

NOTESallANS

Look below for ALL
answers to notes - if you find mistakes, let me know

Trigonometry Unit

Tentative TEST date _____



Reflect – previous TEST mark _____, Overall mark now _____.
Looking back, what can you improve upon?



Big idea/Learning Goals

In this unit you will start with review of primary trigonometric ratios – SOH CAH TOA, and apply them in word problems. SOH CAH TOA and Pythagorean Theorem only work on right (90°) triangles, to work with non-right triangles you will learn Sine and Cosine Laws. (If you took academic grade 10 course – all of this is review, if you took applied course, SOH CAH TOA should be familiar but the laws are new to you.)



Success Criteria

- ☐ I am ready for this unit if I am confident in the following review topics
(circle the topics you are good at & review the ones you left undone before you get too far behind)
SOH CAH TOA, Pythagorean theorem, angles of elevation and depression, simplifying expressions, solving equations
- ☐ I understand the new topics for this unit if I can do the practice questions in the textbook/handouts
(check off the topics for which you have finished the practice)

Date	Topics	Done?
	SOH CAH TOA Section 5.1 p271 #1,2,3,4,5,8,11 & EXTRA Handout	
	Problem Solve with right triangles Section 5.2 p280 #1,6,9,10,13	
	If there is time - Group Presentations	
	Sine Law Section 5.3 p288 #4,5,7,9abc	
	Cosine Law Section 5.4 p299 #2,3,8,10	
	Problem Solve - group Activity Section 5.5 p309 #5,8,9,11,12,14	

- ☐ I am prepared for the test/evaluation if
- ☐ I understand the main concepts from each lesson
 - If not, ask other students in class to help you study or visit the peer tutoring room or ask the teacher for help or get a private tutor
 - also practice "knowledge-understanding" questions from the textbook – look for questions marked by **K**
 - ☐ I can explain/communicate the ideas clearly
 - If not, practice explaining a solved question to someone else or complete the assigned Journal questions
 - also practice "communication" questions from the textbook – look for questions marked by **C**
 - ☐ I can apply these concepts in word problems
 - If not, practice "application" questions from the textbook – look for questions marked by **A**
 - ☐ I did not just memorize steps to do for different types of questions, I understand the ideas behind each concept and therefore can do problems in new contexts
 - If not, practice "thinking-inquiry-problem-solving" questions from the textbook – look for questions marked by **T**
 - ☐ I can do questions independently
 - If not, try redoing an already solved example without looking at solutions
 - ☐ I can complete questions quickly and with confidence
 - If not, try timing yourself for similar type questions to see progress
 - ☐ I completed the review and/or practice test

Corrections for the textbook answers:

SOH CAH TOAuse θ "theta" symbol for angles

1. It is a good idea to start by review the ^{ing} **primary trigonometric ratios**, or SOH CAH TOA, and Pythagorean Theorem from grade 10. Summarize what you should know:

SOH stands for $\sin \theta = \frac{\text{opp}}{\text{hyp}}$ ^{angle goes here} CAH: $\cos \theta = \frac{\text{adj}}{\text{hyp}}$ TDA: $\tan \theta = \frac{\text{opp}}{\text{adj}}$



Pythagorean: $a^2 + b^2 = c^2$ ^{hypotenuse}

2. Try $\sin 90^\circ = 1$ $\cos 90^\circ = 0$ $\tan 90^\circ = \text{error}$. What can you conclude about what angle cannot be used when solving triangles with SOH CAH TOA.

Don't ever use 90° when doing SOH CAH TOA, always choose an acute angle

3. Summarize how and when to round decimals.

→ Round Ratios (ex. $\sin 45^\circ$ or $\frac{3}{4}$) to 4 decimal places

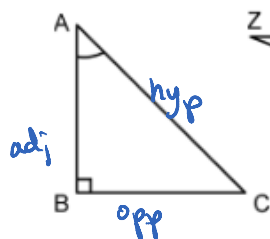
→ Round side lengths to one decimal place

→ Round angles to whole numbers. (unless otherwise requested.)

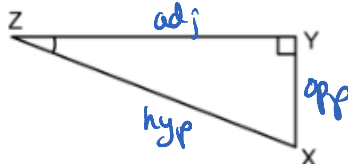
4. Practice labelling triangles using side names: opposite, adjacent, hypotenuse.



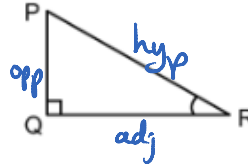
a)



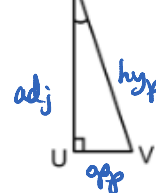
b)



c)



d)



5. Practice using your calculator. MAKE SURE YOUR CALCULATOR IS IN DEGREE MODE

Given the angle find the ratio

a) $\sin 45^\circ \doteq 0.7071$

b) $\cos 98^\circ \doteq -0.1392$

c) $\tan 4^\circ = 0.0699$

d) $\cos 76^\circ = \frac{3}{x}$ ^{adj} _{hyp side}

$x \cos 76^\circ = 3$

$x = \frac{3}{\cos 76^\circ}$

$x \doteq 12.4$

ratios round to 4 decimals

round to one decimal

Given the ^{angle} find the angle (use the SHIFT or 2^{ND} buttons)

e) $\sin A = 0.557$

$A = \sin^{-1}(0.557)$

$A = 34^\circ$

f) $\cos C = 0.705$

$C = \cos^{-1}(0.705)$

$C = 45^\circ$

g) $\tan B = 2.984$

$B = \tan^{-1}(2.984)$

$B = 71^\circ$

h) $\sin x = \frac{5}{6}$

$x = \sin^{-1}\left(\frac{5}{6}\right)$

$x = 56^\circ$

need to see this step shows function notation

angles round to whole #.

Note: \sin^{-1} and \cos^{-1} will not work for #'s bigger than 1



6. Summarize the steps of solving right triangles.

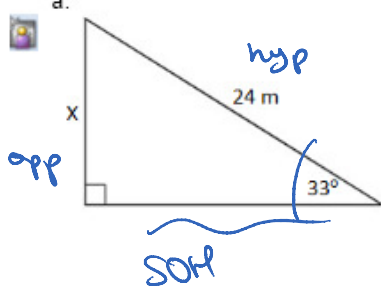
Distinguish how to solve angles and how to solve sides and when to use Pythagorean theorem.

→ If there are no angles in the question use Pythagorean Th.

→ If there are angles use SOH CAH TOA

steps: (1) Label two of the three sides as opp/adj/hyp
 (2) Decide which ratio will work SOH/CAH/TOA
 (3) For sides → cross multiply
 For angles → use \sin^{-1} / \cos^{-1} / \tan^{-1} buttons.

7. Solve the following for X.

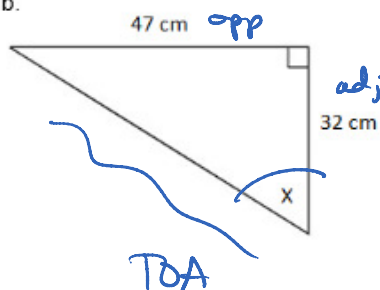


$$\frac{\sin 33^\circ}{1} = \frac{x}{24}$$

$$24 \sin 33^\circ = x$$

$$13.1 \text{ m} = x$$

b.

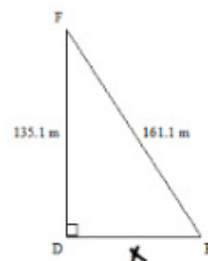


$$\tan X = \frac{47}{32}$$

$$X = \tan^{-1}\left(\frac{47}{32}\right)$$

$$X = 56^\circ$$

c.



no angles!

$$a^2 + b^2 = c^2$$

$$x^2 + 135.1^2 = 161.1^2$$

$$x^2 + 18252.01 = 25953.21$$

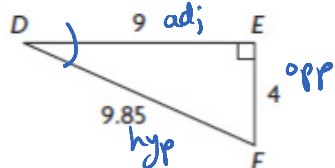
$$x^2 = 7701.2$$

$$x = \pm 87.8$$

choose positive only since length can't be neg.



8. Sometimes there is a choice of what ratio to use. Find all three ratios for the triangle and then find the angle D using more than one ratio. Discuss the fastest way to find the 3rd angle if you know the other two.



$$\sin D = \frac{4}{9.85} \rightarrow D = \sin^{-1}\left(\frac{4}{9.85}\right)$$

$$D = 24^\circ$$

$$\cos D = \frac{9}{9.85} \rightarrow D = \cos^{-1}\left(\frac{9}{9.85}\right)$$

$$D = 24^\circ$$

$$\tan D = \frac{4}{9} \rightarrow D = \tan^{-1}\left(\frac{4}{9}\right)$$

$$D = 24^\circ$$

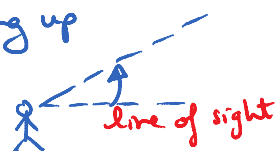
∴ if you're given all 3 sides it doesn't matter what ratio you use.

To Find 3rd angle fastest way is → $F = 180^\circ - 90^\circ - 24^\circ = 66^\circ$

9. Summarize what you should know about the terms angle of elevation and angle of depression.

angle of elevation

looking up



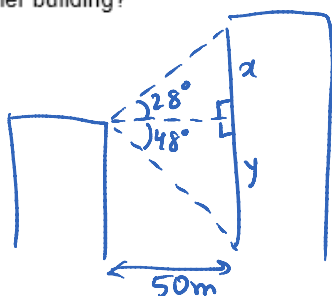
angle of depression

looking down



* always draw the horizontal line of sight first !!

10. From the top of a building, the angle of elevation of the top of a nearby building is 28° and the angle of depression of the bottom of the nearby building is 48° . The distance between the two buildings is 50 m. What is the height of the taller building?



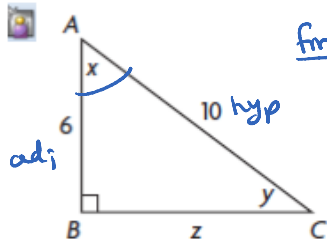
$$\begin{aligned} \frac{28^\circ}{50} & \quad x \\ \text{TOA} \\ \tan 28^\circ &= \frac{x}{50} \\ 50 \tan 28^\circ &= x \\ 26.6 &= x \end{aligned}$$



$$\begin{aligned} \text{TOA} \\ \tan 48^\circ &= \frac{y}{50} \\ 50 \tan 48^\circ &= y \\ 55.5 &= y \end{aligned}$$

\therefore the height of the taller building is $26.6 + 55.5 = 82.1$ metres.

11. Find all the unknown sides and angles.



find x

CAH

$$\cos x = \frac{6}{10}$$

$$x = \cos^{-1}\left(\frac{6}{10}\right)$$

$$x = 53^\circ$$

find y

$$y = 180^\circ - 53^\circ - 90^\circ$$

$$y = 37^\circ$$

find z

$$a^2 + b^2 = c^2$$

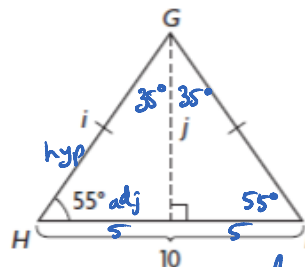
or SOH CAH TOA again

$$6^2 + z^2 = 10^2$$

$$36 + z^2 = 100$$

$$z^2 = 64$$

$$z = 8$$



angles are easy...

since isosceles, base is cut in half.

find i

CAH

$$\cos 55 = \frac{5}{i}$$

$$i(\cos 55) = 5$$

$$i = \frac{5}{\cos 55}$$

$$i = 8.7 \text{ units}$$

find j TOA or Pythag.

$$\tan 55 = \frac{j}{5}$$

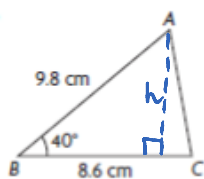
$$j \tan 55 = 5$$

$$j = \frac{5}{\tan 55}$$

$$j = 3.5 \text{ units}$$

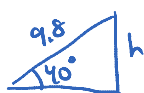
Problem Solve with right triangles

1. Find the area of non right triangles

a. 

$$A = \frac{1}{2}bh$$

Need to find height!



don't know
that side
(less than 8.6
∴ don't use it)

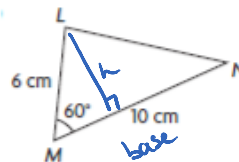
$$\text{SOH} \quad \frac{\sin 40^\circ}{1} \times 9.8 = h$$

$$9.8 \sin 40^\circ = h$$

$$6.3 \doteq h$$

$$\therefore A = \frac{1}{2}(8.6)(6.3)$$

$$A = 27.09 \text{ cm}^2$$

b. 

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(10)(5.2)$$

$$= 26 \text{ cm}^2$$

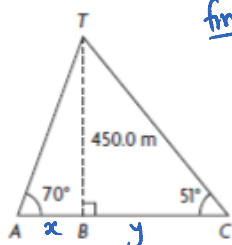
find h SOH

$$\sin 60^\circ = \frac{h}{6}$$

$$6 \sin 60^\circ = h$$

$$5.2 \doteq h$$

2. A mountain is 450 m high. From points A and C, the angles of elevation to the top of the mountain are
- 70°
- and
- 51°
- as shown. Calculate the length of the tunnel from A to C.



find x TOA

$$\frac{\tan 70^\circ}{1} \times \frac{450}{x}$$

$$x(\tan 70^\circ) = 450$$

$$x = \frac{450}{\tan 70^\circ}$$

$$x \doteq 163.8 \text{ m}$$

find y TOA

$$\frac{\tan 51^\circ}{1} \times \frac{450}{y}$$

$$y(\tan 51^\circ) = 450$$

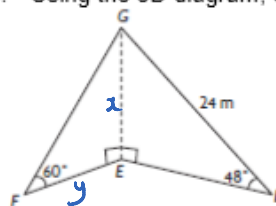
$$y = \frac{450}{\tan 51^\circ}$$

$$y \doteq 364.4$$

∴ distance from A to C

$$\text{is } 364.4 + 163.8 = 528.2 \text{ m}$$

3. Using the 3D diagram, calculate EG and FE.



find x SOH

$$\tan 48^\circ = \frac{x}{24}$$

$$24 \tan 48^\circ = x$$

$$26.7 \text{ m} \doteq x$$

find y TOA

$$\frac{\tan 60^\circ}{1} \times \frac{26.7}{y}$$

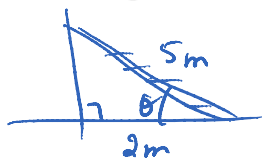
$$y \tan 60^\circ = 26.7$$

$$y = \frac{26.7}{\tan 60^\circ}$$

$$y \doteq 15.4 \text{ m}$$



4. A 5-m ladder is resting against a wall. The base of the ladder is 2 m along the ground from the base of the wall. What angle does the base of the ladder make with the ground? Express your answer to the nearest tenth of a degree.



CAH

$$\cos \theta = \frac{2}{5}$$

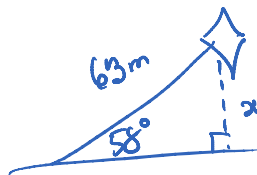
$$\theta = \cos^{-1}\left(\frac{2}{5}\right)$$

$$\theta \approx 66.4^\circ$$

\therefore the ladder makes 66.4° with the ground



5. Jason is flying his kite. He lets out 63 m of string and the wind takes his kite up to a point where the angle of elevation of the kite is 58° . Find the height of the kite to the nearest metre.



SOM

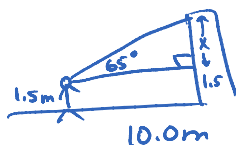
$$\sin 58^\circ = \frac{x}{63}$$

$$63 \sin 58 = x$$

$$53.4 = x$$

\therefore altitude of the kite is approx 53m.

6. Michael stands 10.0 m from the base of a building. He measures the angle of elevation to the top of the building to be 65.0° . Michael's measurement was made from 1.5 m above the ground. Determine the height of the building to the nearest metre.



TOA

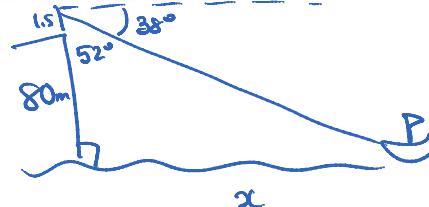
$$\tan 65 = \frac{x}{10}$$

$$10(\tan 65) = x$$

$$21.4 \approx x$$

\therefore height of the building is $21.4 \text{ m} + 1.5 \text{ m} \approx 23 \text{ meters}$

7. The highest point along a cliff is 80 m above the lakeshore. A surveyor stands on the top of the cliff, looking through a 1.5 m tall transit instrument. He spots a boat out on the lake, at an angle of depression of 38° . How far, to the nearest tenth of a metre, is it from the boat to the base of the cliff?



TOA

$$\tan 52^\circ = \frac{x}{81.5}$$

$$81.5 \tan 52^\circ = x$$

$$104.3 \approx x$$

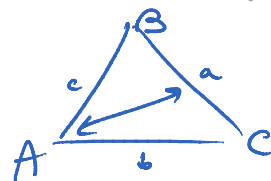
\therefore the boat is 104.3m away from the cliff.

Sine Law

1. It is important to know how triangles are labelled. Summarize when to use small case letters and when capital letters.

Small case letters - used for side lengths
Capital letters - used for angles

* same letters use across from each other as shown:



2. The Sine Law is:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

3. Explain when to use the sine law. (remember if 2 angles are given you can always find the 3rd one)

Use sine law if you have a pair of opposite side + angle

First: if have 2 angles, subtract from 180° to find 3rd angle

4. Practice using algebra and your calculator to solve for the unknown.



a.

$$\frac{\sin 21^\circ}{x} = \frac{\sin 48^\circ}{20}$$

$$20 \sin 21^\circ = x \sin 48^\circ$$

$$\frac{20 \sin 21^\circ}{\sin 48^\circ} = x$$

$$9.2 = x$$

c.

$$\frac{10.2}{\sin 35^\circ} = \frac{6.8}{\sin \theta}$$

$$10.2(\sin \theta) = 6.8(\sin 35^\circ)$$

$$\sin \theta = \frac{6.8(\sin 35^\circ)}{10.2}$$

$$\sin \theta = 0.3824$$

$$\theta = \sin^{-1}(0.3824)$$

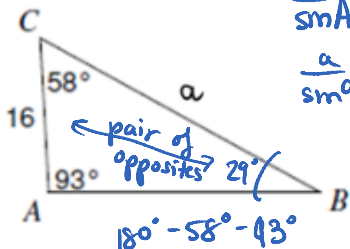
$$\theta = 22^\circ$$

not needed

5. Solve for the unknown side.



a.



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

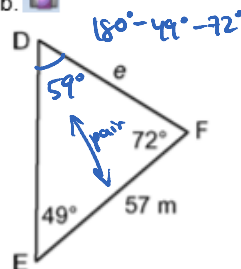
$$\frac{a}{\sin 93^\circ} = \frac{16}{\sin 29^\circ}$$

$$a = \frac{16(\sin 93^\circ)}{\sin 29^\circ}$$

$$a \approx 33.0 \text{ units}$$



b.




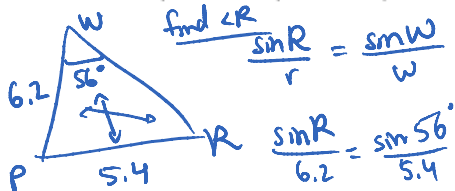
$$\frac{e}{\sin E} = \frac{d}{\sin D}$$

$$\frac{e}{\sin 49^\circ} = \frac{57}{\sin 72^\circ}$$

$$e = \frac{57 \sin 49^\circ}{\sin 72^\circ}$$

$$e \approx 50.2 \text{ m}$$

- 6.
- 
- Solve the triangle means

find all sides + all anglesSolve $\triangle PWR$, $w=5.4$ cm, $r=6.2$ cm, and $\angle W = 56^\circ$.

$$\text{find } \angle R \quad \frac{\sin R}{r} = \frac{\sin W}{w}$$

$$\frac{\sin R}{6.2} = \frac{\sin 56^\circ}{5.4}$$

$$\sin R = \frac{6.2 \sin 56^\circ}{5.4}$$

$$R = \sin^{-1}(0.9518)$$

$$R = 72^\circ$$

find $\angle P$

$$\angle P = 180^\circ - 72^\circ - 56^\circ$$

$$\angle P = 52^\circ$$


find side p

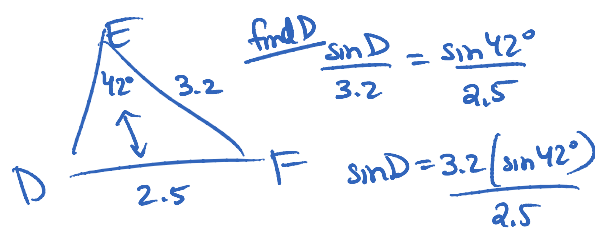
$$\frac{p}{\sin 52^\circ} = \frac{5.4}{\sin 56^\circ}$$

$$p = \frac{5.4 (\sin 52^\circ)}{\sin 56^\circ}$$

$$p = 5.1 \text{ cm}$$

$\angle R = 72^\circ$
 $\angle P = 52^\circ$
 $p = 5.1 \text{ cm}$ Solves all
 the unknowns...

- 7.
- 
- Solve
- $\triangle DEF$
- ,
- $d=3.2$
- m,
- $e=2.5$
- m,
- $\angle E = 42^\circ$



$$\text{find } D \quad \frac{\sin D}{3.2} = \frac{\sin 42^\circ}{2.5}$$

$$\sin D = \frac{3.2 (\sin 42^\circ)}{2.5}$$

$$D = \sin^{-1}(0.8565)$$

$$D = 59^\circ$$

find $\angle F$

$$F = 180^\circ - 59^\circ - 42^\circ$$

$$F = 79^\circ$$

find f

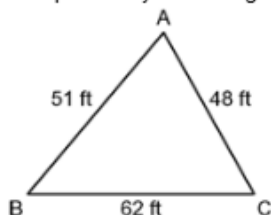
$$\frac{f}{\sin 79^\circ} = \frac{2.5}{\sin 42^\circ}$$

$$f = \frac{2.5 (\sin 79^\circ)}{\sin 42^\circ}$$

$$f = 3.7 \text{ m}$$

Cosine Law

1. Explain why the triangle cannot be solved by sine law.



Don't have a pair of opposites needed by sine law.



2. The Cosine Law is:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

or

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

3. Explain when to use the cosine law.

use when given SAS and looking for 3rd side



use when given SSS and looking for an angle.



4. Practice rewriting the formulas with different letters

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$x^2 = y^2 + z^2 - 2yz \cos X$$

$$\cos X = \frac{y^2 + z^2 - x^2}{2yz}$$

5. Different calculators require you to press buttons in slightly different order. Learn how your calculator works and remember it for the test. Practice using your calculator to get the correct answers.

a. $a^2 = (40)^2 + (25)^2 - 2(40)(25) \cdot \cos 20^\circ$

$$a^2 = 345.614758 \dots$$

$$a \approx 18.6$$

b. $c^2 = (10)^2 + (9)^2 - 2(10)(9) \cdot \cos 66^\circ$

$$c^2 = 107.78740 \dots$$

$$c \approx 10.4$$

c. $\cos B = \frac{12^2 + 14^2 - 11^2}{2(12)(14)}$

$$\cos B = \frac{219}{336}$$

$$B = \cos^{-1}\left(\frac{219}{336}\right)$$

$$B \approx 49^\circ$$

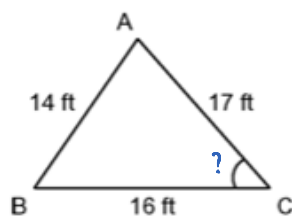
d. $\cos Y = \frac{8^2 + 10^2 - 9^2}{2(8)(10)}$

$$\cos Y = \frac{83}{160}$$

$$Y = \cos^{-1}\left(\frac{83}{160}\right)$$

$$Y \approx 59^\circ$$

6. Find the missing angle or side.

a. 

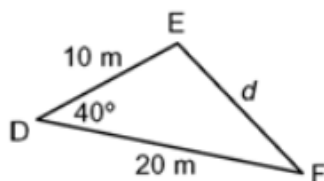
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{16^2 + 17^2 - 14^2}{2(16)(17)}$$

$$\cos C = \frac{349}{544}$$

$$C = \cos^{-1}\left(\frac{349}{544}\right) = 50^\circ$$

c.

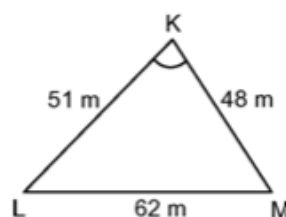


$$d^2 = e^2 + f^2 - 2ef \cos D$$

$$d^2 = 20^2 + 10^2 - 2(20)(10) \cos 40^\circ$$

$$d^2 = 193.58222 \dots$$

$$d = 13.9 \text{ m}$$

b. find $\angle K$

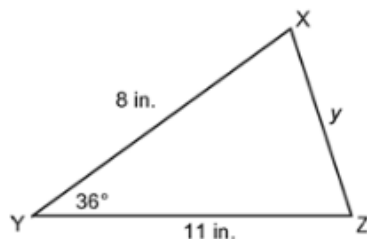
$$\cos K = \frac{m^2 + l^2 - k^2}{2ml}$$

$$\cos K = \frac{51^2 + 48^2 - 62^2}{2(51)(48)}$$

$$\cos K = \frac{1061}{4896} \quad K = \cos^{-1}\left(\frac{1061}{4896}\right)$$

$$K = 77^\circ$$

d.



$$y^2 = x^2 + z^2 - 2xz \cos Y$$

$$y^2 = 11^2 + 8^2 - 2(11)(8) \cos 36^\circ$$

$$y^2 = 42.61300899$$

$$y = 6.5 \text{ inches}$$

Problem Solve – group Activity*Instructions:*

1. Cut out word problems and diagrams. Match each word problem to the correct diagram.
2. On chart paper, create three categories of problem types: Primary Trig Ratios, Sine Law, and Cosine Law. Glue each problem/diagram in the most appropriate category to solve it.
3. For each word problem (i) set up the generalization used to solve it, and (ii) sub the given info in.
4. Solve problems.

<p>In Mexico, one of the Maya pyramids at Chichen Itza has stairs that rise about 64 cm for every 71 cm of run. Find the angle of rise of these stairs.</p>	<p>A golfer hits a tee shot on a 350 m long straight golf hole. The ball is sliced (hit at an angle) 21° to the right. The ball lands 210 m away from the tee. How far is the ball from the hole to the nearest metre?</p>	<p>Gale takes a slap shot that is tipped by Stephane, who is 8 m from Gale. If the puck travels 10.3 m in all and ends up 5° off from its original path, how much did the puck's direction change as a result of the tip?</p>
<p>Two helicopters flying at an altitude of 250 m are 2000 m apart when they spot a life raft below. The raft is directly between two helicopters. The angle of depression from one helicopter to the raft is 45°. The angle of depression from the other helicopter is 60°. Both helicopters are flying at 170 km/h. How long to the nearest second will it take the closer aircraft to reach the raft?</p>	<p>A communications tower 64 m tall has to be supported with cables running from the top of the tower to anchors in the ground on both sides of the tower. The cables must form an angle of 60°. How far from the base of the tower should the anchors be placed?</p>	<p>The police are trying to catch the operator of a pirate radio station. They park two vehicles with directional antennas on the side of a straight section of road 2 km apart. They then aim the antennas to find the pirate station's signal, which is coming from a location somewhere between them. From Constable Abel's vehicle, the angle is 70° and from Constable Baker's vehicle it is 30°. Which police officer is closer to the pirate radio station and how far away is it from the officer?</p>
<p>Jasmine is planning to do some rock climbing. Before she scales the cliff, she paces off 210 m from the base of the cliff and sights the top with a clinometer. The angle of elevation to the top is 21°. How high is the cliff?</p>	<p>A cottage under construction is to be 10.3 m wide. The two sides of the roof are to be equal and supported by rafters that meet at an angle of 45°. How long should the rafters be?</p>	<p>The posts of a hockey goal are 2 m apart. A player tries to score a goal by shooting the puck along the ice from a point 7.1 m from one post and 6.4 m from the other. Within what angle must the player shoot the puck?</p>

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<p>A radio tower is supported by two wires on opposite sides. The wires form an angle of 60° at the top of the post. On the ground, the ends of the wire are 15 m apart, and one wire is at a 45° angle to the ground. How long will the wires be?</p>	<p>A boat leaves Kingston and heads due east for 6.4 km. At the same time, a second boat travels in a direction 30° south of east from Kingston for 7.1 km. How far apart are the boats at this moment when they reach their destinations?</p>	<p>An airplane takes off from a runway near some mountains. The peak of the mountain is on the flight path 2.5 km from the end of the runway. The mountain is 2000 m high. What angle of ascent is needed to clear the mountain top?</p>
