Sinusoidal Functions Unit

Tentative TEST date_____

Reflect – previous TEST mark _____, Overall mark now_____.
Looking back, what can you improve upon?

Big idea/Learning Goals

In this unit you will learn how trigonometry can be used to model wavelike relationships. These wavelike functions are called **sinusoidal functions**. You will study key properties of the sine function and use these properties to sketch these functions, to model real life situations and to solve problems. If you go on studying math you will learn that the cosine function is very similar to sine function, however the tangent function is very different.

Success Criteria

☐ I am ready for this unit if I am confident in the following review topics
(circle the topics you are good at & review the ones you left uncircled before you get too far behind)
Reading graphs, simplifying expressions, solving equations, transformations, function notation, domain & range

☐ I understand the new topics for this unit if I can do the practice questions in the textbook/handouts
(check off the topics for which you have finished the practice)

Date	Topics	Done?
	CBR Investigation of Motion Handout & EXTRA "Roll the Shape" Experiment	
	Periodic Behaviour Section 6.2 p331 #4,5,6,7,8 & EXTRA Handout	
	Sine Function Investigation Section 6.3 p339 #1,3,4,6,7,9 & EXTRA Handout	
	Comparing Sinusoidal Functions Section 6.4 p348 #3,4,5,6,7	
	Translating/Shifting the Sine Function Section 6.5 p365 #3,4,6,9,11	
	Stretching/Compressing the Sine Function Section 6.6 p375 #13,15,17,19 & Extra Handout	
	one EXTRA group Activity – if there is time one EXTRA Handout on KU APP COMM TIPS	

☐ I am prepared for the test/evalutation if
- □ I understand the main concepts from each lesson
  - if not, ask other students in class to help you study or visit the peer tutoring room or ask the teacher for help or get a private tutor
  - also practice "knowledge-understanding" questions from the textbook – look for questions marked by **K**
- □ I can explain/communicate the ideas clearly
  - if not, practice explaining a solved question to someone else or complete the assigned journal questions
  - also practice "communication" questions from the textbook – look for questions marked by **C**
- □ I can apply these concepts in word problems
  - if not, practice "application" questions from the textbook – look for questions marked by **A**
- □ I did not just memorize steps to do for different types of questions, I understand the ideas behind each concept and therefore can do problems in new contexts
  - if not, practice "thinking-inquiry-problem-solving" questions from the textbook – look for questions marked by **T**
- □ I can do questions independently
  - if not, try redoing an already solved example without looking at solutions
- □ I can complete questions quickly and with confidence
  - if not, try timing yourself for similar type questions to see progress
- □ I completed the review and/or practice test

Corrections for the textbook answers:

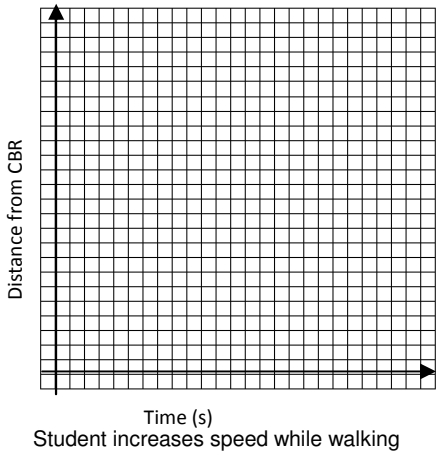
CBR Investigation of Motion

The **Calculator Based Ranger - CBR** is a distance sensor that acts much the same as a submarine or bat to determine its location with respect to an object (it sends out sound waves then determines the time it takes for them to hit the object and bounce back, thus giving its location, or distance from the CBR).

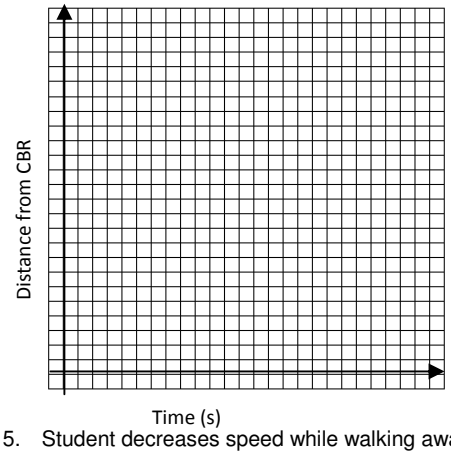
CBR Set-up: (get program to work between calc and ranger) Calc: 2nd, Link, -> , Enter
ranger: 82/83
(get into program) Program, Ranger, Enter
(get sample) Set up, up, start now, … repeat sample
(matching graphs) Applications, meters, dist.match

Instructions: create a clear area for walking in front of CBR, hold a flat surface(ie book) in front yourself as you walk, walk steady, ignore jumps in the graph, the instrument is not very precise.
Sketch the graph first, then walk, then compare if your prediction was correct.

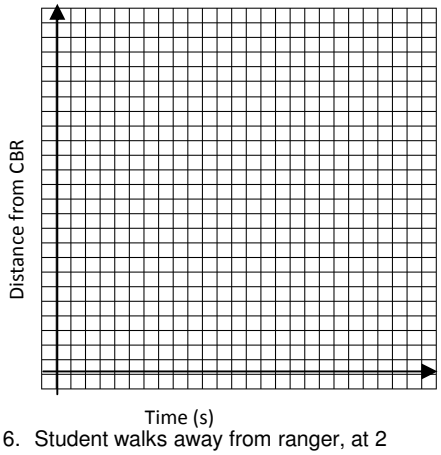
1. Student walks slowly away from CBR



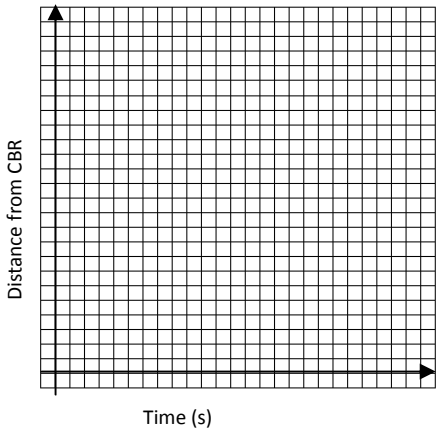
2. Student walks slowly toward the CBR



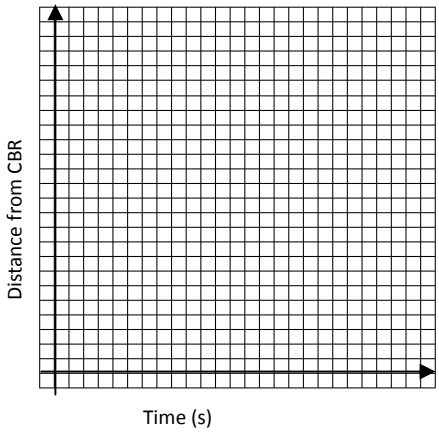
3. Students walks quickly toward the CBR



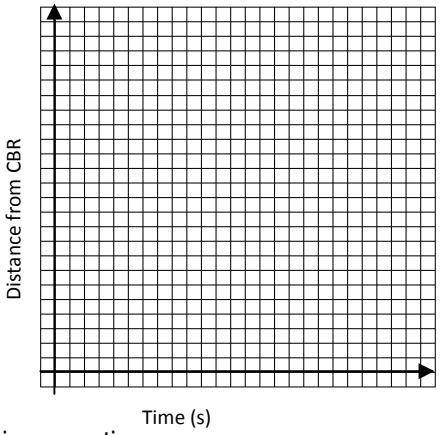
4. Student increases speed while walking toward the CBR



5. Student decreases speed while walking away from CBR



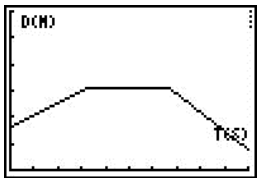
6. Student walks away from ranger, at 2 meters stops for 5 seconds, then returns at the same pace



Do some **match the graph** activities on the CBR for practice, then answer the following questions.

Describe how to create or explain why it is impossible to create the following graphs.

7.



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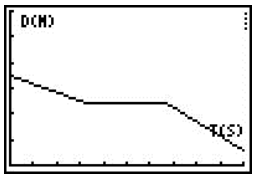

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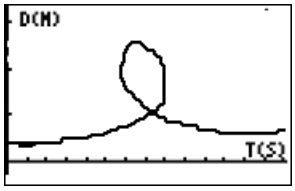

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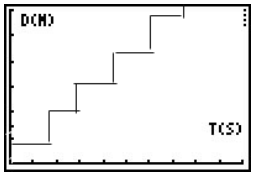

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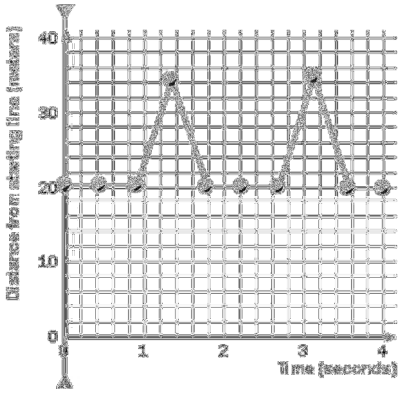

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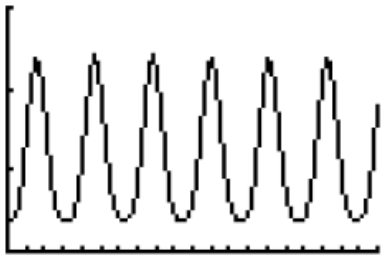
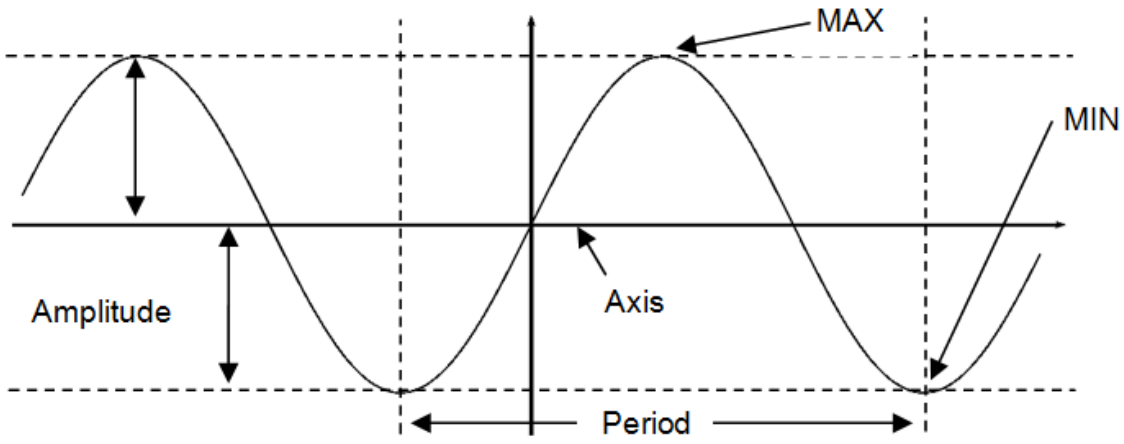


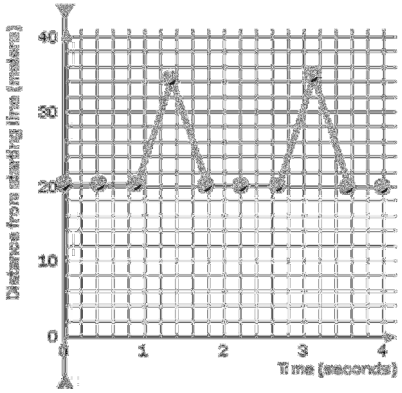
The last two graphs are examples of **periodic motion**, since they repeat in exact same cycles. A **cycle** is a series of events that are regularly repeated, a complete set of changes, starting from one point and returning to the same point in the same way. The **period** is the interval of the independent variable (usually time) needed for one complete cycle. Peaks are _____ values and troughs are _____ values of the graph. The **axis** is the average value of the peaks and troughs, and the **amplitude** is half of the distance between peaks and troughs.

State the formulas that are important to know for the following:

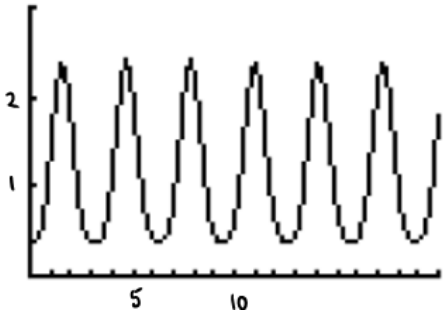


13. Find the period, max & min, axis and amplitude of the these periodic functions.

a.



b.

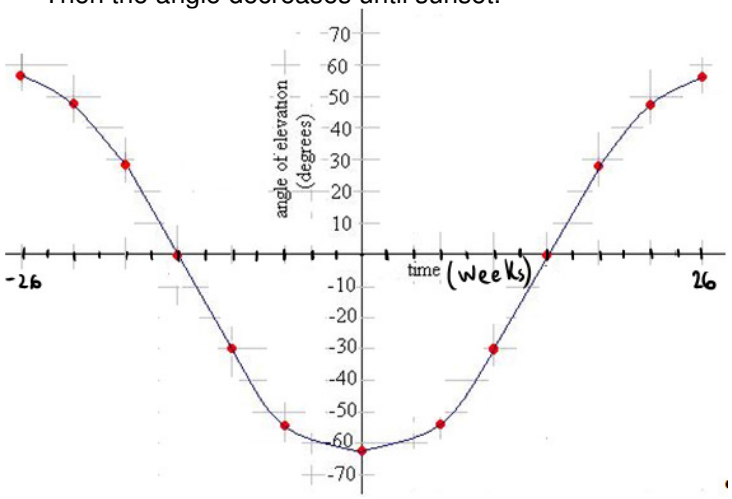


14. What real life relationships can be represented by periodic functions?

Periodic Behaviour

There are many situations in real life that reapeat in cycles.For example, ocean tides, daylight hours, average temperature for the year, heartbeat, volume of air in lungs, rides on ferris wheels, pendulum swings, frequency of musical notes, the list can go on.

1. After the sun rises, its angle of elevation increases rapidly at first, then more slowly, reaching a maximum in 26 weeks. Then the angle decreases until sunset.



a. When does sunrise occur at this time of the year, for this particular spot on Earth?

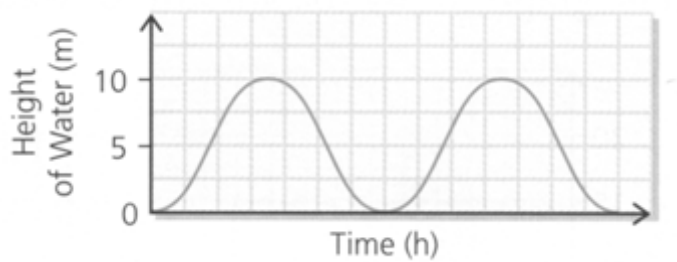
b. What is the period? What is longer the night or the day for this situation?

c. What is the axis? amplitude? range?

d. Extrapolate the angle of elevation in 30 weeks and interpolate the angle of elevation in 20 weeks.

e. During what times is the angle of elevation of the sun above 30 degrees?

2. The Bay of Fundy, which is between New Brunswick and Nova Scotia, has the highest tides in the world. There can be no water on the beach at low tide, while at high tide the water covers the beach.



a. Why can you use periodic functions to model the tides?

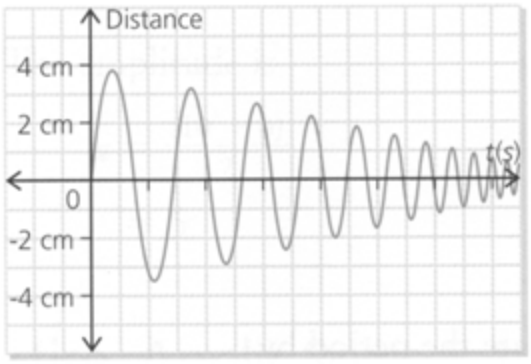
b. What is the change in depth of water from low tide to high tide?

c. Determine the equation of the axis of the curve.

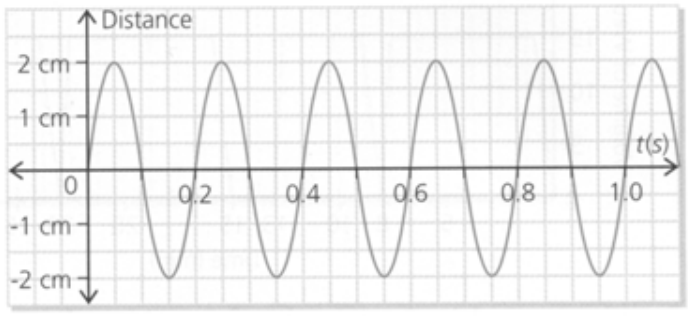
d. What is the amplitude of the curve?

3. Determine if the following represent periodic functions or not.

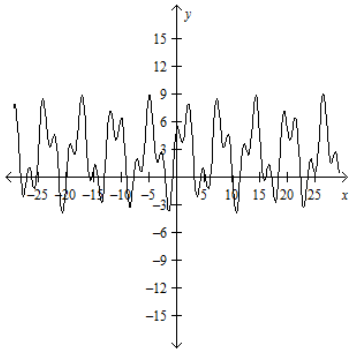
a.



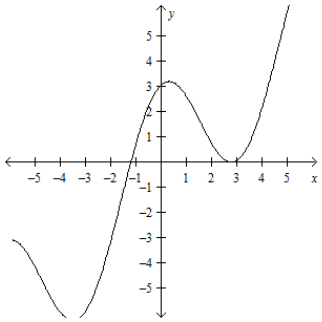
b.



c.



d.



e. Dependent = the horizontal distance travelled by the grandfather clock's pendulum
Independent = time

f. Dependent = cost of riding a taxi
Independent = distance travelled

g. Dependent = interest on the money invested at 5%
Independent = principal deposited

h. Dependent = the height of the pedal on a moving bicycle
Independent = minutes

i.

x	y
-3	7
1	4
5	1
9	4
13	7
17	4
21	1

j.

x	y
0	-2
2	3
4	0
6	-2
8	-4
10	0
12	3

Sine Function Investigation

Introduction: In this investigation you are going to learn how the sine function you used to solve triangles in the last unit will relate to circular motion and how it creates wavelike graphs.

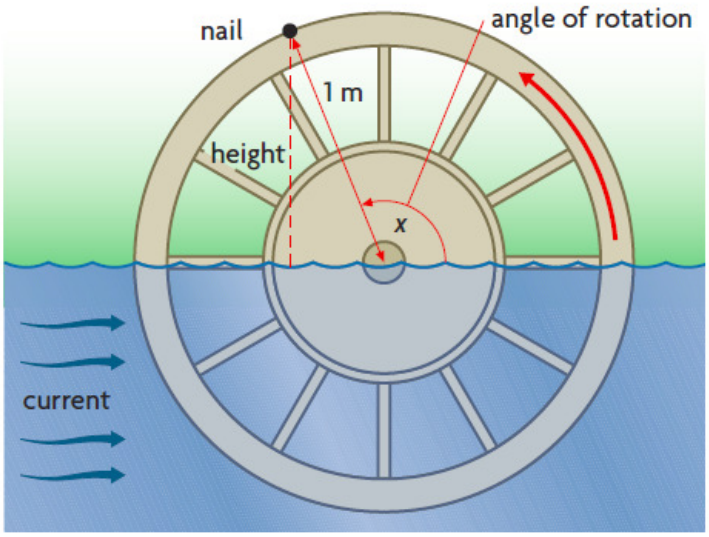
Equipment needed:

- Template of circle with degrees on it shown to be halfway submerged in water.
- Template of circle with a drawn nail on it – to cut out
- Template of the ruler of the same size as the radius of the circle – to cut out
- Cardboard – to tape on it the circle submerged in water
- Tape
- Thumbtack – to pin the circle with nail to the centre of the circle submerged in water.
- Scientific calculator (with sin cos tan buttons)

Steve uses a generator powered by a water wheel to produce his own electricity.

- Half the water wheel is below the surface of the river.
- The wheel has a radius of 1 m.
- The wheel has a nail on its circumference.

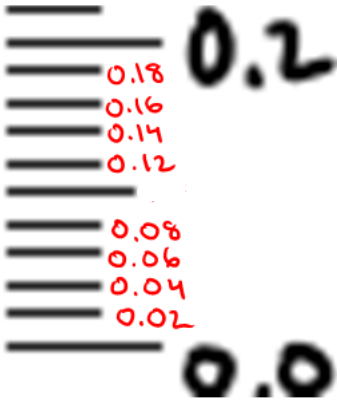
As the current flows, the wheel rotates in a counterclockwise direction to power the generator. The height of the nail, relative to the water level, as the wheel rotates is graphed in terms of the angle of rotation, x.



Performing the Experiment:

1. Tape the circle that is shown to be halfway submerged in water onto the cardboard
2. Cut out the other circle with the nail drawn on it and cut out the ruler along the edge.
3. Pin the centres of the circles together in such a way so that the top circle, with the nail on it, can rotate and so that the degrees on the bigger circle are still visible.
4. Rotate the nail from the initial position at 0° to different degree measurements and record in the table below the heights of the nail above the surface of the water at these different angles.

Reading the ruler



Observations:

Angle	0	30	60	90	120	150	180	210	240	270	300	330	360	390	420	450	480	510
Height																		

5. Now use the calculator in degree mode to calculate:

Angle (x)	0	30	60	90	120	150	180	210	240	270	300	330	360	390	420	450	480	510
Sin(x)																		

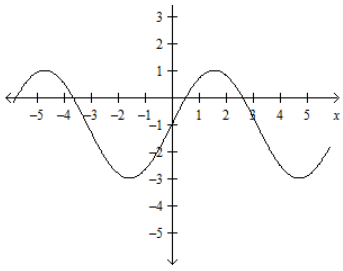
6. What do you notice?

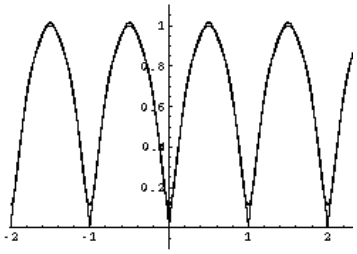
7. Graph the data below

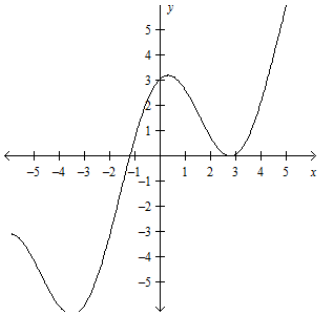
8. What is the period, max&min, axis, and amplitude of this function?

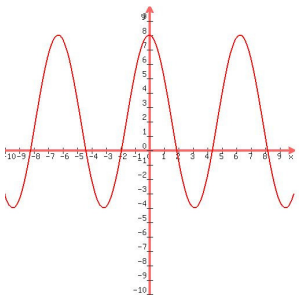
9. You will be studying **sinusoidal functions** – which are created by transforming (stretching/compressing, reflecting and shifting) this parent sine function. Identify the 5 key points on the graph of the parent sine function.

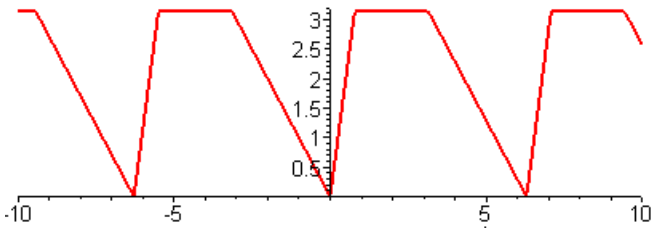
10. Classify each as periodic or nonperiodic and sinusoidal or nonsinusoidal

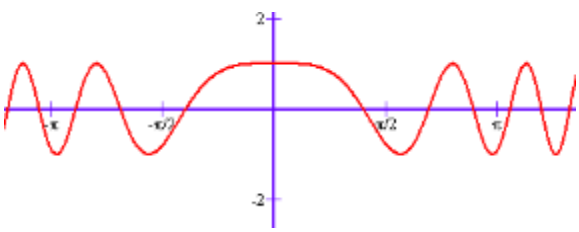
a. 

b. 

c. 

d. 

e. 

f. 

Comparing Sinusoidal Functions

Some of the questions in the textbook require you to graph with technology. There are lots of applets you can use online, or you can download a free program to use on your computer offline.

Online Graphing Calculator
http://my.hrw.com/math06_07/nsmedia/tools/Graph_Calculator/graphCalc.html

Download GeoGebra (offline and online)
http://www.geogebra.org/cms/en/download, select **webstart**, for offline
select **appletstart**, for online

Graphing technology often does not have degrees as independent variable. It has radians – which you will learn in gr.12 (if you go on studying math – college technology then advanced functions)

For now, if the equation has a degree symbol in it

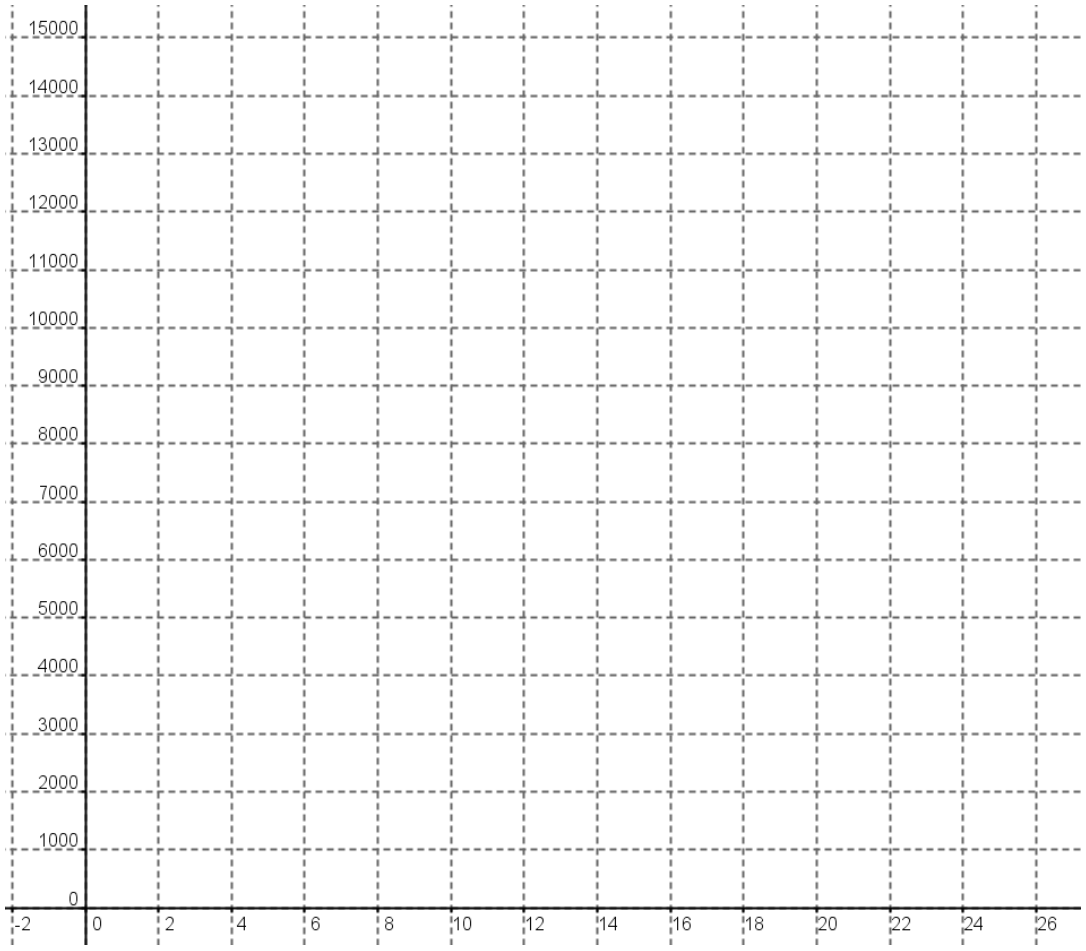
set $k = \frac{\pi}{180^o}$ as a multiple of the given k value

ex. $y = 3\sin 5°x - 2$

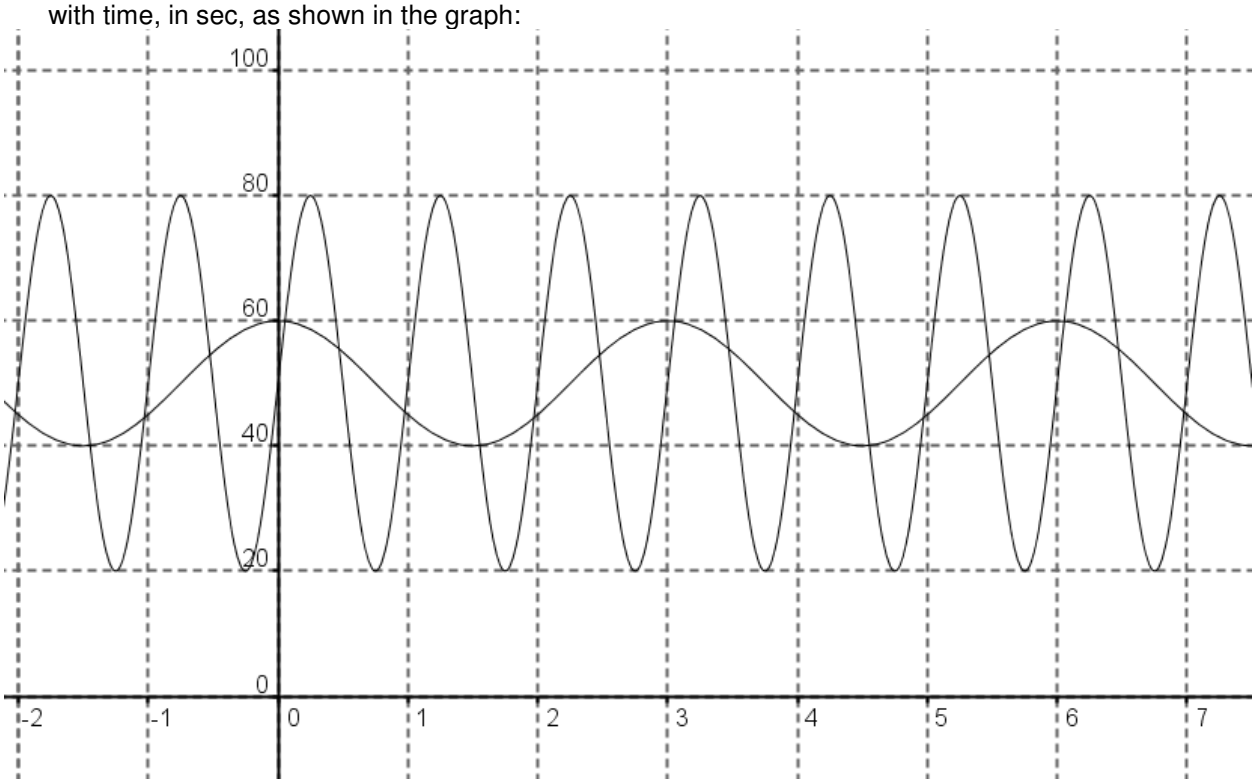
enter as $y = 3\sin\left(5° \cdot \frac{\pi}{180°}x\right) - 2$ to see graph in degrees

1. The population, F, of foxes in the region is modelled by the function $F(t) = 500\sin(15t)° + 1000$, where t is the time in months. The population, R, of rabbits in the same region is modelled by the function, $R(t) = 5000\sin(15t - 30)° + 10000$

   a. Graph F(t) and R(t). Use technology to help you. From technology note where the cycle starts and ends on the axis, what the max and min values are, then use that information to sketch below:



2. Two weights attached to the end of two springs are bouncing up and down. As they bounce their height, in cm, varies with time, in sec, as shown in the graph:



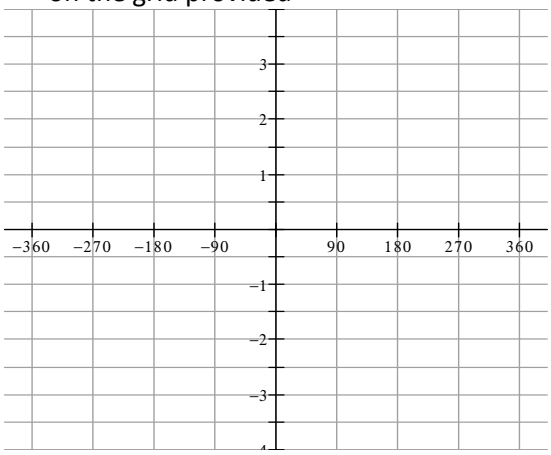
Compare and contrast the bounces of these weights.

Start the homework practice in class.

Translating/Shifting the Sine Function $y = \sin\theta$

Part 1: Vertical Displacements (Up & Down)

1. On the graphing calculator, draw the functions $y = \sin\theta$, $y = \sin\theta + 3$, and $y = \sin\theta - 1$. Sketch the graphs below on the grid provided



STEPS USING THE GRAPHING CALCULATOR:

i. **Put the calculator in degree mode.**
Press MODE. Scroll down and across to **DEGREE**. Press ENTER.

ii. **Adjust the window to correspond to the given domain.** Use the following settings:
Xmin=−360
Xmax=360
Xscl=90
Ymin =−2
Ymax=4

iii. **Enter *y* = sin *x* into the equation editor.**
Press GRAPH. Enter remaining equations

2. Describe how the graphs of $y = \sin\theta$, $y = \sin\theta + 3$, and $y = \sin\theta - 1$ are related.

3. Summarize what the letter c represents in $y = \sin\theta + c$

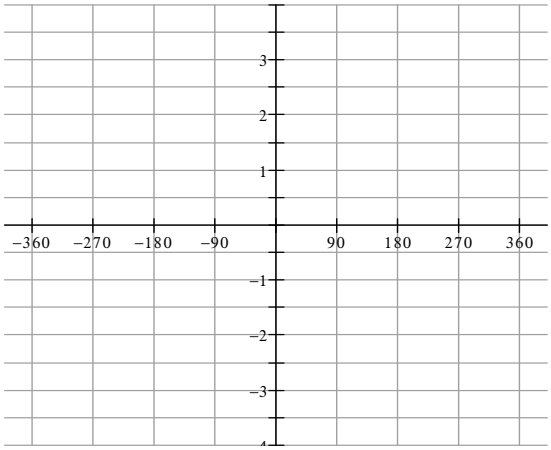
4. Fill in key characteristics of each graph.

	$y = \sin\theta$	$y = \sin\theta + 3$	$y = \sin\theta - 1$
axis			
Max&Min			
Period			
Amplitude			
Where 1st and last pts occur			

5. Out of all the properties (axis, Max&Min, period, amplitude, five points of cycle), which get affected by c?

Part 2: Phase Shifts (Left & Right)

6. On the graphing calculator, draw the functions $y = \sin\theta$, $y = \sin(\theta - 90^\circ)$, and $y = \sin(\theta + 180^\circ)$. Sketch the graphs below on the grid provided.



7. Describe how the graphs of $y = \sin\theta$, $y = \sin(\theta - 90^\circ)$, and $y = \sin(\theta + 180^\circ)$ are related.

8. Summarize what the letter d represents in $y = \sin(\theta - d)$

9. Fill in key characteristics of each graph.

	$y = \sin\theta$	$y = \sin(\theta - 90^\circ)$	$y = \sin(\theta + 180^\circ)$
axis			
Max&Min			
Period			
Amplitude			
Where 1st and last pts occur			

10. Out of all the properties (period, Max&Min, axis, amplitude, five points of the cycle), which get affected by d?

11. Sketch and state the equation for the following then identify the period, axis, amplitude, range, domain for the first cycle.

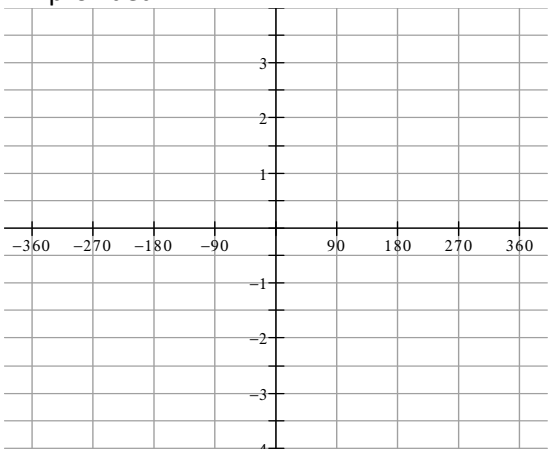
a. the sine function has been moved 39 units down and 23° to the left

b. the sine function has been moved 30 units up and 58° to the right.

Stretching/Compressing the Sine Function $y = \sin\theta$

Part 3: Amplitude

1. On the graphing calculator, draw the functions $y = \sin\theta$ and $y = 4\sin\theta$. Sketch the graphs below on the grid provided.

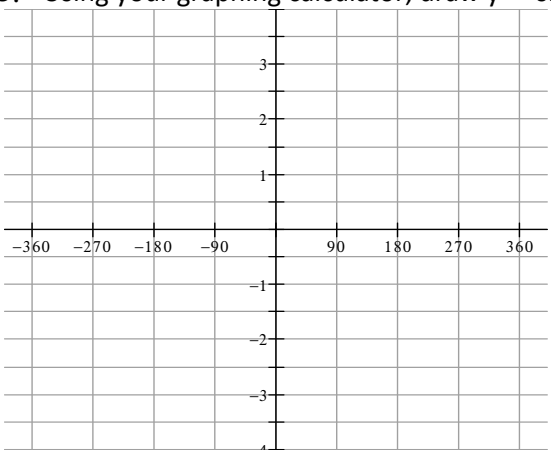


2. a. How do these graphs differ from each other? Calculate the amplitude for both functions

b. What is the domain and range for both graphs for 1st cycle?

3. On your graphing calculator, draw the function y = 0.5sinθ. Sketch the graph on the same grid above.
4. How does y=0.5sinθ differ from y = sinθ?

5. Using your graphing calculator, draw y = -sinθ. Sketch the graph below.



3. Summarize what the letter a represents in $y = a\sin\theta$

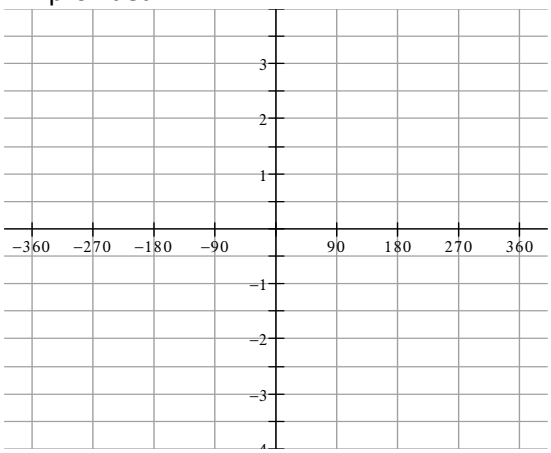
12. Fill in key characteristics of each graph.

	$y = \sin\theta$	$y = 4\sin\theta$	y = 0.5sinθ.
axis			
Max&Min			
Period			
Amplitude			
Where 1st and last pts occur			

4. Out of all the properties (period, Max&Min, axis, amplitude, five points of cycle), which get affected by a?

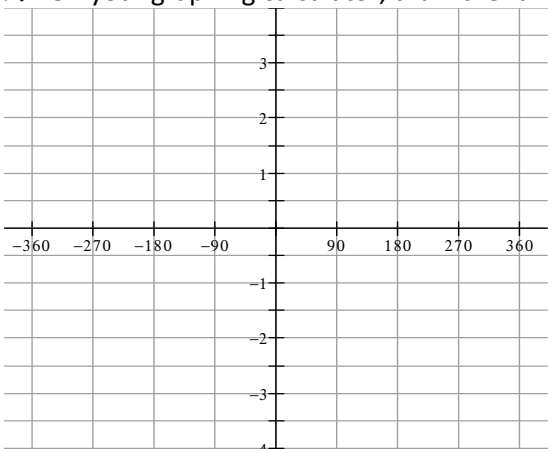
Part 4: Horizontal Stretch or Compression

5. On the graphing calculator, draw the functions $y = \sin\theta$ and $y = \sin 3\theta$. Sketch the graphs below on the grid provided.



6. How do these graphs differ from each other? Calculate the period for both graphs.

7. On your graphing calculator, draw the functions $y = \sin\theta$ and $y = \sin 0.5\theta$. Sketch these graphs below.



8. How do these graphs differ from each other? Calculate the period for both graphs.

9. Summarize what the letter k represents in $y = \sin k\theta$

13. Fill in key characteristics of each graph.

	$y = \sin\theta$	$y = \sin 3\theta$.	$y = \sin 0.5\theta$.
axis			
Max&Min			
Period			
Amplitude			
Where 1st and last pts occur			

10. Out of all the properties (period, Max&Min, axis, amplitude, five points of cycle), which get affected by k?

11. Describe each equations key characteristics then sketch

a. $y = -3\sin(x + 45) + 1$

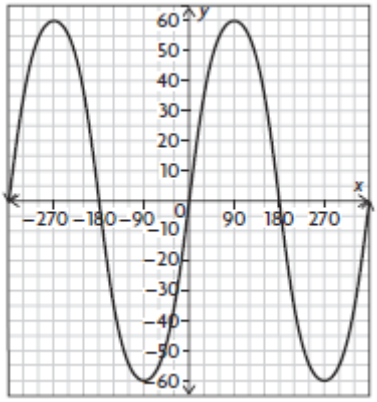
b. $y = \sin\frac{1}{4}(x - 60) - 1$

c. $y = \sin(-3x - 90)$

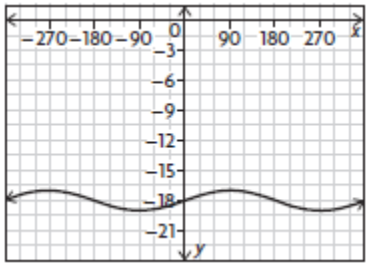
Hint: factor the k out

12. Find the equations for the following

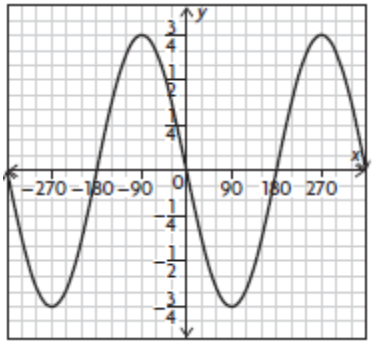
a.

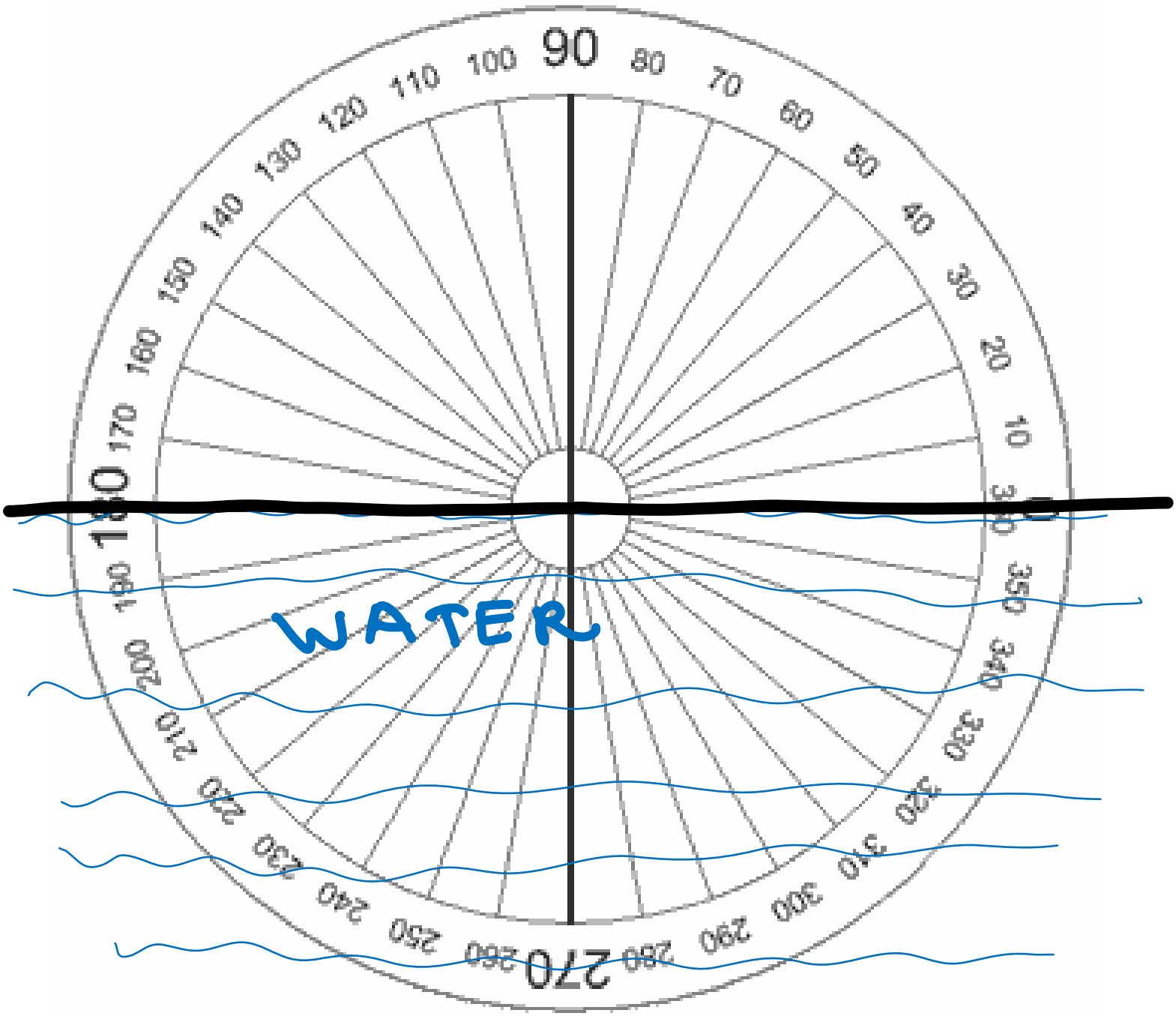


b.



c.




90
80
70
60
50
40
30
20
10
0
350
340
330
320
310
300
290
280
270
260
250
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210
200
190
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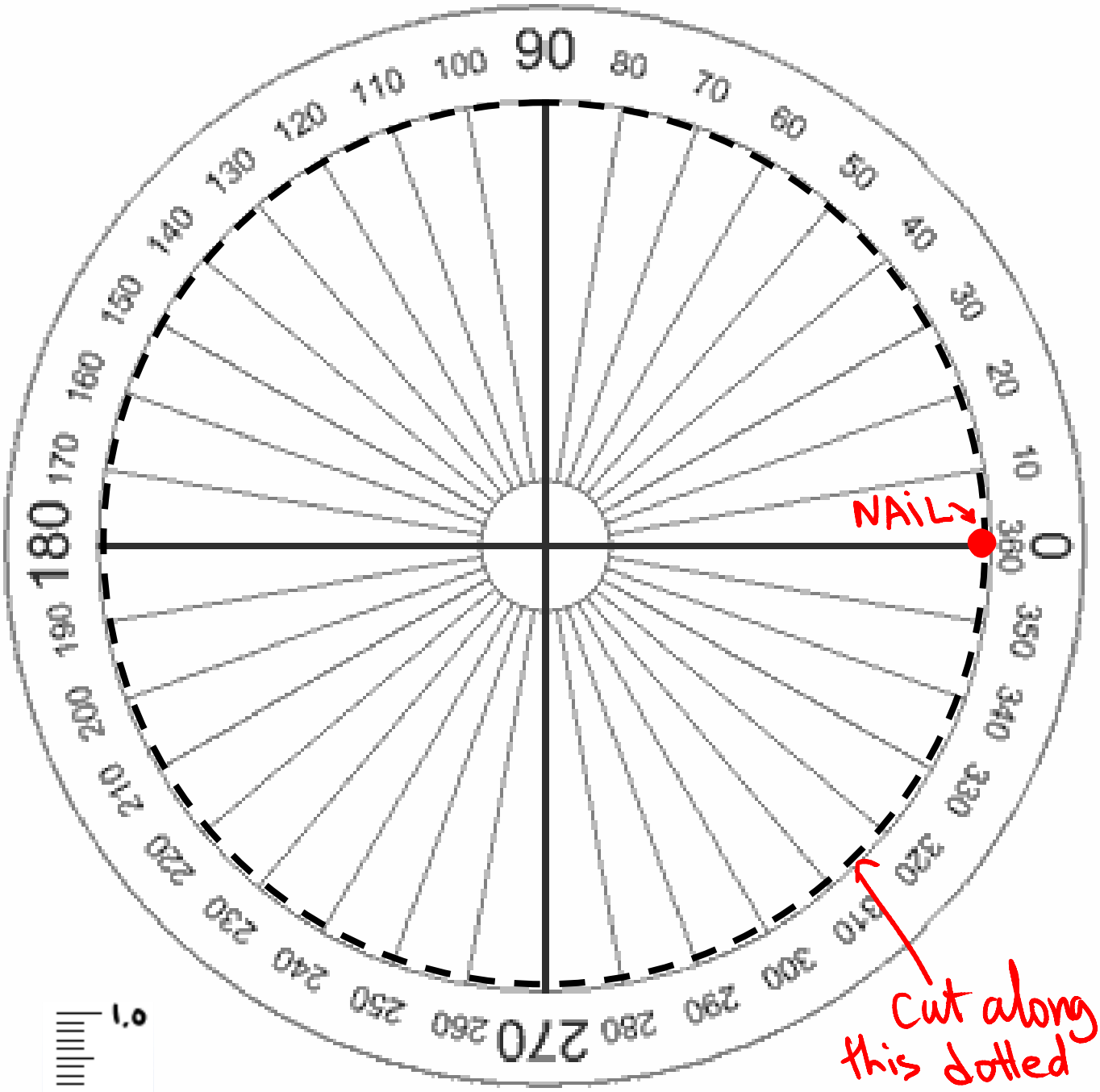
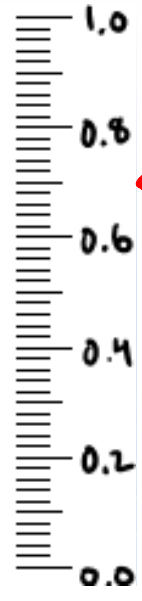
170
160
150
140
130
120
110
100
WATER
WATER



Cut along this dotted circle



← cut this along the edge to use as a ruler

(size should be the same as radius of circle with nail on it)