NOTESallANS

Look below for ALL
answers to notes - if you find mistakes, let me know

Date: ____________________ Name: _________________________

Quadratics in Standard and Vertex Forms Unit

Big idea

This is the last unit of quadratics. In this unit you will concentrate on the standard form and vertex form of quadratics. **Standard form** looks like $y=ax^2+bx+c$, where x^2 term is visible and there are no brackets. **Factored form** looks like $y=a(x-r)(x-t)$, where there is no x^2 term unless you expand, and the equation has brackets. **Vertex form** look like $y=a(x-h)^2+k$. (it can have no brackets , or have only one set of brackets with a square on it).

Identify what forms the following are in, then think of some reasons why vertex form is useful.

$y=x^2+4$ standard + vertex

$y=x^2+3x$ standard

$y=x(x+4)$ factored

$y=(x-5)^2$ vertex + factored.

→ can see the vertex, a.o.s, MAX/MIN value

Because vertex form is commonly used for graphing as well as for problem solving, you must be very comfortable in finding it from standard form by Completing the square (new for gr10 applied students) or from factored form by finding the a.o.s. and opt. val (both academic and applied should have seen this). Also, some quadratics may not be factored over the integers, in this situation you will have to use quadratic formula to find the zeros (this is also new for gr10 applied students).

I know all the prior concepts related to this unit. (If not STOP & complete more review)	Place a ✓ if you are **confident** in that section. Place a ≈ if you are **just ok** in that section. Leave it blank if you are **lost** in that section. If there are gaps in any row, please see the teacher for extra help in that topic.	**Success Criteria** Assessment as Learning for Learning and of Learning I can understand the lesson (If not, ask clarifying questions. Be specific – "what part is unclear?")	I can do a question with an example to follow. (If not, see the teacher for extra help)	I can do questions independently (If not, redo a solved example without looking at solutions)	I can explain/communicate this concept in my own words – JOURNAL (If not, practice explaining steps done in a solved example)	I can apply this concept in other/new contexts/situations (This can be only attained with practice)	I am very confident and am able to complete questions quickly (If not, time yourself to see progress)	I completed the practice in EACH section	I completed the practice test and the review section for this unit.
KU	**Learning Goal**	KU	KU	APP	COMM	TIPS		HW	TEST
Finding equations of and graphing lines, finding equations of and graphing quadratics, simplifying expressions, solving equations, expanding, factoring, problem solving with lines and quadratics	Vertex Form Section 4.1 p204 #4,6,7,9,10,12								P 256- Chapter Self-Test P 254-255 Chapter Review Questions
	Completing the Square Section 4.2 p214 #7,10,11,13 & EXTRA Handout								
	Quadratic Formula Section 4.3 p222 #5,6,8,9 & EXTRA Handout								
	Nature of Roots Section 4.4 p232 #4,5,6,7,9,12								
	Solve Problems Section 4.5 p240 #5,7,8,9,10,11,14								
	Quadratic Models Section 4.6 p251 #6,8,11 & Handout								
	one EXTRA ~~Handout~~ assign on Quad Strategies								

Tentative TEST date_______________

Reflect – TEST mark for this unit___________, Overall mark now___________.

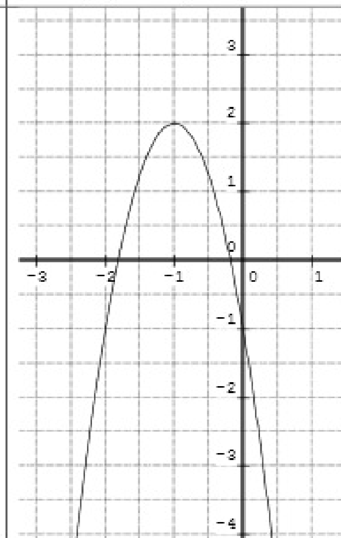
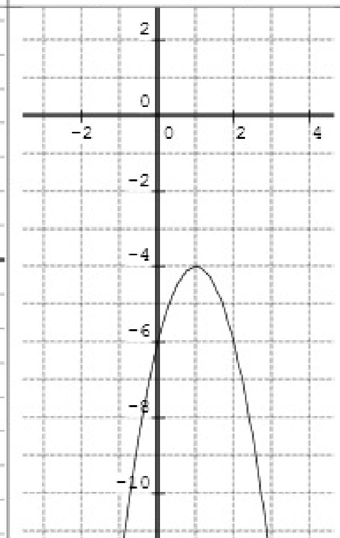
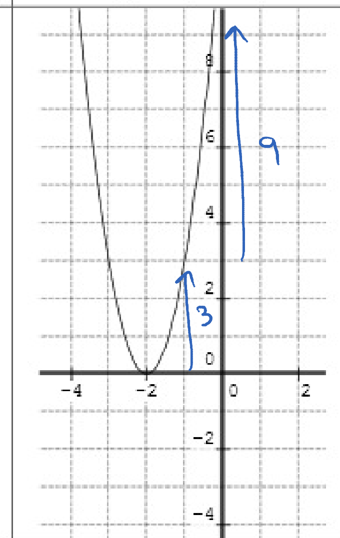
Looking back on this unit, what should you plan to improve upon before the exam?

Corrections for the textbook answers:

Date: ____________________   Name: ________________________

Vertex Form

1. Examine the following functions and their graphs to determine what the vertex form of a quadratic function tells you about its graph.

	$f(x)=-3(x+1)^2+2$	$g(x)=-2(x-1)^2-4$	$h(x)=3(x+2)^2$
			
# of zeros	two	none	one
axis of symmetry	$x=-1$	$x=1$	$x=-2$
vertex	$(-1,2)$	$(1,-4)$	$(-2,0)$
optimal value	$y=2$ MAX	$y=-4$ MAX	$y=0$ MIN
***y*-intercept**	$(0,-1)$	$(0,-6)$	$(0,9)$ use step pattern.

2. Summarize what you should know from vertex forms:

$y=a(x-h)^2+k$

direction of opening

vertex (h,k)

*switch sign for h

also a.of s $=h$

opt.val $=k$ → MAX if "a" is neg → MIN if "a" is pos

domain: $D=\{x\in\mathbb{R}\}$

range $=R=\{y\in\mathbb{R},\ y \geq / \leq k\}$ depends how it opens.

2

Date: ______________ Name: ______________

3. For each quadratic find the vertex, a.of.s, is it max or min, range and sketch.

a.

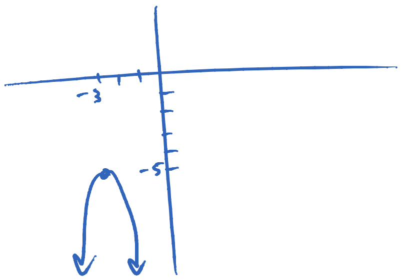
$f(x) = -4(x+3)^2 - 5$

vertex $= (-3, -5)$

a.of.s. $= h = -3$

opens down $\therefore$ MAX value $k = -5$

range $= R = \{y \in \mathbb{R},\ y \le -5\}$



b.

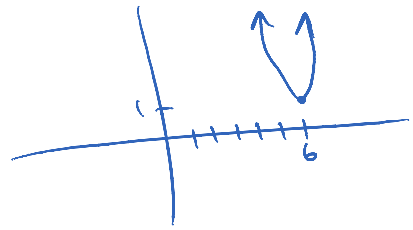
$f(x) = 2(x-6)^2 + 1$

Vertex $(6, 1)$

a.of.s $= h = 6$

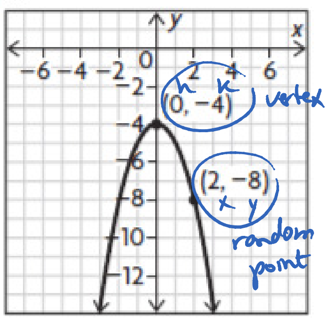
$a = 2$ $\therefore$ opens up $\therefore$ min value $k = 1$

range $= R = \{y \in \mathbb{R},\ y \ge 1\}$



4. For each of the following find the equations in vertex forms.

a.



$y = a(x-h)^2 + k$

$y = a(x-0)^2 - 4$

$-8 = a(2)^2 - 4$

$-8 = 4a - 4$

$+4$

$-4 = 4a$

$-1 = a$

$\therefore y = -1(x-0)^2 - 4$

or

$y = -x^2 - 4$

b. A function has a vertex of (1, – 4) and a y-intercept of 6.

h k

pt. $(0, 6)$

$y = a(x-h)^2 + k$

$6 = a(0-1)^2 - 4$

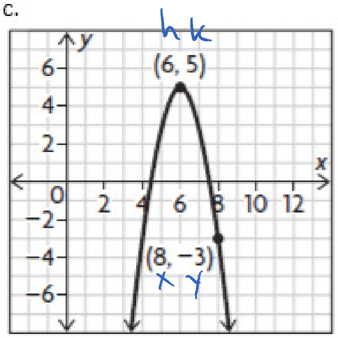
$6 = 1a - 4$

$+4$

$10 = a$

$\therefore y = 10(x-1)^2 - 4$

c.



$y = a(x-h)^2 + k$

$y = a(x-6)^2 + 5$

$-3 = a(8-6)^2 + 5$

$-3 = a(2)^2 + 5$

$-3 = 4a + 5$

$-8 = 4a$

$-2 = a$

$\therefore y = -2(x-6)^2 + 5$

d. A function has a vertex of (1, – 12) and passes through the point (5, 36).

h k

x y

$y = a(x-h)^2 + k$

$36 = a(5-1)^2 - 12$

$36 = a(4)^2 - 12$

$36 = 16a - 12$

$48 = 16a$

$3 = a$

$\therefore y = 3(x-1)^2 - 12$

Date: ____________________ Name: ________________________

5. Summarize all three forms of a quadratic and explain what is easily determined from each

Standard Form

$y = ax^2 + bx + c$

y-int $(0, c)$

Factored Form

$y = a(x-r)(x-t)$

zeros $(r, 0)$ and $(t, 0)$

Vertex Form

$y = a(x-h)^2 + k$

vertex (h, k)

6. A rocket travels according to the equation $h = -4.9(t-6)^2 + 182$, where h is the height, in metres, above the ground and t is the time, in seconds.
   a. When does the rocket reach its maximum height?
   b. What is maximum height?
   c. What is the height at launch?
   d. When did the rocket reach the height of 170 m?

vertex $(6, 182)$
x y
time height

$a = -4.9$ ∴ opens down ∴ MAX at vertex.

ⓐ rocket reaches MAX height at 6 sec.

ⓑ rockets MAX height is 182 m.

ⓒ at launch time starts $t = 0$

$h = -4.9(0-6)^2 + 182$

$h = -4.9(36) + 182$

$h = -176.4 + 182$

$h = 5.6$

∴ launched from a height of 5.6m off the ground.

ⓓ $170 = -4.9(t-6)^2 + 182$

can isolate

$-12 = -4.9(t-6)^2$

$2.44 \doteq (t-6)^2$

$\pm 1.56 \doteq t - 6$

$+1.56 + 6 = t$ or $-1.56 + 6 = t$

$7.56 = t$ or $4.44 = t$

reach at 4.44 sec then at 7.56 sec back down to 170m

7. Given that the parabola has zeros at (-1, 0) and (3, 0) and goes through a point (4, 5) find the vertex form, ~~then convert it to standard form~~

$y = a(x-r)(x-t)$

$y = a(x--1)(x-3)$ sub pt. $(4,5)$

$5 = a(4+1)(4-3)$

$5 = a(5)(1)$

$1 = a$

a.o.s. $= \frac{-1+3}{2} = \frac{2}{2} = 1 = h$

$y = a(x-h)^2 + k$

$y = 1(x-1)^2 + k$ sub pt. $(4,5)$

$5 = 1(4-1)^2 + k$

$5 = 3^2 + k$

$5 = 9 + k$

$-4 = k$

∴ $y = 1(x-1)^2 - 4$

Date: ____________________ Name: ________________________

Completing the Square

Completing the square is a process used to change standard form to vertex form by creating a perfect square in the expression, and then factoring the square.

INSTRUCTIONS	EXAMPLE #1 $f(x)=2x^2+12x-3$	EXAMPLE #2 $f(x)=-5x^2+20x+2$	EXAMPLE #3 $f(x)=-3x^2+42x-129$
1. Factor out the constant a from both x^2 and the x terms.	$f(x)=2(x^2+6x)-3$	$f(x)=-5(x^2-4x)+2$	$f(x)=-3(x^2-14x)-129$
2. Find the constant that must be added <u>and</u> subtracted to create a perfect square. *(The value equals the square of half of the coefficient of the x term found in step 1.)* $\left(\frac{b}{2}\right)^2$ Rewrite the expression by adding, then subtracting this value after the x-term inside the brackets.	*The constant to be added and subtracted is* $\left(\frac{b}{2}\right)^2=\left(\frac{6}{2}\right)^2=3^2=9.$ $f(x)=2(x^2+6x+9-9)-3$	$\left(\frac{b}{2}\right)^2=\left(\frac{-4}{2}\right)^2=(-2)^2=4$ $f(x)=-5(x^2-4x+4-4)+2$	$\left(\frac{b}{2}\right)^2=\left(\frac{-14}{2}\right)^2=(-7)^2=49$ $f(x)=-3(x^2-14x+49-49)-129$
3. Group the three terms that form the perfect square. Move the subtracted value outside the brackets by multiplying it by the a.	$f(x)=2(x^2+6x+9)-9(2)-3$	$f(x)=-5(x^2-4x+4)-4(-5)+2$ -2 -2	$f(x)=-3(x^2-14x+49)-49(-3)-129$ $f(x)=-3(x-7)(x-7)+147-129$
4. Factor the perfect square and collect like terms.	$f(x)=2(x+3)^2-21$	$f(x)=-5(x-2)(x-2)+20+2$ $=-5(x-2)^2+22$	$f(x)=-3(x-7)^2+18$

Date: ____________________ Name: ________________________

Completing the Square

1. Complete the square to express each function in vertex form. Then graph each, and state the domain and range.

a. $f(x) = -\frac{1}{2}x^2 + 20x + 8$

$f(x) = -0.5x^2 + 20x + 8$

$f(x) = -0.5(x^2 - 40x) + 8$

$\left(\frac{b}{2}\right)^2 = \left(-\frac{40}{2}\right)^2 = (-20)^2 = 400$

$f(x) = -0.5(x^2 - 40x + 400 - 400) + 8$

$f(x) = -0.5(x^2 - 40x + 400) - 400(-0.5) + 8$

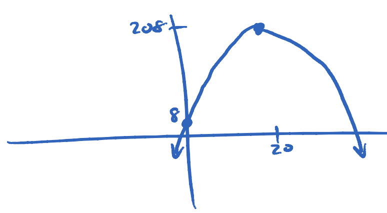
-20, -20

$f(x) = -0.5(x-20)(x-20) + 200 + 8$

$f(x) = -0.5(x-20)^2 + 208$

vertex $(20, 208)$

$a = -0.5$ opens down and vertically compressed



y-int $(0, 8)$

$D = \{x \in \mathbb{R}\}$

$R = \{y \in \mathbb{R}, y \le 208\}$

b. $g(x) = 3x^2 - 15x + 75$

$g(x) = 3(x^2 - 5x \quad) + 75$

$\left(\frac{b}{2}\right)^2 = \left(-\frac{5}{2}\right)^2 = \frac{25}{4}$

$g(x) = 3\left(x^2 - 5x + \frac{25}{4} - \frac{25}{4}\right) + 75$

$g(x) = 3\left(x^2 - 5x + \frac{25}{4}\right) - \frac{25}{4}(3) + 75$

$-\frac{5}{2}$, $-5/2$

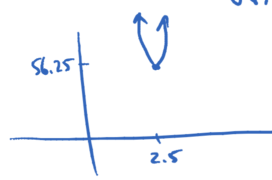
$g(x) = 3(x - 5/2)(x - 5/2) - \frac{75}{4} + \frac{75}{1}\frac{\times 4}{\times 4}$

$\frac{-75 + 300}{4}$

$g(x) = 3(x - 5/2)^2 + \frac{225}{4}$

vertex $\left(5/2, \frac{225}{4}\right) = (2.5, 56.25)$

$a = 3$ opens up and vertically stretched



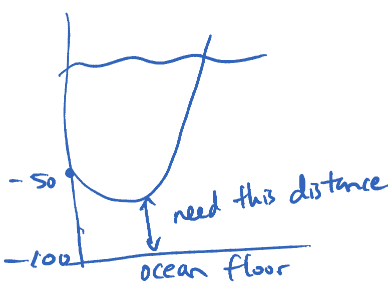
y-int $= (0, 75)$

$D = \{x \in \mathbb{R}\}$

$R = \{y \in \mathbb{R}, y \ge 56.25\}$

A submarine traveling in a parabolic arc ascends to the surface. The path of the submarine is described by $y = 2x^2 - 10x - 50$, where x represents the time in minutes nad y represents the submarines depth in meters. What is the minimum distance from the ocean's floor that the submarine ever reaches. Assume the ocean floor in that area is 100m.

need "k"



$y = 2(x^2 - 5x \ ) - 50$

$\left(\frac{b}{2}\right)^2 = \left(-\frac{5}{2}\right)^2 = \frac{25}{4}$

$y = 2\left(x^2 - 5x + \frac{25}{4} - \frac{25}{4}\right) - 50$

$y = 2\left(x^2 - 5x + \frac{25}{4}\right) - \frac{25}{4}\left(\frac{2}{1}\right) - 50$

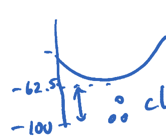
$-5/2$, $-5/2$

$y = 2(x - 5/2)(x - 5/2) - \frac{50}{4} - \frac{50}{1}\frac{\times 4}{\times 4}$

$\frac{-50 - 200}{4} = \frac{-250}{4} = \frac{-125}{2}$

$y = 2(x - 5/2)^2 - \frac{125}{2}$

$y = 2(x - 2.5)^2 - 62.5$



∴ closest distance is 37.5m

6

Date: ____________________ Name: ____________________

A certain 120V electrical circuit has a resistance of 12 amps. The power P in watts that can be produces in the circuit when a current i in amperes is flowing is given by $P(i) = -12i^2 + 120i$. Find the maximum power that can be produced in the circuit.

find vertex

$P(i) = -12(i^2 - 10i \quad)$

$\left(\frac{b}{2}\right)^2 = \left(\frac{-10}{2}\right)^2 = (-5)^2 = 25$

$P(i) = -12(i^2 - 10i + 25 - 25)$

$P(i) = -12(i^2 - 10i + 25) - 25(-12)$

$P(i) = -12(i-5)(i-5) + 300$

$P(i) = -12(i-5)^2 + 300$

$\therefore$ vertex $(5, 300)$

x — current, y — power

$a = -12$ opens down

$\therefore$ MAX at vertex

$\therefore$ MAX power is 300 watts.

at current of 5 amperes.

State the transformations of $y = x^2$ to produce the graph of $y = -3x^2 + 12x - 9$

only seen from vertex form

$y = -3(x^2 - 4x \quad) - 9$

$\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$

$y = -3(x^2 - 4x + 4 - 4) - 9$

$y = -3(x^2 - 4x + 4) - 4(-3) - 9$

$y = -3(x-2)(x-2) + 12 - 9$

$y = -3(x-2)^2 + 3$

$\therefore$ reflect in x-axis

Vertically stretched

shift right by 2

shift up by 3

7

Date: ____________________ Name: ________________________

Quadratic Formula

1. Summarize the quadratic formula and when you are allowed to use it.

Use quadratic formula when looking for zeros
*must have equals zero on one side!
$0 = ax^2 + bx + c$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

2. Find the roots of the following

a. $x^2 - x = 5$

$x^2 - x - 5 = 0$
$a=1 \quad b=-1 \quad c=-5$
$x = \frac{--1 \pm \sqrt{(-1)^2 - 4(1)(-5)}}{2(1)}$
$x = \frac{1 \pm \sqrt{21}}{2}$
$x \doteq 2.79$
$x \doteq -1.79$

b. $x(2x-3) = 7$ not standard, not equals zero

$2x^2 - 3x = 7$
$2x^2 - 3x - 7 = 0$
$\therefore x = \frac{--3 \pm \sqrt{(-3)^2 - 4(2)(-7)}}{2(2)}$
$x = \frac{+3 \pm \sqrt{65}}{4}$
$x \doteq 2.8$
$x \doteq -1.3$

3. There are several methods that you can use to find the roots of an equation. One of these methods always works, however there are shortcuts that can be used in some cases. Summarize what they are and when to use them.

need one side $= 0$

(1.) Quadratic Formula - always works on standard form
(2.) Common Factor - if two terms with x
(3.) Criss Cross Factor - if trinomial and can easily see the combination

(4) Isolate x - if only one term with x is present.

4. Identify a method that could be used to determine the roots of the given equations. Then use it to find the roots

a. $3x^2 = 18x$ common factor

$3x^2 - 18x = 0$
$3x(x-6) = 0$
$3x = 0 \quad\quad x - 6 = 0$
$x = 0 \quad\quad x = 6$

b. $x^2 = 40$ can isolate

$x = \pm\sqrt{40}$
$x \doteq 6.3$
or
$x \doteq -6.3$

c. $(x-1)(x+2) = (3x+2)(x+2)$

expand $x^2 + 2x - x - 2 = 3x^2 + 6x + 2x + 4$
$0 = 2x^2 + 7x + 6$
try factorings: $\binom{2}{1}$ $\quad \frac{2}{3}\binom{3}{2}$
$0 = (2x+3)(x+2)$
$2x+3=0$
$2x = -3$
$x = -\frac{3}{2}$
$x+2=0$
$x=-2$

for all these can also use quadratic formula as long as have
• equals zero
• standard form - no brackets.

Date: ____________________ Name: ____________________

d. $-x^2+5x=3$ quad formula

$0=x^2-5x+3$

$x=\frac{+5\pm\sqrt{(-5)^2-4(1)(3)}}{2(1)}$

$x=\frac{5\pm\sqrt{13}}{2}$ → $x \doteq 4.3$, $x \doteq 0.7$

e. $-2(x+4)^2+98=0$ can isolate

$-2(x+4)^2=-98$

$(x+4)^2=49$

$x+4=\pm 7$

$x+4=7$, $x=3$ $x+4=-7$, $x=-11$

f. $x^2-6x-7=0$ factor

$(x-7)(x+1)=0$

$x-7=0$, $x=7$ $x+1=0$, $x=-1$

5. A baseball player throws a ball into the air. If the equation that represents the ball path is $h=-2t^2+6t+8$, where h represents height in feet and t represents time in seconds.

a. What is the initial height of the ball?
b. How long was the ball in the air?
c. What is the maximum height of the ball?
d. What is the height of the ball after 1 sec?
e. When did the ball reach the height of 10 feet on its way down?

(a) initial height at $t=0$

$h=-2(0)^2+6(0)+8$

$h=8$

∴ initial height is 8 feet.

(b) ball no longer in the air when it drops to the ground where height $h=0$

$0=-2(t^2-3t-4)$

$0=-2(t-4)(t+1)$

$t-4=0$ or $t+1=0$

$t=4$ ~~$t=-1$~~ time can't be neg.

∴ The ball was in the air for 4 sec.

(c) MAX at vertex.

a.o.s $=\frac{4+-1}{2}=\frac{3}{2}=1.5$

opt.val $=-2(1.5)^2+6(1.5)+8$

$=-2(2.25)+9+8$

$=-4.5+9+8$

$=12.5$

∴ vertex $(1.5, 12.5)$
x = time, y = height

$a=-2$ ∴ opens down ∴ MAX

∴ Max height is 12.5 feet.

(d) $h=-2(1)^2+6(1)+8$

$h=12$ feet

(e) $10=-2t^2+6t+8$

$0=-2t^2+6t-2$

$t=\frac{-6\pm\sqrt{6^2-4(-2)(-2)}}{2(-2)}$

$t=\frac{-6\pm\sqrt{20}}{-4}$ → $t \doteq 0.38$ on way up, $t \doteq 2.6$ on way down

Date: ____________________ Name: ________________________

6. A rocket ship is attempting to land on the moon. The ship's computers calculate that the height of the ship above the moon's surface can be modelled by the equation $h(t) = -1.6t^2 - 1.45t + 200$, where h(t) is in meters and t is in seconds

a. The ship's pilot must decide whether the current spot is suitable for landing. He must make the decision before the ship is less than 50 meters off the ground or else it will be too late to change course. How long does he have?

b. Assuming that the pilot chooses to land in the current place, how long from the initial reading will it be before the ship touches down?

(a) sub $h = 50$

doesn't say find MAX/MIN
∴ not complete sq.
make equal zero
solve by factoring
or quad. formula

$50 = -1.6t^2 - 1.45t + 200 \quad -50$

$0 = -1.6t^2 - 1.45t + 150$

$t = \dfrac{--1.45 \pm \sqrt{(-1.45)^2 - 4(-1.6)(150)}}{2(-1.6)}$

$t = \dfrac{+1.45 \pm \sqrt{902.1025}}{-3.2}$

$t \doteq \cancel{-9.8}$ or $t \doteq 8.9$

can't be neg. for time

∴ He has 8.9 sec left before reaching 50m high

(b) touches down $h = 0$

* use original!!

$0 = -1.6t^2 - 1.45t + 200$

$t = \dfrac{+1.45 \pm \sqrt{(-1.45)^2 - 4(-1.6)(200)}}{2(-1.6)}$

$t = \dfrac{1.45 \pm \sqrt{1282.1025}}{-3.2}$

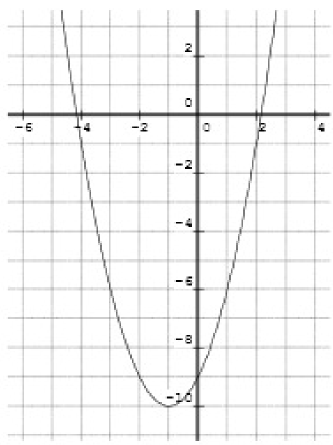
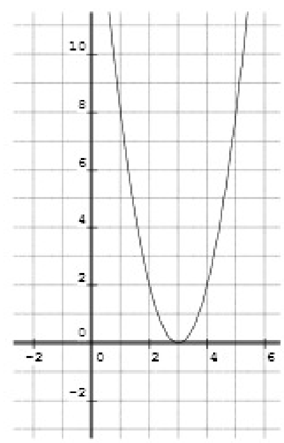
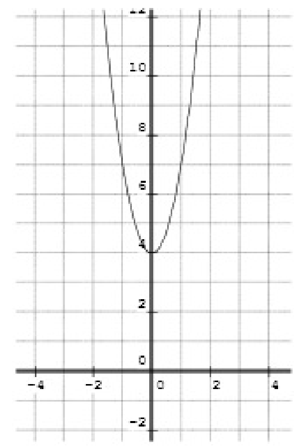
$t \doteq \cancel{-11.6}$ or $t \doteq 10.7$

∴ Will land in 10.7 sec

Date: ____________________ Name: ________________________

Nature of Roots

1. Complete the table

	$f(x) = x^2 + 2x - 9$	$g(x) = 2x^2 - 12x + 18$	$h(x) = 3x^2 + 4$
			
# of Zeros	two	one	none
Calculate the Roots (use the quadratic formula)	$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-9)}}{2(1)}$ $x = \frac{-2 \pm \sqrt{40}}{2}$ $x \doteq 2.2$ or $x \doteq -4.2$	$x = \frac{--12 \pm \sqrt{(-12)^2 - 4(2)(18)}}{2(2)}$ $x = \frac{+12 \pm \sqrt{0}}{4}$ $x = 3$ only one!	$x = \frac{-0 \pm \sqrt{0^2 - 4(3)(4)}}{2(3)}$ $x = \frac{0 \pm \sqrt{-48}}{6}$ get error if negative under root!! can't do it.
Value Under $\sqrt{}$ (+,–, or 0)	positive	zero	negative

2. Summarize how to tell how many roots the equation will have if you are given the following forms

STANDARD FORM

Use Discriminant = under the root of quadratic formula

$b^2 - 4ac > 0$ (pos) → two roots
$b^2 - 4ac < 0$ (neg) → no roots
$b^2 - 4ac = 0$ → one root

VERTEX FORM

Use signs of "a" and "K"

- a and K same sign → no roots
  - opens up + shift up
  - opens down + shift down
- a and K opposite sign → two roots
- if K=0 → one root

12

Date: ______________________ Name: ___________________________

3. Determine the number of real solutions each equation has. Do not solve.

a. $2x = x^2 + 3$

$0 = x^2 - 2x + 3$

$b^2 - 4ac$

$= (-2)^2 - 4(1)(3)$

$= 4 - 12$

$=$ neg $\therefore$ no solutions

b. $3(x-4)^2 - 1 = 0$

opens up

shift down

$\therefore$ Two solutions

c. $2x^2 + 5x = 6$

$2x^2 + 5x - 6 = 0$

$b^2 - 4ac$

$= 5^2 - 4(2)(-6)$

$= 25 + 48$

$=$ pos $\therefore$ Two solutions

4. Determine the number of x-intercepts the function has. Do not solve.

a. $f(x) = 100x^2 + 60x + 9$

$b^2 - 4ac$

$= 60^2 - 4(100)(9)$

$= 3600 - 3600$

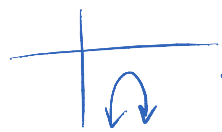
$= 0$

$\therefore$ ONE solution for x-int

b. $f(x) = -2(x+1)^2 - 5$

$a = -2$ opens

$k = -5$ shift down



$\therefore$ no x-int.

c. $f(x) = -4(x-9)^2$

$k = 0$ no shift up/down on the x-axis!

$\therefore$ ONE x-int.

5. For what value(s) of k does the function have no zeros $f(x) = kx^2 + 6x + k$

no zeros if $b^2 - 4ac < 0$

$6^2 - 4(k)(k)$

$36 - 4k^2 < 0$

$k = 4, 5$ etc

$k > 3$

$k < -3$

6. For what value(s) of k does the equation have two solutions $4x^2 - 2x + k = 0$

$b^2 - 4ac$ must be positive

$(-2)^2 - 4(4)(k)$

$4 - 16k$

choose $k < -4$

$k = -5$ or $k = -6$ etc.

7. For what value(s) of k does the function have one x-intercept $f(x) = \underset{a}{1}x^2 + \underset{b}{k}x \underset{c}{- k + 2}$

$b^2 - 4ac$ must be zero

$k^2 - 4(1)(-k+2) = 0$

$k^2 - 4(-k+2) = 0$

$k^2 + 4k - 8 = 0$

quad formula

$k = \dfrac{-4 \pm \sqrt{4^2 - 4(1)(-8)}}{2(1)}$

$k = \dfrac{-4 \pm \sqrt{48}}{2}$

$k \doteq 1.46$

$k \doteq -5.46$

The function $P(x) = -25x^2 + 2500x + 825$ models the profit earned by a dance studio on the basis of the cost of a dance lesson, x. Does the dance studio ever break even? → ever zero?

ie. does it touch x-axis?

ie. How many x-int does it have?

look at discriminant

$b^2 - 4ac$

$= 2500^2 - 4(-25)(825)$

$= 6332500$ positive $\therefore$ there will be 2 x-int

$\therefore$ the studio can break even for two different prices of lessons.

Date: ____________________ Name: ________________________

Solve Problems

1. There are several strategies that allow you to solve a quadratic word problem. They are listed below; identify which list refers to finding maximum/minimum values and which list solves for the zeros/roots of the equation.

In order to find Zeros / Roots use
- Table of values
- Graphing
- Factoring
- Quadratic formula

In order to find MAX / MiN values use
- Table of values
- Graphing
- Find zeros then find a.of s. and opt val
- Completing the Square

These lists are not complete. There are other things that you can do too, like substitute into the equation the given values before you solve, or expand the equation to get another form first… etc.

Now practice identifying what strategy is most efficient to solve the following if you have no access to graphing technology

2. STANDARD form

$y = 2x^2 + 5x - 3$

a. Find min value
Complete sq. look for "k"

b. Find y if x = – 1
Sub in

c. Find when min value occurs
Complete sq. look for "h"

d. Find x if y=0
Quad. Formula / Factor

e. Find x if y= – 3
Sub in, move to one side
Quad. Formula / Factor

3. FACTORED form

$y = (2x+1)(x-3)$

a. Find vertex a.of s = $\frac{\text{add zeros}}{2}$
opt.val = sub in ...

b. Find x-intercepts
Solve each bracket = 0 separately

c. Find initial value
sub x=0

4. VERTEX form

$y = 2(x - \frac{5}{4})^2 - \frac{49}{8}$

a. Find axis of symmetry
look at h = 5/4 *switch sign

b. Find zeros
sub y=0 can isolate since x appears once

c. Find y-intercept
sub x=0

5. A farmer is building a new pig sty on the side of his barn. He has 60 m of fencing. The area that can be enclosed is modelled by the function $A(x) = -2x^2 + 60x$, where x is the width of the sty in metres, and A(x) is the area in square metres. What is the maximum area that can be enclosed?

Complete sq. look at "k"

$A(x) = -2(x^2 - 30x\ )$

$\left(\frac{b}{2}\right)^2 = \left(\frac{-30}{2}\right)^2 = (-15)^2 = 225$

$A(x) = -2(x^2 - 30x + 225 - 225)$

$A(x) = -2(x^2 - 30x + 225) - 225(-2)$

-15
-15

$A(x) = -2(x-15)(x-15) + 450$

$A(x) = -2(x-15)^2 + 450$

$\therefore$ vertex $(15, 450)$
x: width
y: Area

a=-2
∴ opens down
∴ MAX at vertex
∴ MAX area is 450m²

Date: ____________________ Name: ________________________

6. The manager of a grocery store sells 1250 bags of milk for $2 each. He wants to know how much money he will earn if he increases the price in 10¢ increments, which lower the quantity sold by 20 bags. A model of the revenue function is

$R(x) = (price)(quantity)$

expand

$= (2 + 0.10x)(1250 - 20x)$,

$= -2x^2 + 85x + 2500$

where x is the number of 10¢ increments and R(x) is the revenue in dollars.

a. Explain how the equation can be set up from the wording of the problem.
b. What is the maximum revenue?
c. What price yields the maximum revenue? x
d. What is the revenue when the price of milk is $2.40.

(a) $\boxed{price = \$2 + 0.10x}$ original prize is $2 plus - since increase by 10 cents. x represents # of times you change price.

$\boxed{quantity = 1250 - 20x}$ original quantity is 1250 bags. Minus - since lower quantity is sold by 20 bags.

(b) MAX at vertex

$R(x) = -2(x^2 - 42.5x + 451.5625 - 451.5625) + 2500$

$\left(\frac{b}{2}\right)^2 = \left(-\frac{42.5}{2}\right)^2 = (-21.25)^2 = 451.5625$

$R(x) = -2(x^2 - 42.5x + 451.5625) - 451.5625(-2) + 2500$

$R(x) = -2(x - 21.25)(x - 21.25) + 903.125 + 2500$

$R(x) = -2(x - 21.25)^2 + 3403.125$

vertex = (21.25, 3403.125)

x: # of times change price; Y: revenue

∴ MAX revenue is $3403.13

(c) price = 2 + 0.10x

= 2 + 0.10(21.25)

= 2 + 2.125

≐ $4.13

(d) 2.40 = 2 + 0.10x

0.40 = 0.10x

4 = x

∴ $R(4) = -2(4)^2 + 85(4) + 2500$

$= -2(16) + 340 + 2500$

$= -32 + 2840$

$= 2808$

∴ Revenue is $2808 at price of $2.40 per bag.

Date: ____________________ Name: ________________________

7. The population of a small town is modelled by the function $P(t) = 5t^2 + 120t + 20000$, where P(t) is the population and t is the time in years since 2000.

a. When will the population be 25 000? sub in $P(t) = 25000$
b. What will the population be in 2025? sub in $t = 25$
c. When does minimum population occur? complete sq. look for "h"
d. What is the minimum population? complete sq look for "k"
e. Will the population ever be zero? Explain. check discriminant.

ⓐ $25000 = 5t^2 + 120t + 20000$ can't isolate for t appears twice ∴ factor or quad. formula

$0 = 5t^2 + 120t^2 - 5000$

$$t = \frac{-120 \pm \sqrt{120^2 - 4(5)(-5000)}}{2(5)}$$

$$t = \frac{-120 \pm \sqrt{114400}}{10}$$

$$t \doteq \frac{-120 \pm 338.2}{10}$$

$t \doteq 21.82$

$t \doteq -\cancel{45.82}$

∴ in the year 2021 towards the end the population will be 25 000

ⓑ $P(25) = 5(25)^2 + 120(25) + 20000$

$= 5(625) + 3000 + 20000$

$= 3125 + 3000 + 20000$

$= 26125$ is the population.

ⓒ $P(t) = 5(t^2 + 24t + 144 - 144) + 20000$

$\left(\frac{b}{2}\right)^2 = \left(\frac{24}{2}\right)^2 = 12^2 = 144$

$P(t) = 5(t^2 + 24t + 144) - 144(5) + 20000$

$P(t) = 5(t+12)(t+12) - 720 + 20000$

$P(t) = 5(t+12)^2 + 19280$

∴ at $t = -12$ or in the year 1988 the population was

ⓓ Minimum of 19 280

ⓔ $b^2 - 4ac$ → of original!

$= 120^2 - 4(5)(20000)$

$= 14400 - 400000$

$=$ negative ∴ never at zero population

Date: ____________________ Name: ____________________

To find "a": $a = \frac{2^{nd} \text{ difference \#}}{2(\Delta x)^2}$ "delta" change in x values

Quadratic Models

1. A thrown ball has the following heights at various times.

t	h
0	10
1	10
2	9
3	7
4	4

$\Delta x = 1$ $h = 0.5$

)0)−1
)−1)−1
)−2)−1
)−3

$\therefore a = \frac{-1}{2(1)^2}$
$= -\frac{1}{2}$

Vertex at (0.5, ?)

$y = a(x-h)^2 + k$
$y = -\frac{1}{2}(x-0.5)^2 + k$ sub pt. (0,10)
$10 = -\frac{1}{2}(0-0.5)^2 + k$
$10 = -\frac{1}{2}(0.25) + k$
$10 = -0.125 + k$
$10.125 = k$

$\therefore h = -0.5(t-0.5)^2 + 10.125$

a. Find an equation, in vertex form, that represents this data
b. When does the ball land on the ground?

(b) $0 = -0.5(t-0.5)^2 + 10.125$
can isolate
$-10.125 = -0.5(t-0.5)^2$
$20.25 = (t-0.5)^2$
$\pm 4.5 = t - 0.5$
$4.5 = t - 0.5$ or $-4.5 = t - 0.5$
$5 = t$ or $\cancel{-4 = t}$

$\therefore$ lands on the ground in 5 sec.

2. Kim practices the javelin throw. The distance and height of one of the javelin throws are given in the table

d	h
1	4.2
3	7.4
5	9
7	9
9	~~9~~ 7.4

2(2(

) 3.2) −1.6
) 1.6) −1.6
) 0) −1.6
) −1.6

Vertex at h = 6
don't know k.

$\therefore a = \frac{2^{nd} \text{ diff. in "y"}}{2(\Delta x)^2} = \frac{-1.6}{2(2)^2} = -0.2$

a. Find an equation, in vertex form, that represents this data
b. How far does the javelin travel before hitting the ground?

$y = a(x-h)^2 + k$
$y = -0.2(x-6)^2 + k$ sub in a pt. (1, 4.2)
$4.2 = -0.2(1-6)^2 + k$
$4.2 = -0.2(-5)^2 + k$
$4.2 = -0.2(25) + k$
$4.2 = -5 + k$
$9.2 = k$
$\therefore$ vertex form
$y = -0.4(x-6)^2 + 9.2$

(b) hit ground at h = 0 = y
$0 = -0.2(x-6)^2 + 9.2$
can isolate x
$-9.2 = -0.2(x-6)^2$
$\sqrt{46} = \sqrt{(x-6)^2}$
$\pm 6.8 \doteq x - 6$

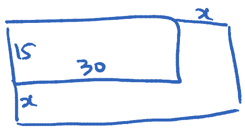
$-6.8 \doteq x - 6$ or $+6.8 \doteq x - 6$
$-0.8 \doteq x$ or $12.8 = x$

$\therefore$ Travelled from 12.8m in total.

16

Date: ____________________ Name: ________________________

3. The community garden club has a vegetable garden that measures 15 m by 30 m. One of the members has donated a new piece of land for a larger garden. They plan to increase the garden by 250 m^2. However, because of the dimension of the new land, both dimensions of the original garden must increase by the same amount. Determine the dimensions of the new garden.



$A_{old} = lw$
$= 15 \times 30$
$= 450$

$A_{new} = 450 + 250$
$= 700$

$A = lw$
$700 = (30+x)(15+x)$
$700 = 450 + 30x + 15x + x^2$
$0 = x^2 + 45x - 250$

$x = \frac{-45 \pm \sqrt{45^2 - 4(1)(-250)}}{2(1)}$

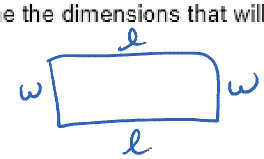
$x = \frac{-45 \pm \sqrt{3025}}{2}$

$x = \frac{-45 \pm 55}{2}$ → $x = 5$ or $\cancel{x = -50}$ can't have neg. increase

∴ dimensions of new garden are
$l = 30 + x$
$= 30 + 5 = 35m$

$w = 15 + x$
$= 15 + 5 = 20m$

4. A farmer wishes to enclose a rectangular field with 48 metres of fencing.
   a. Write down, in function notation, the area of the field in terms of length
   b. Determine the dimensions that will make the area maximum and find the maximum area



$P = 2l + 2w$
$48 = 2l + 2w$ → $48 - 2l = 2w$
$24 - l = w$

isolate w. to get in terms of l.
sub into Area

(a)
$A = lw$
$A = l(24 - l)$
$A(l) = -l^2 + 24l$

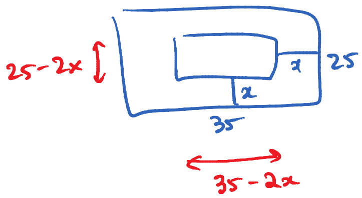
(b) MAX at vertex.
$A(l) = -(l^2 - 24l + 144 - 144)$
$\left(\frac{b}{2}\right)^2 = \left(-\frac{24}{2}\right)^2 = (-12)^2 = 144$

$A(l) = -(l^2 - 24l + 144) - 144(-1)$
$A(l) = -(l-12)(l-12) + 144$
$A(l) = -(l-12)^2 + 144$
∴ vertex $(12, 144)$
x: length, y: area

∴ length = 12m
MAX area = 144 m^2
width = $24 - l$
$= 24 - 12$
$= 12m$

Date: ______________________ Name: ____________________________

5. A framed picture has length 35 cm and width 25 cm. The picture itself has area 375 cm². How far is it from the edge of the picture to the edge of the frame if this distance is uniform around the picture?



$A = lw$

$375 = (35-2x)(25-2x)$

$375 = 875 - 70x - 50x + 4x^2$

$0 = 4x^2 - 120x + 500$

can't isolate for x factor / quad formula

$4(x^2 - 30x + 125)$

-5, -25

$0 = 4(x-5)(x-25)$

$4 \neq 0$ never true

$x - 5 = 0$ or $x - 25 = 0$

$x = 5$ $x = 25$

if $x = 5$

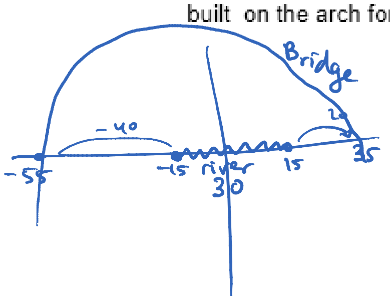
$l = 35 - 2x = 35 - 2(5) = 25$

but if $x = 25$

$l = 35 - 2(25) = 35 - 50 = -15$

$\therefore$ Frame border is 5 cm wide

6. A parabolic arch is built over a river. The bottoms of the arch touch the ground 40 meteres from the left bank of the river and 20 metres from the right bank. The river is 30 metres wide. The arch is 200 metres tall at its highest point
   a. Write an equation which models this arch, using the centre of the river for x=0
   b. A daredevil wishes to dive into the river from a height of 150 metres. At what x-position should a platform be built on the arch for this stunt? Note that the platform must be directly above the river.



(a) $y = a(x-r)(x-t)$

$y = a(x - -55)(x-35)$ sub pt. $(-10, 200)$

$200 = a(-10+55)(-10-35)$

$200 = a(45)(-45)$

$200 = -2025a$

$\frac{-200}{2025} = a$

$-\frac{8}{81} = a$

a.of s $= -\frac{55+35}{2} = -\frac{20}{2} = -10$

$\therefore y = -\frac{8}{81}(x+55)(x-35)$

(b) height $= 150 = y$

$150 = -\frac{8}{81}(x+55)(x-35)$ - can't isolate x - can't separate brackets to solve (not equals zero!!)

$150 = -\frac{8}{81}(x^2 - 35x + 55x - 1925)$

$150 \doteq -0.0988(x^2 + 15x - 1925)$

$0 \doteq -0.0988x^2 - 1.482x + 190.19 - 150$

$0 = -0.0988x - 1.482x + 40.19$

$x \doteq \frac{+1.482 \pm \sqrt{1.482^2 - 4(-0.0988)(40.19)}}{2(-1.482)}$

$x \doteq \frac{1.482 \pm \sqrt{1590.505}}{-2.964}$

$x \doteq -13.96 \sim -14$ on left.

$x \doteq 12.96 \sim 13$ on right. Both ok (over water)

18