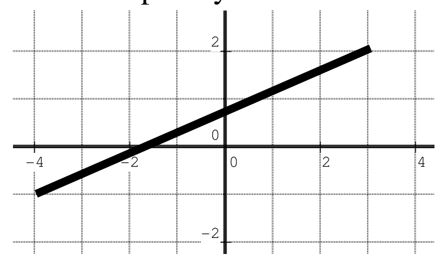
STRAND REVIEW: QUADRATICS

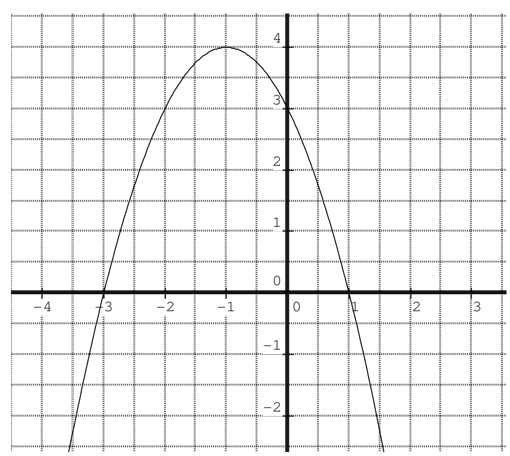
Answer each of the following questions in the space provided. Show all steps of work.

1. State the domain and range for each. Is the relation a function? Explain your answer.
   a. {(-5, -5), (-5, -4), (-8, -4), (-3, -5)}
   b.



2. For the functions $f(x) = -2x^2 + 10x - 14$ and $g(x) = -9x + 3$,
   a. find $g(2)$
   b. find $f(-1)$
   c. find $g(f(1))$

3. For the graph of the quadratic function, state



   a. the zero(s) __________________
   b. the axis of symmetry __________________
   c. the optimum value __________________
   d. the vertex __________________
   e. the domain and range __________________

   ____

   f. the equation of the function in standard form

4. Given the equation of a quadratic function in standard form, $f(x) = x^2 + 8x + 12$, state
   a. the equation in factored form

   b. the zero(s)

   c. the axis of symmetry

   d. the optimum value

   e. the vertex

   f. the y-intercept

   g. the domain and range

   h. whether the parabola has a maximum or minimum and explain why

5. Given the equation of a quadratic function in standard form, $f(x) = 3x^2 + 12x + 15$, state
   a. the equation in vertex form

   b. the number of zeros and explain how you know

   c. the axis of symmetry

   d. the optimum value

   e. the vertex

   f. the domain and range

   g. the y-intercept

   h. the transformations of the graph $y = x^2$

6. Given the equation of a quadratic function in standard form, $f(x) = -x^2 + 8x + 3$, state
   a. the y-intercept

   b. the number of zeros and explain how you know

   c. whether the parabola has a maximum or minimum and explain why

7. Find the equation in standard form of each quadratic function.
   a. Find the equation in factored form for a quadratic function that has zeros at – 8 and 2 and has a y-intercept of 32.

   b. Find the equation in vertex form for a quadratic function that has its vertex at (– 1, 6) and passes through the point (1, 2).

8. Find the roots of each quadratic equation using the most appropriate method.

a. $5x^2+11x+2=0$

b. $4x^2+6x=6x+100$

c. $x^2+4x+15=-4x$

d. $x^2-4x=21$

e. $x^2+5x=1$

f. $2x^2-8x+15=0$

g. $4x^2+12x+9=0$

9. Put each quadratic equation into vertex form by completing the square and state the vertex.

a. $5x^2+10x+2=0$

b. $4x^2+24x+9=0$

c. $2x^2-8x+3=0$

10. Connor is trying out a mini remote controlled submarine in a swimming pool. As the submarine ascends to the water's surface and then dives, its depth can be modeled by the function $d(t) = -0.5t^2 + 10t - 50$, where $d(t)$ is the depth of the submarine, in centimeters, at t seconds.

    a. What is the initial depth of the mini submarine?

    b. How long will it take the mini submarine to reach the water's surface?

11. A dolphin jumps out of the water. Its height, in metres, above the water is modeled by the quadratic function $h(t) = -0.2t^2 + 2t$, where t is in seconds. When will the dolphin reach a height of 1.8 m?

12. The quadratic function $P(x) = -30x^2 + 360x + 785$ models the profit earned by a theatre, where P is the profit, in dollars, and x is the ticket price, in dollars.

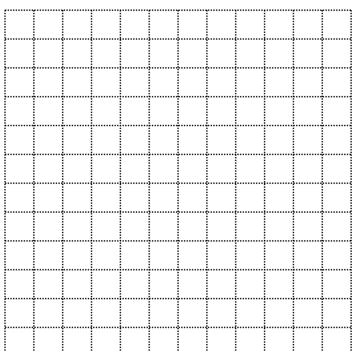
    a. What is the maximum profit?

    b. How much should each ticket cost to maximize the profit?

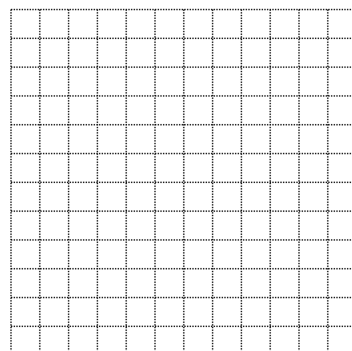
13. The profit of a bicycle manufacturer can be modeled by the quadratic function $P(c) = -2c^2 + 14c - 20$, where P is the profit, in hundreds of thousands of dollars, and c is the number of bicycles produced, in thousands. Find the maximum profit the company can earn.

14. Sketch a graph of each quadratic function investigated in questions 4 and 5.

a.



5.



15. A framed picture has length 23 cm and width 25 cm. The picture itself has area 360 cm^2. How far is it from the edge of the picture to the edge of the frame if this distance is uniform around the picture?

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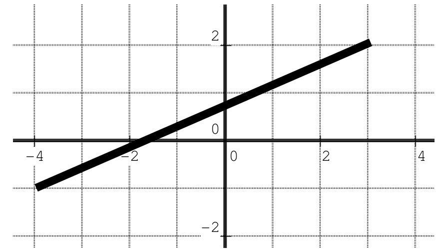
1. State the domain and range for each. Is the relation a function? Explain your answer.

   a. {(-5, -5), (-5, -4), (-8, -4), (-3, -5)}

   Domain = {-5, -8, -3}
   Range = {-5, -4}
   Not a function — one x goes to 2 different y's

   b.



   D = {$x \in \mathbb{R}$, $-4 \le x \le 3$}
   R = {$y \in \mathbb{R}$, $-1 \le y \le 2$}

2. For the functions $f(x) = -2x^2 + 10x - 14$ and $g(x) = -9x + 3$,

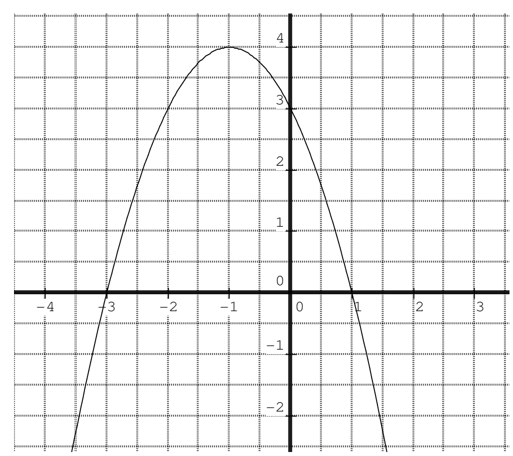
   a. find $g(2)$ b. find $f(-1)$ c. find $g(f(1))$

   a. $g(2) = -9(2) + 3$
   $= -18 + 3$
   $= -15$

   b. $f(-1) = -2(-1)^2 + 10(-1) - 14$
   $= -2 - 10 - 14$
   $= -26$

   c. $f(1) = -2(1)^2 + 10(1) - 14$
   $= -2 + 10 - 14$
   $= -6$
   $g(-6) = -9(-6) + 3$
   $= 54 + 3$
   $= 57$

3. For the graph of the quadratic function, state



   a. the zero(s) $(-8, 0)$ $(2, 0)$
   b. the axis of symmetry $x = -1$
   c. the optimum value $y = 4$ MAX
   d. the vertex $(-1, 4)$
   e. the domain and range $x \in \mathbb{R}$; $y \in \mathbb{R}, y \le 4$
   f. the equation of the function in standard form

$y = a(x - r)(x - t)$
$y = a(x - -8)(x - 2)$ sub pt. $(-1, 4)$
$4 = a(1 + 8)(1 - 2)$
$4 = a(9)(-1)$
$4 = -9a$
$-\frac{4}{9} = a$

factored form $y = -\frac{4}{9}(x + 8)(x - 2)$
FOIL for standard
$y = -\frac{4}{9}(x^2 - 2x + 8x - 16)$
$y = -\frac{4}{9}(x^2 + 6x - 16)$

4. Given the equation of a quadratic function in standard form, $f(x) = x^2 + 8x + 12$, state

a. the equation in factored form

$y = (x+2)(x+6)$

b. the zero(s)

$(-2,0)$ and $(-6,0)$

c. the axis of symmetry

$\frac{-2+-6}{2} = \frac{-8}{2} = -4$

d. the optimum value

$(-4)^2 + 8(-4) + 12$
$= 16 - 32 + 12$
$= -4$

e. the vertex

$(-4,-4)$

f. the y-intercept

c - value $(0,12)$
or
sub $x=0$

g. the domain and range

$D = \{x \in \mathbb{R}\}$
$R = \{y \in \mathbb{R},\ y \geq -4\}$ opens up

h. whether the parabola has a maximum or minimum and explain why

$a = 1$ $\therefore$ opens up $\therefore$ MIN

(OR opt. val is neg and it has two zeros)

5. Given the equation of a quadratic function in standard form, $f(x) = 3x^2 + 12x + 15$, state

a. the equation in vertex form

$3(x^2 + 4x \qquad) + 15$

$\left(\frac{b}{2}\right)^2 = \left(\frac{4}{2}\right)^2 = 2^2 = 4$

$3(x^2 + 4x + 4 - 4) + 15$

$\rightarrow 3(x^2 + 4x + 4) - 4(3) + 15$

$3(x+2)(x+2) - 12 + 15$

$3(x+2)^2 + 3$

b. the number of zeros and explain how you know

# of zeros $b^2 - 4ac$ from standard

OR sign of "a" and "k" of vertex form

$b^2 - 4ac$
$= 12^2 - 4(3)(15)$
$= 144 - 180$
$= -36$ $\therefore$ no zeros

OR a and k BOTH positive $\rightarrow$ opens up and shift up $\therefore$ no zeros

c. the axis of symmetry

$h = -2$

d. the optimum value

$k = 3$

e. the vertex

$(-2, 3)$

f. the domain and range

$D = \{x \in \mathbb{R}\}$
$R = \{y \in \mathbb{R},\ y \geq 3\}$

g. the y-intercept

c value $(0,15)$
OR
sub $x=0$

h. the transformations of the graph $y = x^2$

only seen from vertex form

$y = 3(x+2)^2 + 3$

left

up

vertical stretch

6. Given the equation of a quadratic function in standard form, $f(x) = -x^2 + 8x + 3$, state

   a. the y-intercept

   c-value   OR sub $x=0$

   $(0,3)$

   b. the number of zeros and explain how you know

   $b^2 - 4ac$

   $= 8^2 - 4(-1)(3)$

   $= 64 + 12$

   pos $\therefore$ TWO zeros

   c. whether the parabola has a maximum or minimum and explain why

   MAX   since $a = -1$ opens down

7. Find the equation in standard form of each quadratic function.

   a. Find the equation in factored form for a quadratic function that has zeros at – 8 and 2 and has a y-intercept of 32.

   $y = a(x - r)(x - t)$

   $y = a(x - -8)(x - 2)$ $(0, 32)$ (x y)

   $32 = a(0 + 8)(0 - 2)$

   $32 = -16a$

   $-2 = a$ $\therefore y = -2(x + 8)(x - 2)$

   b. Find the equation in vertex form for a quadratic function that has its vertex at (– 1, 6) and passes through the point (1, 2).

   (h k)

   $y = a(x - h)^2 + k$

   $y = a(x - -1)^2 + 6$ pt. $(1,2)$

   $2 = a(1 + 1)^2 + 6$

   $2 = 4a + 6$

   $-4 = 4a$

   $-1 = a$ $\therefore y = -1(x + 1)^2 + 6$

make $=0$, Factor or Quad. Formula

8. Find the roots of each quadratic equation using the most appropriate method.

a. $5x^2+11x+2=0$

$(5x+1)(x+2)=0$

$5x+1=0$ $\quad$ $x+2=0$

$5x=-1$ $\quad$ $x=-2$

$x=-\frac{1}{5}$

b. $4x^2+6x=6x+100$

$4x^2+6x-6x-100=0$

$4x^2-100=0$

diff. of sq.

$(2x+10)(2x-10)=0$

$2x+10=0$ $\quad$ $2x-10=0$

$2x=-10$ $\quad$ $2x=10$

$x=-5$ $\quad$ $x=5$

c. $x^2+4x+15=-4x$

$x^2+4x+15+4x=0$

$x^2+8x+15=0$

$(x+3)(x+5)=0$

$x=-3$ $\quad$ $x=-5$

d. $x^2-4x=21$

$x^2-4x-21=0$

$(x+3)(x-7)=0$

$x=-3$ $\quad$ $x=7$

e. $x^2+5x=1$

$x^2+5x-1=0$

can't

$x=\frac{-5\pm\sqrt{5^2-4(1)(-1)}}{2(1)}$

$x=\frac{-5\pm\sqrt{29}}{2}$

$x=\frac{-5+\sqrt{29}}{2}\doteq 0.19$

OR

$x=\frac{-5-\sqrt{29}}{2}\doteq -5.19$

f. $2x^2-8x+15=0$

can't

$x=\frac{+8\pm\sqrt{(-8)^2-4(2)(15)}}{2(2)}$

$x=\frac{8\pm\sqrt{-56}}{4}$

can't

$\therefore$ no solutions

h. $4x^2+12x+9=0$

9. Put each quadratic equation into vertex form by completing the square and state the vertex.

a. $5x^2+10x+2=0$

$5(x^2+2x\qquad)+2$

$\left(\frac{2}{2}\right)^2=1^2=1$

$5(x^2+2x+1-1)+2$

$5(x^2+2x+1)-1(5)+2$

$5(x+1)(x+1)-5+2$

$5(x+1)^2-3$

vertex $(-1,-3)$

b. $4x^2+24x+9=0$

$4(x^2+6x\qquad)+9$

$\left(\frac{6}{2}\right)^2=3^2=9$

$4(x^2+6x+9-9)+9$

$4(x^2+6x+9)-9(4)+9$

$4(x+3)(x+3)-36+9$

$4(x+3)^2-27$

$\therefore$ vertex $(-3,-27)$

c. $2x^2-8x+3=0$

$2(x^2-4x\qquad)+3$

$\left(\frac{-4}{2}\right)^2=(-2)^2=4$

$2(x^2-4x+4-4)+3$

$2(x^2-4x+4)-4(2)+3$

$2(x-2)(x-2)-8+3$

$2(x-2)^2-5$

$\therefore$ vertex $(2,-5)$

10. Connor is trying out a mini remote controlled submarine in a swimming pool. As the submarine ascends to the water's surface and then dives, its depth can be modeled by the function $d(t) = -0.5t^2 + 10t - 50$, where $d(t)$ is the depth of the submarine, in centimeters, at t seconds.

a. What is the initial depth of the mini submarine?

y-int at $t=0$ c-value

-50 or 50 meters of depth

b. How long will it take the mini submarine to reach the water's surface? $d=0$

no words "MAX/MIN" ∴ Quad. Formula must be $= 0$

$$0 = -0.5t^2 + 10t - 50$$

$$t = \frac{-10 \pm \sqrt{10^2 - 4(-0.5)(-50)}}{2(-0.5)}$$

$$t = \frac{-10 \pm \sqrt{0}}{-1}$$

one solution

$t = \frac{-10}{-1} = 10$ sec to reach surface

11. A dolphin jumps out of the water. Its height, in metres, above the water is modeled by the quadratic function $h(t) = -0.2t^2 + 2t$, where t is in seconds. When will the dolphin reach a height of 1.8 m?

$h = 1.8$

no words "MAX/MIN" ∴ Quad. Formula must be $= 0$

$$1.8 = -0.2t^2 + 2t$$

$$0 = -0.2t^2 + 2t - 1.8$$

$$t = \frac{-2 \pm \sqrt{(2)^2 - 4(-0.2)(-1.8)}}{2(-0.2)}$$

$$t = \frac{-2 \pm \sqrt{2.56}}{-0.4}$$

$t = 1$ ← reach in 1 sec

$t = 9$ ← on way down

12. The quadratic function $P(x) = -30x^2 + 360x + 785$ models the profit earned by a theatre, where P is the profit, in dollars, and x is the ticket price, in dollars.

a. What is the maximum profit?

vertex form, opt. val $= k$ ∴ complete the square

$$-30(x^2 - 12x \quad\quad) + 785$$

$$\left(\frac{-12}{2}\right)^2 = (-6)^2 = 36$$

$$-30(x^2 - 12x + 36 - 36) + 785$$

$$-30(x^2 - 12x + 36) - 36(-30) + 785$$

$$-30(x-6)(x-6) + 1080 + 785$$

$$-30(x-6)^2 + 1865$$

∴ max profit is \$1865

b. How much should each ticket cost to maximize the profit?

↳ x part of vertex

\$6 for each ticket.

13. The profit of a bicycle manufacturer can be modeled by the quadratic function $P(c) = -2c^2 + 14c - 20$, where P is the profit, in hundreds of thousands of dollars, and c is the number of bicycles produced, in thousands. Find the maximum profit the company can earn.

complete sq.

$-2(c^2 - 7c \qquad) - 20$

$\left(\frac{-7}{2}\right)^2 = (-3.5)^2 = 12.25$

$-2(c^2 - 7c + 12.25 - 12.25) - 20$

$-2(c^2 - 7c + 12.25) - 12.25(-2) - 20$

-3.5
-3.5

$-2(c - 3.5)(c - 3.5) + 24.5 - 20$

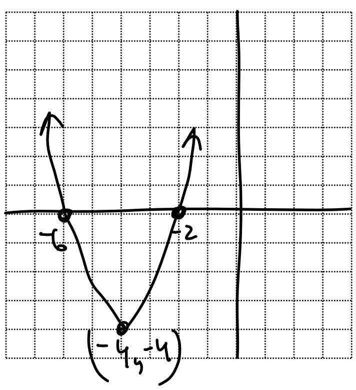
$-2(c - 3.5)^2 + 4.5$ ∴ vertex (c, P)

$(3.5, 4.5)$ ∴ max profit \$4.5 hundred thousand or \$450 000

14. Sketch a graph of each quadratic function investigated in questions 4 and 5.

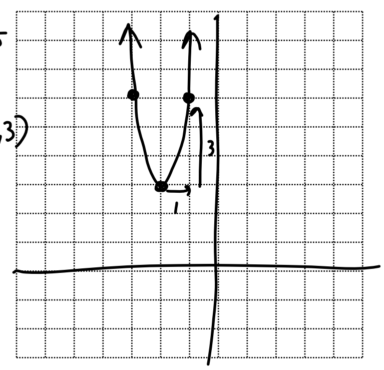
a.

$x^2 + 8x + 12$
$(x+2)(x+6)$
zeros
$(-2, 0)$ $(-6, 0)$
vertex
$(-4, -4)$



5.

$3x^2 + 12x + 15$
$3(x+2)^2 + 3$
vertex $(-2, 3)$
step
3, 15, 27



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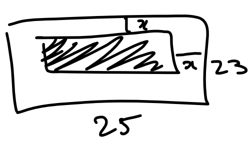

$A = lw$

$360 = (25 - 2x)(23 - 2x)$

$360 = 575 - 50x - 46x + 4x^2$

$0 = 4x^2 - 96x + 215$

$x = \dfrac{--96 \pm \sqrt{(-96)^2 - 4(4)(215)}}{2(4)}$

$x = \dfrac{96 \pm \sqrt{5776}}{8}$

$x = \dfrac{96 \pm 76}{8}$

$x = 21.5$ x can't be so large

$x = 2.5$ ∴ Frame is 2.5 cm wide.