$ away from building

b. How high is the flagpole?

many ways to do this

← easiest method

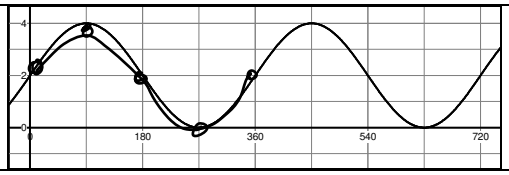
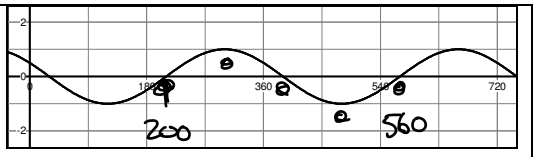
$\tan 43^\circ = \frac{z}{43.2}$

$43.2 \tan 43^\circ \doteq z$

$40.3 \doteq z$

∴ Flagpole is $40.3 - 27 = 13.3\text{m}$ tall

9. Complete the chart for each graph.

		
max & min values	MAX=4 MIN=0	Max=1 MIN=-1
axis of the curve	$\frac{4+0}{2}=2$	$\frac{-1+1}{2}=\frac{0}{2}=0$
amplitude	2	1
Phase shift	1st point = sine starts at axis then goes up $=0^\circ$	200°
period	$360^\circ-0^\circ=360^\circ$	$560^\circ-200^\circ=360^\circ$
equation of the function	$y=2\sin[1(x-0^\circ)]+2$	$y=1\sin[1(x-200^\circ)]+0$
	$y=2\sin x+2$	$y=\sin(x-200^\circ)$

$c=$ $a=$ $d=$ $K=\frac{360^\circ}{360^\circ}$ $K=1$

10. Complete the chart for each function.

	$-3\sin(2(\theta+90))-4$	$y=\frac{1}{3}\sin 3\theta-5$	$y=\sin 5(\theta-30^\circ)+7$
amplitude	3	$\frac{1}{3}$	1
period	$\frac{360^\circ}{2}=180^\circ$	$\frac{360^\circ}{3}=120^\circ$	$\frac{360^\circ}{5}=72^\circ$
phase shift	-90° (left)	0°	30° (right)
vertical translation	-4 (down)	-5 (down)	7 (up)

11. Write an equation to represent each of the transformations of the function $y=\sin\theta$.

a. a reflection in the x-axis, a phase shift right 60° and a vertical translation down 2

$$y=-\sin(\theta-60^\circ)-2$$

b. an amplitude of 3 and a vertical translation up 6

$$y=3\sin\theta+6$$

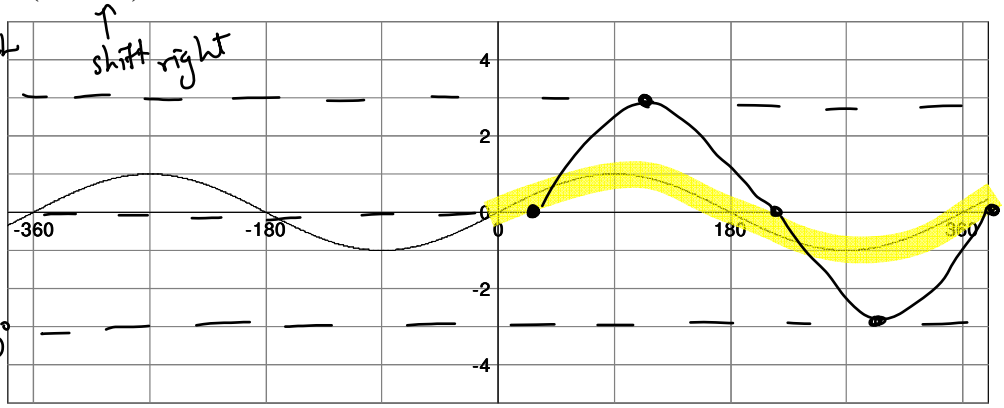
c. an amplitude of ½, a period of 180° and a vertical translation down 1

$K=\frac{360^\circ}{180^\circ}=2$

$$y=\frac{1}{2}\sin[2\theta]-1$$

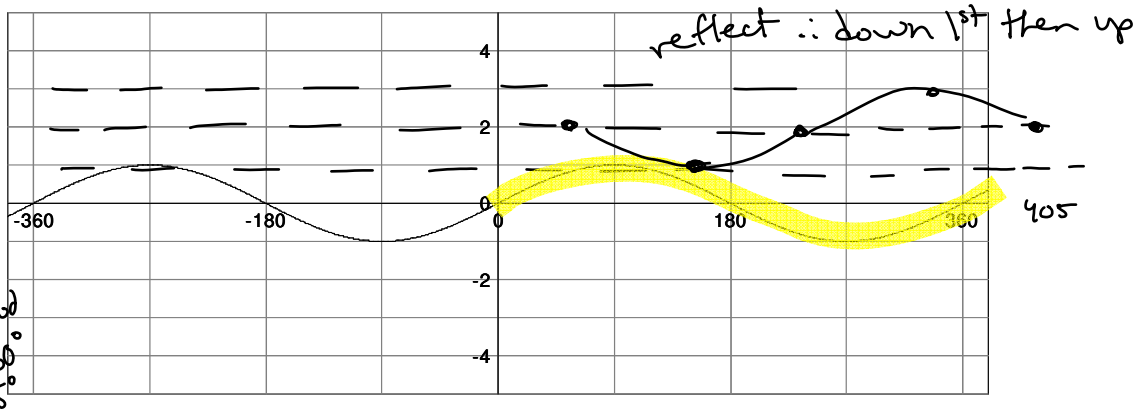
12. Sketch each of the following functions.

a. $y = 3\sin(\theta - 20^\circ)$



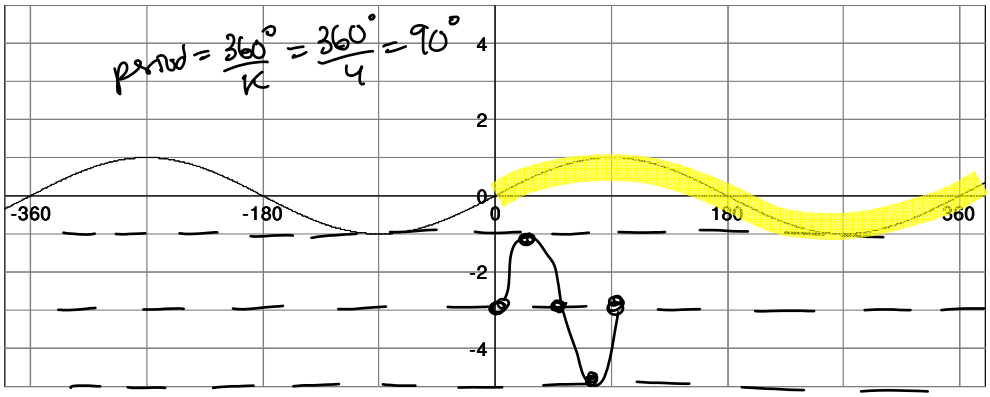
MAX = 3
C = 0
MIN = -3
1st pt = d = 20°
last pt = d + period
= 20° + 360°
= 380°

b. $y = -\sin(\theta - 45^\circ) + 2$



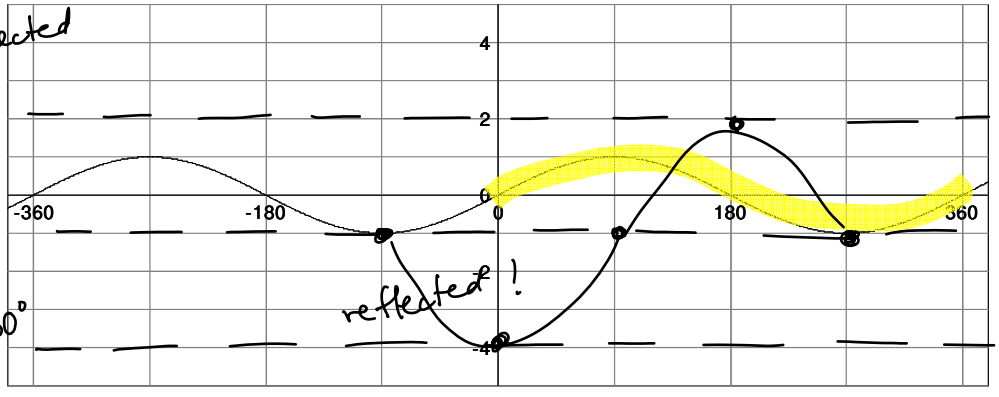
reflected
MAX = 2 + 1 = 3
C = 2
MIN = 2 - 1 = 1
1st pt = d = 45°
last pt = d + period
= 45° + 360°
= 405°

c. $y = 2\sin 4\theta - 3$



MAX = -3 + 2 = -1
C = -3
MIN = -3 - 2 = -5
1st pt = d = 0°
last pt = d + period
= 0° + 90°
= 90°

d. $y = -3\sin(\theta + 90^\circ) - 1$



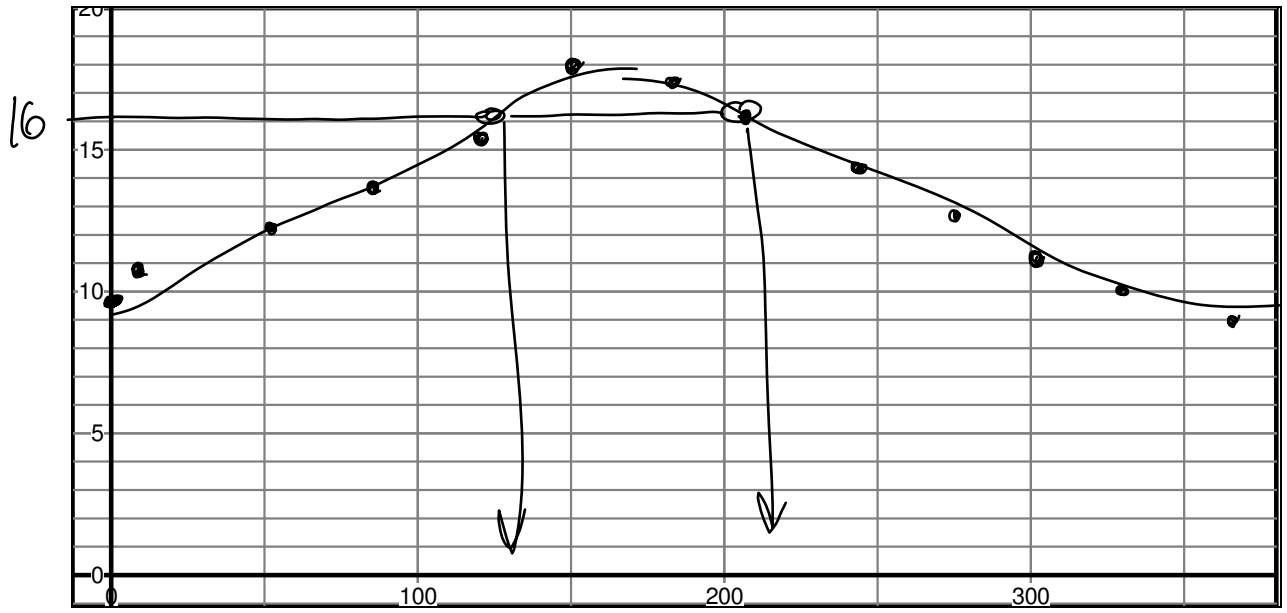
MAX = -1 + 3 = 2
C = -1
MIN = -1 - 3 = -4
1st pt = -90° =
last pt = -90° + 360°
= 270°

13. The table below shows how the number of hours of daylight observed at Trois-Rivières, Québec varies over several days. Trois-Rivières is located at 72º 33' W and 46º 21' N. Day 0 represents January 1st.

Day Number	# of hrs of daylight
0	9.84
31	10.72
59	12.08
90	13.79
120	15.45
151	16.79
181	17.04

Day Number	# of hrs of daylight
212	16.02
243	14.39
273	12.74
304	11.15
334	10.03
365	9.82

a. Draw a scatter plot of the data and then sketch the curve of best fit.



b. What is the possible domain for this situation, if the data continued for many days?

day # = d $\quad 0 \leq d \leq 365$

c. What is the range of this function?

# of Daylight hrs = h $\quad 9.82 \leq h \leq 17.04$

d. Write an equation for the axis of the curve.

$\frac{9.82 + 17.04}{2} = \frac{26.86}{2} = 13.43$

e. On the next season of the Amazing Race, they want to run a detour in Trois-Rivières. The detour will require exactly 16 hours of daylight. When should they plan to have the teams arrive in Trois-Rivières? Express your answer(s) in terms of month, i.e. early January, mid February, late March, etc. Explain your reasoning.

Begin of May ↙ end of June

between days 125 and 205

Jan=31 Feb=28 Mar=31 Apr=30 May=31 June=30 July=31

14. A clock pendulum swings back and forth with the amplitude of 1 cm.

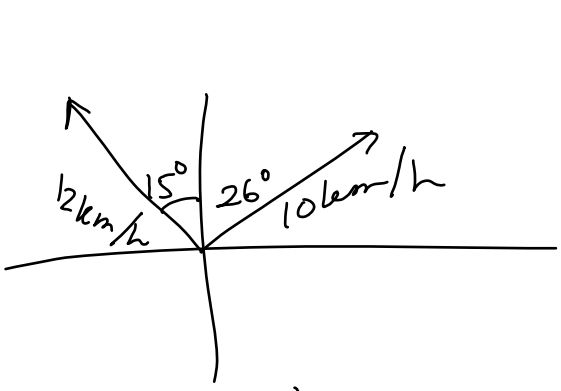
a. In order to generate a sine curve, what measurements should be used for the horizontal and vertical axes?

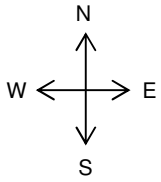
horizontal axis = time
vertical axis = horiz. distance from centre

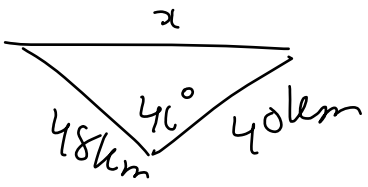
b. Explain why the sine function would be an appropriate model.

Sine is a wave with MAX + MIN and pendulum goes to the right up to MAX then to perfect horiz. then to left MIN then back ... etc

15. Two boats depart from the same dock in a marina. A sailboat sails at 12 km/h in a direction of 15° west of north. At the same time, a powerboat travels 10 km/h in a direction of 26° east of north. How far apart are the boats after 4 hours?







after 4 hrs distance graph is

$12 \times 4 = 48$

$10 \times 4 = 40$

$x^2 = 48^2 + 40^2 - 2(48)(40)\cos 46^\circ$

$x^2 \doteq 1005.9\ldots$

$x \doteq 31.7$ km apart