NOTESalIANS

Look below for ALL answers to notes - if you find mistakes, let me know

1 l 11 n	it 6	11IIC D	ate:	Namer					
1101	111 0	HOCD	alc	Name:					
	Tontati	ve TES1	Γ data	Sinusoidal Functions Unit					
5000				EST mark, Overall mark now					
				you improve upon?					
?	Big idea/Learning Goals In this unit you will learn how trigonometry can be used to model wavelike relationships. These wavelike functions are called sinusoidal functions. You will study key properties of the sine function and use these properties to sketch these functions, to model real life situations and to solve problems. If you go on studying math you will learn that the cosine function is very similar to sine function, however the tangent function is very different.								
	Succ	·ess (riteria	3					
				his unit if I am confident in the following review topics					
		(circle the to	opics you are o Reading	good at & review the ones you left undroied before you get too far behind) graphs, simplifying expressions, solving equations, transformations, function nota	tion, domain & range				
		l under	stand the	e new topics for this unit if I can do the practice questions in the te	extbook/handouts				
				hich you have finished the practice) Topics	Done?				
			Date	CBR Investigation of Motion	Bollo:				
				Handout & EXTRA "Roll the Shape" Experiment Periodic Behaviour					
				Section 6.2 p331 #4,5,6,7,8 & EXTRA Handout					
				Sine Function Investigation Section 6.3 p339 #1,3,4,6,7,9 & EXTRA Handout					
				Comparing Sinusoidal Functions					
				Section 6.4 p348 #3,4,5,6,7 Translating/Shifting the Sine Function					
				Section 6.5 p365 #3,4,6,9,11					
				Stretching/Compressing the Sine Function Section 6.6 p375 #13,15,17,19 & Extra Handout					
				one EXTRA group Activity – if there is time one EXTRA Handout on KU APP COMM TIPS					
		l am pr	epared f	or the test/evalutation if					
				stand the main concepts from each lesson					
			:	If not, ask other students in class to help you study or visit the peer tutoring room or ask the teacher for help also practice "knowledge-understanding" questions from the textbook – look for questions marked by sc .	or get a private tutor				
				plain/communicate the ideas clearly					
			:	If not, practice explaining a solved question to someone else or complete the assigned journal questions also practice "communication" questions from the textbook – look for questions marked by c					
			I can ap	oply these concepts in word problems					
			l did no	If not, practice "application" questions from the textbook – look for questions marked by A t just memorize steps to do for different types of questions, I unde	erstand the ideas behin				
				oncept and therefore can do problems in new contexts					
			l can do	If not, practice "thinking-inquiry-problem-solving" questions from the textbook – look for questions marked by questions independently	ут				
			•		ут				

Corrections for the textbook answers:

CBR Investigation of Motion

The Calculator Based Ranger - CBR is a distance sensor that acts much the same as a submarine or bat to determine its location with respect to an object (it sends out sound waves then determines the time it takes for them to hit the object and bounce back, thus giving its location, or distance from the CBR).

CBR Set-up: (get program to work between calc and ranger) Calc: 2nd, Link, -> , Enter

ranger: 82/83

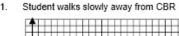
Program, Ranger, Enter (get into program)

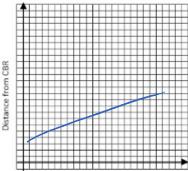
(get sample) Set up, up, start now, ... repeat sample (matching graphs) Applications, meters, dist.match



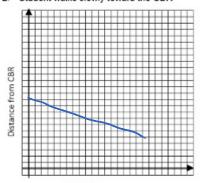
create a clear area for walking in front of CBR, hold a flat surface(ie book) in front yourself as you walk, walk steady, ignore jumps in the graph, the instrument is not very precise.

Sketch the graph first, then walk, then compare if your prediction was correct.

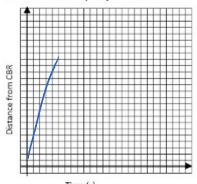




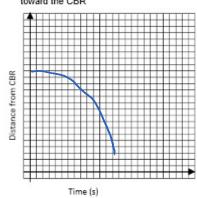
2. Student walks slowly toward the CBR



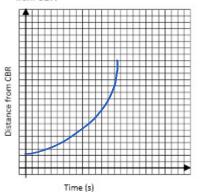
3. Students walks quickly toward the CBR



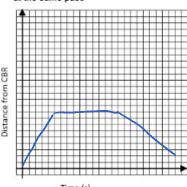
Time (s) Student increases speed while walking toward the CBR



Time (s) Student decreases speed while walking away



Time (s) 6. Student walks away from ranger, at 2 meters stops for 5 seconds, then returns



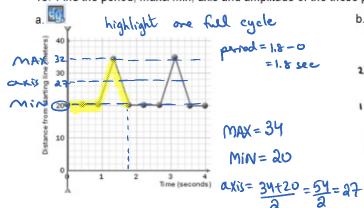
Do some match the graph activities on the CBR for practice, then answer the following questions.

3 Unit 6 11UC Date:		Name:
Describe how to create or explain why it is in 7.		
Walk away stop walk tower	s, fuster	walk toward walk toward
9. impossible can't trave time	back in [DON)	impossible con't appear suddenly futher away at the same time
11.	12.	
Stophers 20 - Stophers 20 - walk - wa	away the toward opent	-gradually charge of walk away then toward then toward then toward then toward then toward then toward walk constantly in a circle
The last two graphs are examples of period events that are regularly repeated, a complet the same way. The period is the interval of the arem_x_ values and troughs aremin values and the amplitude is half of the distance bet State the formulas that are important to know	te set of changes, starting from one he independent variable (usually tin /alues of the graph. The axis is the ween peaks and troughs	point and returning to the same point in ne) needed for one complete cycle. Peaks average value of the peaks and troughs,
		axis + amplitude (use vertical) MIN = axis - amplitude
Amplitude = (MAX-axis or = axis- Mix	Axis = MAX+ Min	
! ◀	Period - last pt - 1	pt. (horizontal values)

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amplitude = 1

13. Find the period, max& min, axis and amplitude of the these periodic functions.



MAX = 34 MIN = 20 last pt = 7

amplitude = 27-20

: prived = 2

MAX = 2.2 4.0 = Min 0xis = 2.2+0.2 = 3.4 = 1.2

🞆 14. What real life relationships can be represented by periodic functions?

anything that repeats in a cycle

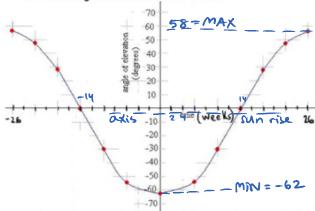
- breathing (volume in lungs)
- earth circling sun (seasons)
- earth circling on its axis (day + night)
- tides (high is low - caused by the moon)

-riding a Ferris wheel

Periodic Behaviour

There are many situations in real life that reapeat in cycles. For example, ocean tides, daylight hours, average temperature for the year, heartbeat, volume of air in lungs, rides on ferris wheels, pendulum swings, frequency of musical notes, the list can go on.

After the sun rises, its angle of elevation increases rapidly at first, then more slowly, reaching a maximum in 26 weeks. Then the angle decreases until sunset.



When does sunrise occur at this time of the year, for this particular spot on Earth?

after 14 weeks (North pole has 2 year day)
this maybe somewhere close to
North / South poles What is the period? What is longer the night or the day for this situation?

Night = 28 weeks perN=26+26=52 weeks Day = 24 weeks ... Night is longer

c. What is the axis? amplitude? range?

axis = 58+-62 amplitude = 60° Ronge fOLIR, -62° 40 458°

d. "Extrapolate the angle of elevation in 🕏 weeks and

read beyond given data -read within given data $\theta(30) = 48$ 8(30) = 48° 8(20) = 28° 0 (W)

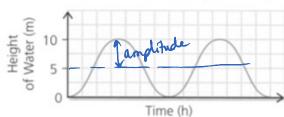
e. During what times is the angle of elevation of the sun above 30 degrees?

approximately between

30° at -20 weeks -> add provid = 52

and 20 weeks " between 20 weeks and 32 weeks the sun is higher than 300

2. The Bay of Fundy, which is between New Brunswick and Nova Scotia, has the highest tides in the world. There can be no water on the beach at low tide, while at high tide the water covers the beach.



Why can you use periodic functions to model the tides?

b. What is the change in depth of water from low tide to high tide?

10 meters ot a

Determine the equation of the axis of the curve.

$$axis = \frac{MAX + MiN}{2} = 0 + 10 = 5$$

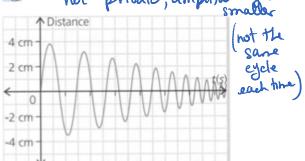
d. What is the amplitude of the curve?

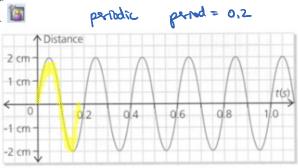
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3. Determine if the following represent periodic functions or not.

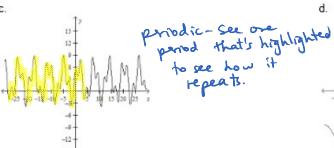


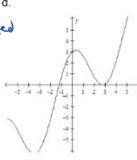
↑ Distance





C.





not periodic no constant MAX/MiN

e. Dependent = the horizontal distance travelled by the grandfather clock's pendulum

Independent = time

periodic-repeats the same way all the time - has MAX + MIN

f. Dependent = cost of riding a taxi Independent = distance travelled

not periodic - linear - the more you travel the more it cash

g. Dependent = interest on the money invested at 5% Independent = principal deposited

ust 1	periodic -	- lir	ear - Y	to more
You	deposit	, He	merk	intrest
0	•	•	you'	d get.

i.	x	у	
check ? se values ore in order	-3	7	XAM
x values	1	4	
ove in	5	1	MiM
Orac	9	4	
+ same	13	7	XAM
ship	17	4	
	21	1	WW

h. Dependent = the height of the pedal on a moving bicycle Independent = minutes

perodic - has MAX + MIN

j.	X	у
	0	-2
	2	3 WHX
144	4	0
	6	-2
	8	-4 MiN
	10	0
	12	3 MAX

symmetre : not die

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Sine Function Investigation



Introduction: In this investigation you are going to learn how the sine function you used to solve triangles in the last unit will relate to circular motion and how it creates wavelike graphs.

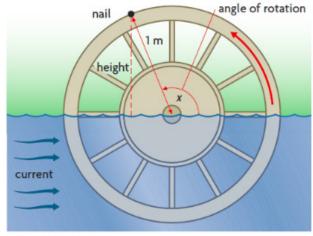
Equipment needed:

- Template of circle with degrees on it shown to be halfway submerged in water.
- Template of circle with a drawn nail on it to cut out
- Template of the ruler of the same size as the radius of the circle to cut out
- Cardboard to tape on it the circle submerged in water
- Thumbtack to pin the circle with nail to the centre of the circle submerged in water.
- Scientific calculator (with sin cos tan buttons)

Steve uses a generator powered by a water wheel to produce his own electricity.

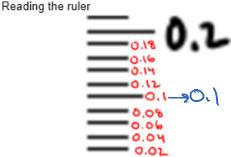
- · Half the water wheel is below the surface of the river.
- The wheel has a radius of 1 m.
- The wheel has a nail on its circumference.

As the current flows, the wheel rotates in a counterclockwise direction to power the generator. The height of the nail, relative to the water level, as the wheel rotates is graphed in terms of the angle of rotation, x.



Performing the Experiment:

- 1. Tape the circle that is shown to be halfway submerged in water onto the cardboard
- 2. Cut out the other circle with the nail drawn on it and cut out the ruler along the edge.
- 3. Pin the centres of the circles together in such a way so that the top circle, with the nail on it, can rotate and so that the degrees on the bigger circle are still visible.
- Rotate the nail from the initial position at 0° to different degree measurements and record in the table below the heights of the nail above the surface of the water at these different angles.



480

Observations: record height using rules 120 | 150 | 180 | 210 | 240 | 270 | 300 | 330 | 360 | 390 | 420 | 450 | Angle 0 0.86 0.5 0 -0.5 -0.86 -1 -0.86 -0.5 0 Height

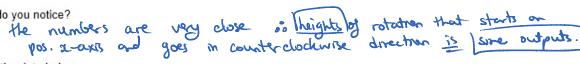
from water to middle of nail once you see the pattern, fill in without measuring

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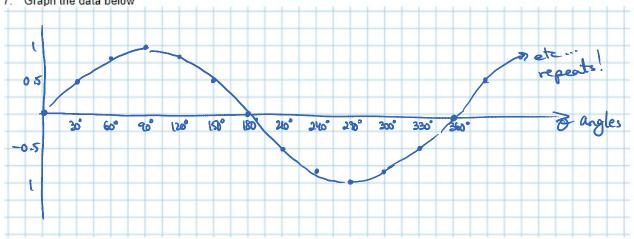


Now	use the	e calcu	ılator ir	n degr	ee mo	de to c	alcula	te:										
Angle (x)	0	30	60	90	120	150	180	210	240	270	300	330	360	390	420	450	480	510
Sin(x)	0	0.5	680	1	6.87	0.5			20									

6. What do you notice?



7. Graph the data below



8. What is the period, max&min, axis, and amplitude of this function?

priod = 360°

MAX=1

MIN = -1 axiz = 0 amplitude =)

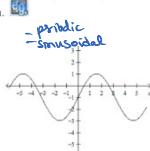
9. You will be studying sinusoidal functions - which are created by transforming (stretching/compressing, reflecting and shifting) this parent sine function. Identify the 5 key points on the graph of the parent sine function.

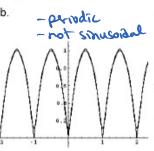


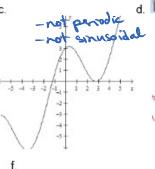
5 pomb :

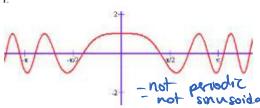
1. First point
2. Last point = 1st last Jakis 5. Three Questers in = Min

10. Classify each as periodic or nonperiodic and sinusoidal or nonsinusoidal









Comparing Sinusoidal Functions



Some of the questions in the textbook require you to graph with technology. There are lots of applets you can use online, or you can download a free program to use on your computer offline.

Online Graphing Calculator

http://my.hrw.com/math06 07/nsmedia/tools/Graph Calculator/graphCalc.html

Download GeoGebra (offline and online) http://www.geogebra.org/cms/en/download,

select webstart, for offline select appletstart, for online ex. $y=3\sin 5^{\circ}x-2$ enter as $y=3\sin(5^{\circ}\cdot\frac{\pi}{180^{\circ}}x)-2$ to see graph in degrees

Graphing technology often does not have degrees as independent variable. It has radians - which you will learn in gr.12 (if you go on studying math - college technology then advanced functions)

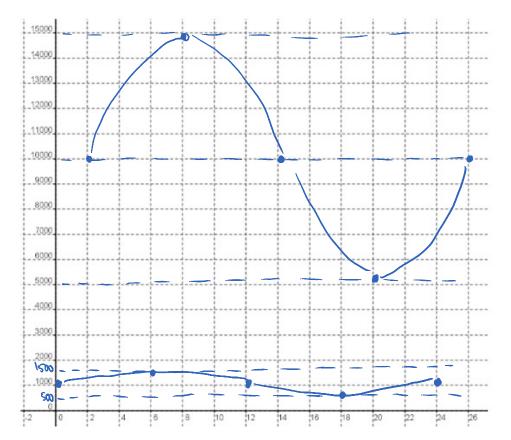
For now, if the equation has a degree symbol in it

set
$$k = \frac{\pi}{180^{\circ}}$$
 as a multiple of the given k value

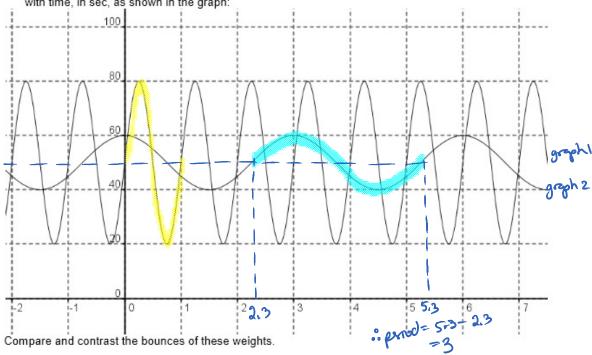


- \overline{m} 1. The population, F, of foxes in the region is modelled by the function $F(t) = 500 \sin(15t)^{\circ} + 1000$, where t is the time in months. The population, R, of rabbits in the same region is modelled by the function, $R(t) = 5000 \sin(15t - 30)^{\circ} + 10000$
 - a. Graph F(t) and R(t). Use technology to help you. From technology note where the cycle starts and ends on the axis, what the max and min values are, then use that information to sketch below:





🔯 2. Two weights attached to the end of two springs are bouncing up and down. As they bounce their height, in cm, varies with time, in sec, as shown in the graph:



Same - both periodic (repeat in a pattern)
- both simusoidal (wavelike)
- both have the same axi3 = 50

different:

graph 1 | graph 2

perrod = 1 | perrod = 3

amplitude = 30 | amplitude = 10

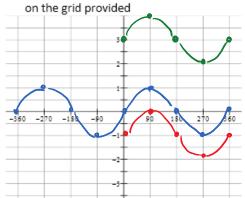
1st point = 0 | 1st point = 2.3

Start the homework practice in class.

Translating/Shifting the Sine Function ${\it F-sin}\,\theta$

Part 1: Vertical Displacements (Up & Down)

1. On the graphing calculator, draw the functions $y = \sin \theta$, $y = \sin \theta + 3$, and $y = \sin \theta - 1$. Sketch the graphs below on the grid provided



STEPS USING THE GRAPHING CALCULATOR:

- i. Put the calculator in degree mode.

 Press MODE. Scroll down and across to DEGREE. Press
- ii. Adjust the window to correspond to the given domain. Use the following settings:

Xmin=-360 Xmax=360 Xscl=90 Ymin =-2 Ymax=4

iii. Enter y = sin x into the equation editor. Press GRAPH. Enter remaining equations

2. Describe how the graphs of $y = \sin \theta$, $y = \sin \theta + 3$, and $y = \sin \theta - 1$ are related.

3. Summarize what the letter c represents in $y = \sin \theta + c$

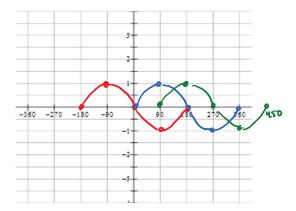
4. Fill in key characteristics of each graph.

	$y = \sin \theta$	$y = \sin \theta + 3$	$y = \sin \theta - 1$			
axis	0	3	-(
Max&Min	MAX=1 MIN=-1	MAX=4 MIN=2	MAX=0 MM=-2			
Period	360°	7	A			
Amplitude	1	٧	> SAME			
Where 1 st and last pts occur	14=0° last=360°	J	·			

5. Out of all the properties (axis, Max&Min, period, amplitude, five points of cycle), which get affected by c?

Part 2: Phase Shifts (Left & Right)

6. On the graphing calculator, draw the functions $y = \sin \theta$, $y = \sin(\theta - 90^\circ)$, and $y = \sin(\theta + 180^\circ)$. Sketch the graphs below on the grid provided.



7. Describe how the graphs of $y = \sin \theta$, $y = \sin(\theta - 90^\circ)$, and $y = \sin(\theta + 180^\circ)$ are related.

y=sind is the parent y=sm(0-90°) is the parent shifted 90° right y=sm(0+180°) is parent shifted 180° left

- - 8. Summarize what the letter d represents in $y = \sin(\theta d)$

9. Fill in key characteristics of each graph.

	$y = \sin \theta$	$y = \sin(\theta - 90^{\circ})$	$y = \sin(\theta + 180^\circ)$
axis	0	7	
Max&Min	MAX= MiN=1	4	> SAME
Period	360		
Amplitude		U	
Where 1 st and last pts occur	1st=0° last= 360°	1st= 90° last= 450°	1st = -180° last=180°

10. Out of all the properties (period, Max&Min, axis, amplitude, five points of the cycle), which get affected by d?

- Sketch and state the equation for the following then identify the period, axis, amplitude, range, domain for the first cycle.
- a. the sine function has been moved 39 units down and 23° to the left b. the sine function has been moved 30 units up and 58° to the right.

$$y = \sin(0 + 23^{\circ}) - 39$$

period = 360° | 1st pt = -23°

axis = -39 | last pt = -23° + 360° = 337°

amplifuele = |

range fyer, -40 = ye-38°

domain $f = 0 = 10^{\circ}$, -23° = $0 = 337^{\circ}$.

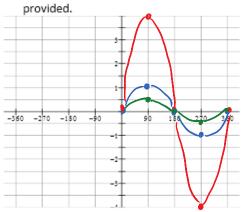
 $y = \sin(\theta - 58^{\circ}) + 30$ period = 360° axis = 30amplitude = 1

range = $\{y \in R, -1 \le y \le 1\}$ domain = $\{0 \in R, 56^{\circ} \le 0 \le 48^{\circ}\}$

Stretching/Compressing the Sine Function $y = sin \theta$

📆 Part 3: Amplitude

1. On the graphing calculator, draw the functions $y = \sin \theta$ and $y = 4\sin \theta$. Sketch the graphs below on the grid



2. a. How do these graphs differ from each other? Calculate the amplitude for both functions

y=45m0 is a stretch of the parent

amplitude of parent = 1
amplitude of stretched = 4

b. What is the domain and range for both graphs
for 1" cycle? D=SDER, 0° ± 0 € 360° }

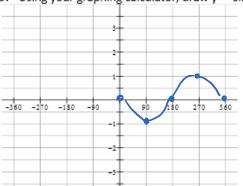
Rpopert = { yER, -1 = y = 1}

3. On your graphing calculator, draw the function y = 0.5sin θ. Sketch the graph on the same grid above.

4. How does y=0.5sin θ differ from v = sin θ?

4. How does $y=0.5\sin\theta$ differ from $y=\sin\theta$? Vertically compressed, amplitude is now 0.5

5. Using your graphing calculator, draw $y = -\sin \theta$. Sketch the graph below.



remember parent sine - starts at axis - then goes up

this one got reflected in x-axis

- storts at axis

- Hen goes down and inde B

3. Summarize what the letter a represents in $y = a \sin \theta$ "a" controls amplitude (stretch/compress) and reflections in x-axis (stretch/compress) montant to state!

12. Fill in key characteristics of each graph.

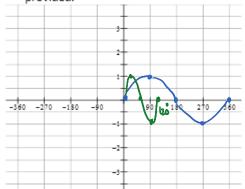
	$y = \sin \theta$	$y = 4 \sin \theta$	$y = 0.5 \sin \theta$.
axis	0 ~		-> sure
Max&Min	MAX=1 MN=-1	Y-=MM Y=XAM	MAX=0,5 MN=-05
Period	360° —		-> same
Amplitude		4	0.5
Where 1st and last pts occur	1st=0° last=360°		- same

4. Out of all the properties (period, Max&Min, axis, amplitude, five points of cycle), which get affected by a?

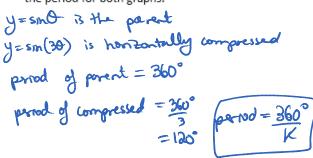
MAX + MIN & get affected by "a" amplitude

Part 4: Horizontal Stretch or Compression

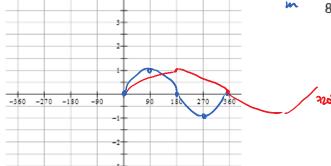
5. On the graphing calculator, draw the functions $y = \sin \theta$ and $y = \sin \theta$. Sketch the graphs below on the grid provided.



6. How do these graphs differ from each other? Calculate the period for both graphs.



7. On your graphing calculator, draw the functions $y = \sin \theta$ and $y = \sin 0.5 \theta$. Sketch these graphs below.



8. How do these graphs differ from each other? Calculate the period for both graphs.

9. Summarize what the letter k represents in $y = \sin k\theta$ K stretches/compresses the function horizontally. (Note if K is regative - reflects in y-axis

13. Fill in key characteristics of each graph.

	$y = \sin \theta$	$y = \sin 3\theta$.	y = $\sin 0.5 \theta$.
axis	0 —		-> same
Max&Min	MAX=1 MIN=-1-	>	sane
Period	360°	120°	720°
Amplitude			-> same
Where 1 st and last pts occur	1st=0° last = 360°	1st=0° last=120°	1st=0° last=720°

10. Out of all the properties (period, Max&Min, axis, amplitude, five points of cycle), which get affected by k?

period = 360° and 5 point of the cycle get affected by """

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11. Describe each equations key characteristics then sketch



a.
$$y = -3\sin(x + 45) + 1$$

a=-3 K=1 d=-45 c=1
b. $y = \sin\frac{1}{4}(x - 60) - 1$

axis=1

b.
$$y = \sin \frac{1}{4}(x - 60) - 1$$

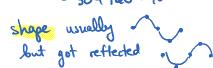
a=1 K=4=0.25 d=60 c=1

 $y = \sin (-3(x + 30)) + 0$

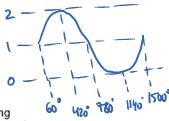
c.
$$y = \sin(-3x - 90)$$

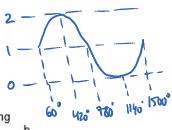
Hint: factor the k out

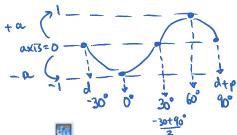




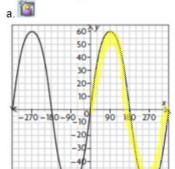
reflected!

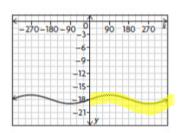


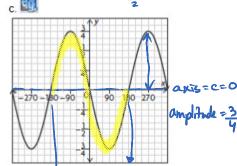




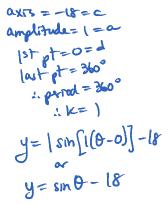
12. Find the equations for the following

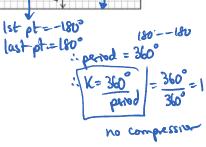




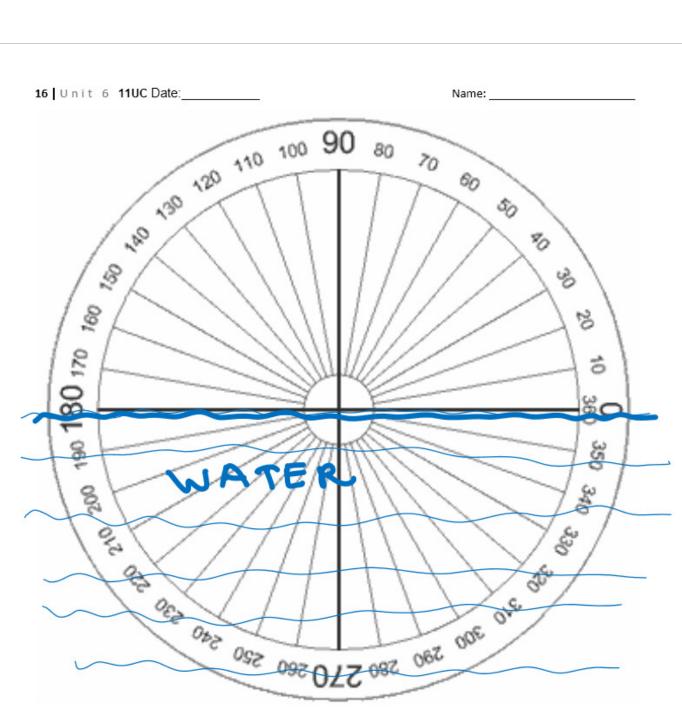


0x13 = 0 = c amplitude=60 = a ..y= 60sin [1 (0-0)]+0 7=60 sin 0





00 y= 3 sm[1(2+180)]+0



WATER

17 Unit 6 11UC Date:	Name:

