 11UC Date:__________ Name: ______________________

Quadratics in Standard and Vertex Forms – Unit

Tentative TEST date_______________

Reflect – previous TEST mark ___________, Overall mark now___________.

Looking back, what can you improve upon?

Big idea/Learning Goals

This is the last unit of quadratics. In this unit you will concentrate on the standard form and vertex form of quadratics. **Standard form** looks like ______________________________, where x^2 term is visible and there are no brackets. **Factored form** looks like _________________________________, where there is no x^2 term unless you expand, and the equation has brackets. **Vertex form** look like ______________________________, (it can have no brackets , or have only one set of brackets with a square on it).

Identify what forms the following are $y = x^2 + 4$ $\quad y = x^2 + 3x$ $\quad y = x(x+4)$ $\quad y = (x-5)^2$

Because vertex form is commonly used for graphing as well as for problem solving, you must be very comfortable in finding it from standard form by ______________________________ (new for gr10 applied students) or from factored form by finding the ___________________________________ (both academic and applied should have seen this). Also, some quadratics may not be factored over the integers, in this situation you will have to use _________________________________ to find the zeros (this is also new for gr10 applied students).

Success Criteria

- ☐ I am <u>ready for this unit</u> if I am confident in the following review topics
  (circle the topics you are good at & review the ones you left uncircled before you get too far behind)
  Finding equations of and graphing lines, finding equations of and graphing quadratics, simplifying expressions, solving equations, expanding, factoring, problem solving with lines and quadratics

- ☐ I <u>understand the new topics</u> for this unit if I can do the practice questions in the textbook/handouts
  (check off the topics for which you have finished the practice)

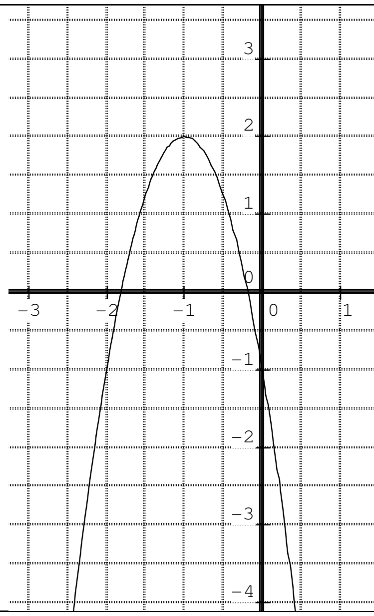
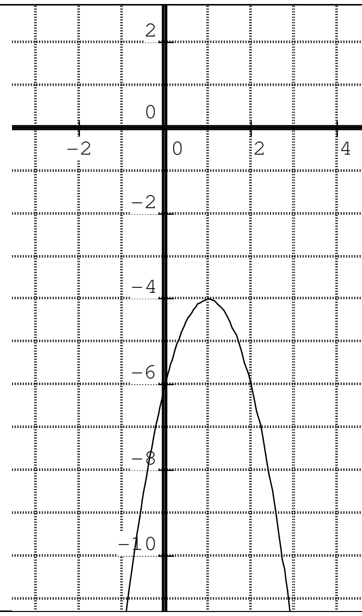
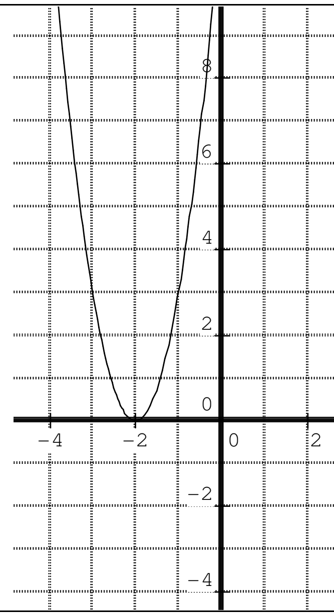
Date	Topics	Done?
	Vertex Form Section 4.1 p204 #4,6,7,9,10,12	
	Completing the Square Section 4.2 p214 #7,10,11,13 & EXTRA Handout	
	Quadratic Formula Section 4.3 p222 #5,6,8,9 & EXTRA Handout	
	Nature of Roots Section 4.4 p232 #4,5,6,7,9,12	
	Solve Problems Section 4.5 p240 #5,7,8,9,10,11,14	
	Quadratic Models Section 4.6 p251 #6,8,11 & Handout	
	one EXTRA assignment on Quad Strategies	

- ☐ I am <u>prepared for the test/evaluation</u> if
  - ☐ I understand the main concepts from each lesson
    - if not, ask other students in class to help you study or visit the peer tutoring room or ask the teacher for help or get a private tutor
    - also practice "knowledge-understanding" questions from the textbook – look for questions marked by **K**
  - ☐ I can explain/communicate the ideas clearly
    - if not, practice explaining a solved question to someone else or complete the assigned journal questions
    - also practice "communication" questions from the textbook – look for questions marked by **C**
  - ☐ I can apply these concepts in word problems
    - If not, practice "application" questions from the textbook – look for questions marked by **A**
  - ☐ I did not just memorize steps to do for different types of questions, I understand the ideas behind each concept and therefore can do problems in new contexts
    - If not, practice "thinking-inquiry-problem-solving" questions from the textbook – look for questions marked by **T**
  - ☐ I can do questions independently
    - if not, try redoing an already solved example without looking at solutions
  - ☐ I can complete questions quickly and with confidence
    - If not, try timing yourself for similar type questions to see progress
  - ☐ I completed the review and/or practice test

<u>Corrections for the textbook answers:</u>

Vertex Form

1. Examine the following functions and their graphs to determine what the vertex form of a quadratic function tells you about its graph.

	$f(x) = -3(x+1)^2 + 2$	$g(x) = -2(x-1)^2 - 4$	$h(x) = 3(x+2)^2$
			
# of zeros			
axis of symmetry			
vertex			
optimal value			
***y*-intercept**			

2. Summarize what you should know from vertex forms:

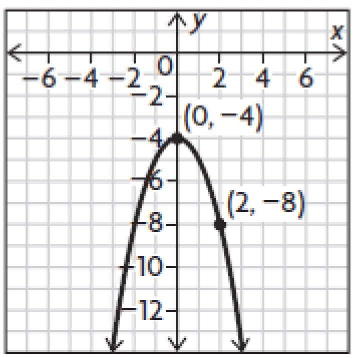
3. For each quadratic find the vertex, a.of.s, is it max or min, range and sketch.

a. $f(x) = -4(x+3)^2 - 5$

b. $f(x) = 2(x-6)^2 + 1$

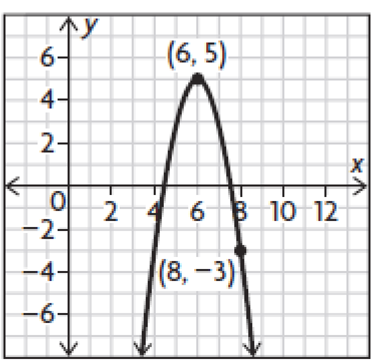
4. For each of the following find the equations in vertex forms.

a.



b. A function has a vertex of (1, – 4) and a *y*-intercept of 6.

c.



d. A function has a vertex of (1, – 12) and passes through the point (5, 36).

5. Summarize all three forms of a quadratic and explain what is easily determined from each

6. A rocket travels according to the equation $h = -4.9(t-6)^2 + 182$, where h is the height, in metres, above the ground and t is the time, in seconds.
   a. When does the rocket reach its maximum height?
   b. What is maximum height?
   c. What is the height at launch?
   d. When did the rocket reach the height of 170 m?

7. Given that the parabola has zeros at (-1, 0) and (3, 0) and goes through a point (4, 5) find the vertex form

COMPLETING THE SQUARE

Completing the square is a process used to change standard form to vertex form by creating a perfect square in the expression, and then factoring the square.

INSTRUCTIONS	EXAMPLE #1 $f(x)=2x^2+12x-3$	EXAMPLE #2 $f(x)=-5x^2+20x+2$	EXAMPLE #3 $f(x)=-3x^2+42x-129$
1. Factor out the constant a from both x^2 and the x terms.	$f(x)=2(x^2+6x)-3$		
2. Find the constant that must be added <u>and</u> subtracted to create a perfect square. *(The value equals the square of half of the coefficient of the x term found in step 1.)* $\left(\frac{b}{2}\right)^2$ Rewrite the expression by adding, then subtracting this value after the x-term inside the brackets.	*The constant to be added and subtracted is* $\left(\frac{b}{2}\right)^2=\left(\frac{6}{2}\right)^2=3^2=9.$ $f(x)=2(x^2+6x+9-9)-3$		
3. Group the three terms that form the perfect square. Move the subtracted value outside the brackets by multiplying it by the a.	$f(x)=2(x^2+6x+9)-9(2)-3$		
4. Factor the perfect square and collect like terms.	$f(x)=2(x+3)^2-21$		

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Completing the Square

1. Complete the square to express each function in vertex form. Then graph each, and state the domain and range.

a. $f(x) = -\frac{1}{2}x^2 + 20x + 8$

b. $g(x) = 3x^2 - 15x + 75$

A submarine traveling in a parabolic arc ascends to the surface. The path of the submarine is described by $y = 2x^2 - 10x - 50$, where x represents the time in minutes nad y represents the submarines depth in meters. What is the minimum distance from the ocean's floor that the submarine ever reaches. Assume the ocean floor in that area is 100m.

A certain 120V electrical circuit has a resistance of 12 amps. The power P in watts that can be produces in the circuit when a current i in amperes is flowing is given by $P(i) = -12i^2 + 120i$. Find the maximum power that can be produced in the circuit.

State the transformations of $y = x^2$ to produce the graph of $y = -3x^2 + 12x - 9$

Quadratic Formula

1. Summarize the quadratic formula and when you are allowed to use it.

2. Find the roots of the following

a. $x^2 - x = 5$

b. $x(2x-3) = 7$

3. There are several methods that you can use to find the roots of an equation. One of these methods always works, however there are shortcuts that can be used in some cases. Summarize what they are and when to use them.

4. Identify a method that could be used to determine the roots of the given equations. Then use it to find the roots

a. $3x^2 = 18x$

b. $x^2 = 40$

c. $(x-1)(x+2) = (3x+2)(x+2)$

d. $-x^2+5x=3$ e. $-2(x+4)^2+98=0$ f. $x^2-6x-7=0$

5. A baseball player throws a ball into the air. If the equation that represents the ball path is $h=-2t^2+6t+8$, where h represents height in feet and t represents time in seconds.
   a. What is the initial height of the ball?
   b. How long was the ball in the air?
   c. What is the maximum height of the ball?
   d. What is the height of the ball after 1 sec?
   e. When did the ball reach the height of 10 feet on its way down?

6. A rocket ship is attempting to land on the moon. The ship's computers calculate that the height of the ship above the moon's surface can be modelled by the equation $h(t) = -1.6t^2 - 1.45t + 200$, where h(t) is in meters and t is in seconds
   a. The ship's pilot must decide whether the current spot is suitable for landing. He must make the decision before the ship is less than 50 meters off the ground or else it will be too late to change course. How long does he have?
   b. Assuming that the pilot chooses to land in the current place, how long from the initial reading will it be before the ship touches down?

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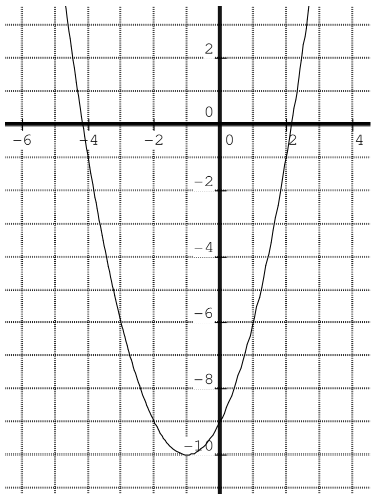
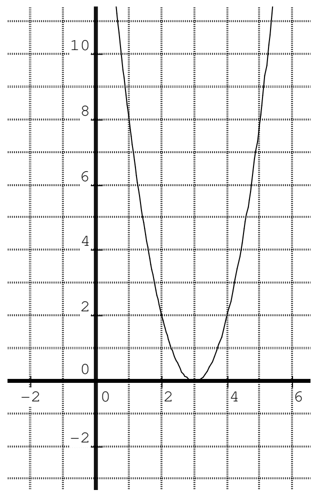
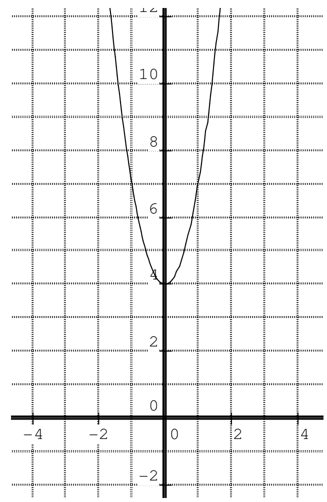
Nature of Roots

1. Complete the table

	$f(x)=x^2+2x-9$	$g(x)=2x^2-12x+18$	$h(x)=3x^2+4$
			
# of Zeros			
Calculate the Roots (use the quadratic formula)			
Value Under $\sqrt{}$ (+,–, or 0)			

2. Summarize how to tell how many roots the equation will have if you are given the following forms

STANDARD FORM

VERTEX FORM

3. Determine the number of real solutions each equation has. Do not solve.

a. $2x = x^2 + 3$ b. $3(x-4)^2 - 1 = 0$ c. $2x^2 + 5x = 6$

4. Determine the number of x-intercepts the function has. Do not solve.

a. $f(x) = 100x^2 + 60x + 9$ b. $f(x) = -2(x+1)^2 - 5$ c. $f(x) = -4(x-9)^2$

5. For what value(s) of k does the function have no zeros $f(x) = kx^2 + 6x + k$

6. For what value(s) of k does the equation have two solutions $4x^2 - 2x + k = 0$

7. For what value(s) of k does the function have one x-intercept $f(x) = x^2 + kx - k + 2$

The function $P(x) = -25x^2 + 2500x + 825$ models the profit earned by a dance studio on the basis of the cost of a dance lesson, x. Does the dance studio ever break even?

Solve Problems

1. There are several strategies that allow you to solve a quadratic word problem. They are listed below; identify which list refers to finding maximum/minimum values and which list solves for the zeros/roots of the equation.

In order to find_________________________________ use
- Table of values
- Graphing
- Factoring
- Quadratic formula

In order to find_________________________________ use
- Table of values
- Graphing
- Find zeros then find a.of s. and opt val
- Completing the Square

These lists are not complete. There are other things that you can do too, like substitute into the equation the given values before you solve, or expand the equation to get another form first… etc.

Now practice identifying what strategy is most efficient to solve the following if you have no access to graphing technology

2. STANDARD form
$$y = 2x^2 + 5x - 3$$
   a. Find min value
   b. Find y if x= – 1
   c. Find when min value occurs
   d. Find x if y=0
   e. Find x if y= – 3

3. FACTORED form
$$y = (2x+1)(x-3)$$
   a. Find vertex
   b. Find x-intercepts
   c. Find initial value

4. VERTEX form
$$y = 2(x - \frac{5}{4})^2 - \frac{49}{8}$$
   a. Find axis of symmetry
   b. Find zeros
   c. Find y-intercept

5. A farmer is building a new pig sty on the side of his barn. He has 60 m of fencing. The area that can be enclosed is modelled by the function $A(x) = -2x^2 + 60x$, where x is the width of the sty in metres, and A(x) is the area in square metres. What is the maximum area that can be enclosed?

6. The manager of a grocery store sells 1250 bags of milk for $2 each. He wants to know how much money he will earn if he increases the price in 10¢ increments, which lower the quantity sold by 20 bags. A model of the revenue function is

$$
\begin{aligned}
R(x) &= (price)(quantity) \\
&= (2+0.10x)(1250-20x), \\
&= -2x^2+85x+2500
\end{aligned}
$$

where x is the number of 10¢ increments and R(x) is the revenue in dollars.

a. Explain how the equation can be set up from the wording of the problem.
b. What is the maximum revenue?
c. What price yields the maximum revenue?
d. What is the revenue when the price of milk is $2.40.

7. The population of a small town is modelled by the function $P(t) = 5t^2 + 120t + 20000$, where P(t) is the population and t is the time in years since 2000.
   a. When will the population be 25 000?
   b. What will the population be in 2025?
   c. When does minimum population occur?
   d. What is the minimum population?
   e. Will the population ever be zero? Explain.

Quadratic Models

1. A thrown ball has the following heights at various times.

t	h
0	10
1	10
2	9
3	7
4	4

a. Find an equation, in vertex form, that represents this data
b. When does the ball land on the ground?

2. Kim practices the javelin throw. The distance and height of one of the javelin throws are given in the table

d	h
1	4.2
3	7.4
5	9
7	9
9	7.4

a. Find an equation, in vertex form, that represents this data
b. How far does the javelin travel before hitting the ground?

3. The community garden club has a vegetable garden that measures 15 m by 30 m. One of the members has donated a new piece of land for a larger garden. They plan to increase the garden by 250 m^2. However, because of the dimension of the new land, both dimensions of the original garden must increase by the same amount. Determine the dimensions of the new garden.

4. A farmer wishes to enclose a rectangular field with 48 metres of fencing.
    a. Write down, in function notation, the area of the field in terms of length
    b. Determine the dimensions that will make the area maximum and find the maximum area

5. A framed picture has length 35 cm and width 25 cm. The picture itself has area 375 cm^2. How far is it from the edge of the picture to the edge of the frame if this distance is uniform around the picture?

6. A parabolic arch is built over a river. The bottoms of the arch touch the ground 40 meteres from the left bank of the river and 20 metres from the right bank. The river is 30 metres wide. The arch is 200 metres tall at its highest point
   a. Write an equation which models this arch, using the centre of the river for $x=0$
   b. A daredevil wishes to dive into the river from a height of 150 metres. At what x-position should a platform be built on the arch for this stunt? Note that the platform must be directly above the river.