NOTESsomeANS

Notes that are done in class will be updated online periodically

(Any questions left blank you are responsible to try yourself → get help if needed.)

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Quadratics in Standard and Factored Forms Unit

Big idea

You will continue working with quadratics functions for two more units. This unit concentrates on the standard form and the factored form of a quadratic. **Standard form** looks like $f(x)= ax^2+bx+c$, where x^2 term is visible and there are no brackets. **Factored form** looks like $f(x)= a(x-r)(x-t)$, where there is no x^2 term visible, unless you expand, and the equation has brackets (btw it can have only one set of brackets – as long as there is nothing squared). Think of some reasons why standard form is useful and think of some reasons why factored form is useful. Jot the ideas down here.

ex. $y=x(x+2)$ is factored form.

Standard shows y-int = $(0,c)$

Factored shows x-int = zeros = $(r,0)$ and $(t,0)$

This unit will also involve graphing quadratic functions as well as problem solving. There are a lot of real life applications for quadratics. For example, revenue and profit made if you have your own business is modeled by a quadratic relationship, any object that is pulled down by gravity can be represented by parabolas when graphed against time, as well as areas of some shapes can be related to quadratics. There are more applications but you will mainly study these.

Success Criteria
Assessment **as** Learning for Learning and of Learning

Place a ✓ if you are **confident** in that section.
Place a ≈ if you are **just ok** in that section.
Leave it blank if you are **lost** in that section.

If there are gaps in any row, please see the teacher for extra help in that topic.

I know all the prior concepts related to this unit. (If not STOP & complete more review)		I can understand the lesson (If not, ask clarifying questions. Be specific – "what part is unclear?")	I can do a question with an example to follow. (If not, see the teacher for extra help)	I can do questions independently (If not, redo a solved example without looking at solutions)	I can explain/communicate this concept in my own words – JOURNAL (If not, practice explaining steps done in a solved example)	I can apply this concept in other/new contexts/situations (This can be only attained with practice)	I am very confident and am able to complete questions quickly (If not, time yourself to see progress)	I completed the practice in EACH section	I completed the practice test and the review section for this unit.
KU	**Learning Goal**	KU	KU	APP	COMM	TIPS		HW	TEST
Finding equations of and graphing lines, finding equations of and graphing quadratics, simplifying expressions, solving equations, expanding, factoring, problem solving with lines and quadratics	Exploring Situations that involve Quadratics Handout								P184 - Chapter Self-Test P182-183 Chapter Review Questions
	Relating Standard and Factored Forms – 2 days Section 3.2 p139 #2,3,8,11,12 & EXTRA Handout								
	Solving Quadratics by Graphing Section 3.3 p150 #4,6,7,9,10,11 – using technology								
	Solving Quadratics by Factoring – 2 days Section 3.4 p162 #7,9,10,11,13								
	Problem Solving Section 3.5 p168 #1,2,3,9,10								
	Creating Quadratic Models Section 3.6 p176 #1,4,5,7,9								
	one EXTRA Handout on APP&TIPS								

Tentative TEST date________________

Reflect – TEST mark for this unit___________, Overall mark now___________.

Looking back on this unit, what should you plan to improve upon before the exam?

Corrections for the textbook answers:

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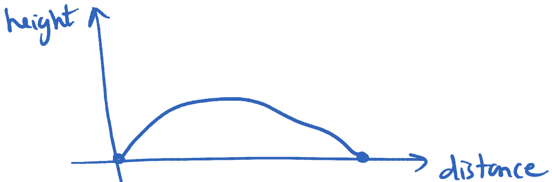
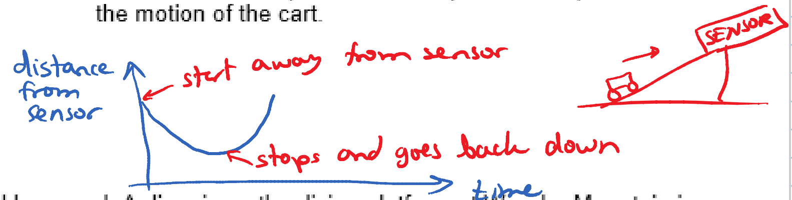
Exploring Situations that involve Quadratics

1. Draw a sketch for each scenario

a. The holder places the football on the ground and holds it for the place kicker. The ball is kicked up in the air and lands down field.



b. A four-wheeled cart is held at the bottom of a ramp. It is given a gentle push so that is rolls part of the way up the ramp, slows, stops and then rolls back down the ramp. A motion detector is placed at the top of the ramp to detect the motion of the cart.

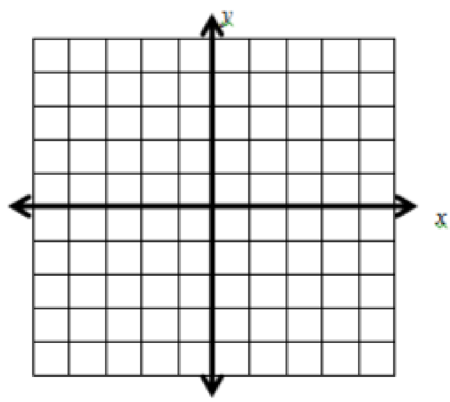


c. A student stands facing a motion detector. He quickly walks toward the detector, slows down, stops and then slowly walks away from the detector. He speeds up as he gets farther away from the detector.

d. A diver is on the diving platform at Wonder Mountain in Canada's Wonderland. She jumps up and dives into the water at the base of the mountain.

2. Graph the data provided in the table of values

x	*y*
−1	2
0	−1
1	−2
2	−1
3	2



3. Label each of the following on the parabola and provide a definition in the space provided. A **parabola** is the name used to describe the shape of a quadratic function.

- Zeros = x-intercepts where the graph crosses the x-axis written as (#, 0)
- Vertex = turning point of the parabola (highest or lowest point)
- Axis of symmetry = vertical line though the vertex equation written as $x = \#$
- Optimal value = MAX or MIN value written as $y = \#$
- Direction of opening for maximum opens down and for minimum opens up

"a" is neg "a" is pos.

Vertex (h, k)

a.of s. opt. val.

$\boxed{\frac{\text{add zeros}}{2}}$ $\boxed{\text{sub in a.of s. into eqn.}}$

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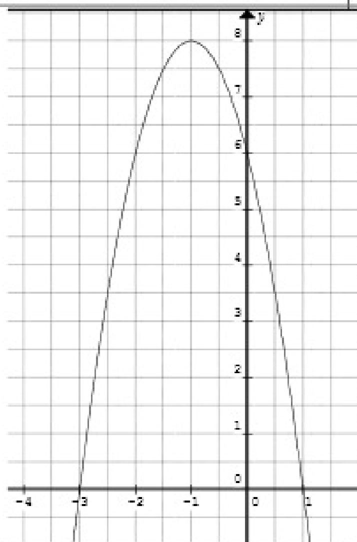
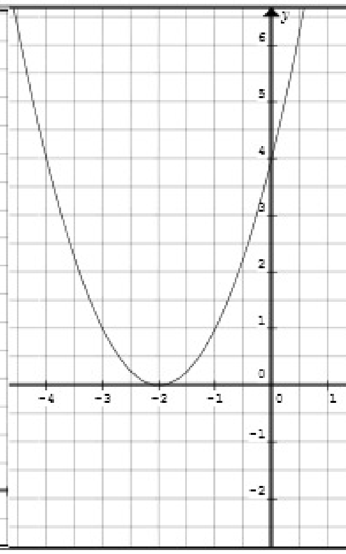
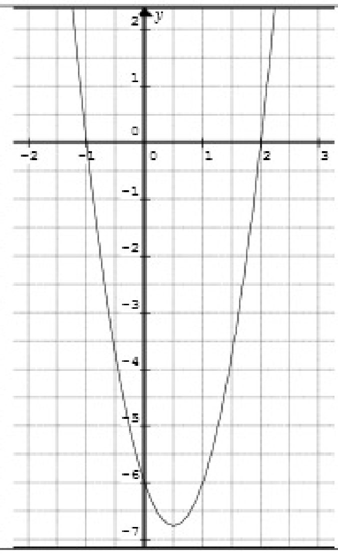
4. Examine the following functions and their graphs to determine what the factored form of a quadratic function tells you about its graph.

	$g(x)=2(1-x)(x+3)$	$h(x)=(x+2)(x+2)$	$f(x)=-(3x+3)(2-x)$
			
zeros			
axis of symmetry			
vertex			
optimal value			
***y*-intercept**			

5. Summarize what you should know from factored forms:

Factored Form : $y=a(x-r)(x-t)$

direction of opening

x-int = zeros = $(r,0)$ and $(t,0)$

Note: Be careful if x has a coefficient on it

ex. $y=2(3-x)(4x+8)$

looks like x-int are $(3,0)$ and $(-8,0)$ But that's also wrong!

looks like $a=2$

But it's actually $a=-8$

Either factor out the coefficients from the x's to create TRUE factored form or think what # for x will make each bracket zero?

$y=2(-x+3)(4x+8)$

$y=2(-1)(x-3)(4)(x+2)$

$y=-8(x-3)(x+2)$

$\therefore$ x-int $(3,0)$ $(-2,0)$

$3-x=0$

$3=x$

$4x+8=0$

$4x=8$

$x=-2$

3

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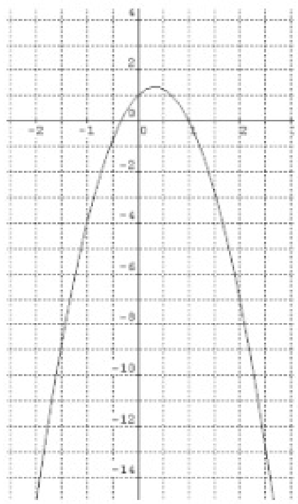
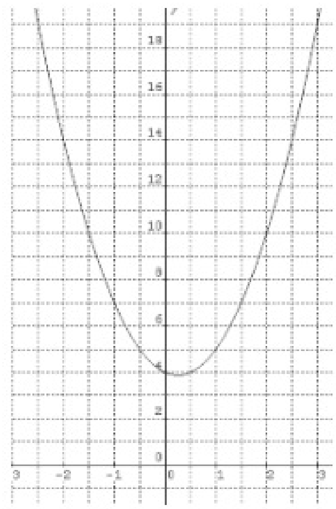
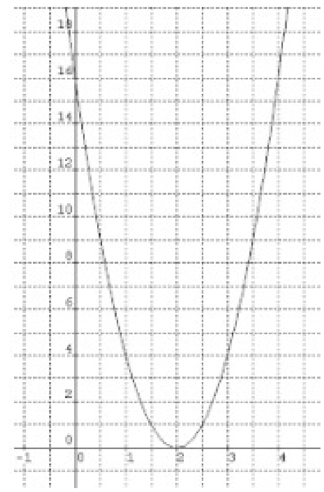
6. Examine the following functions and their graphs to determine what the standard form of a quadratic function tells you about its graph

	$f(x)=-3x^2+2x+1$	$g(x)=2x^2-x+4$	$h(x)=4x^2-16x+16$
			
# of zeros			
optimal value			
***y*-intercept**			

7. Summarize what you should know about standard forms:

Standard Form : $y = ax^2 + bx + c$

direction of opening

y-int $(0,c)$

usually does not mean anything.
If it is a falling object problem then b = initial velocity

* For x-int sub y=0 and solve (each factor separately)

* For y-int sub x=0 and solve

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* a.of s = $\frac{\text{add zero's}}{2}$

* opt.val = sub a.of s. # in.

Relating Standard and Factored Forms

1. Graph each quadratic by finding zeros, vertex, y-intercept

Factored given

a. $f(x) = -(2x+1)(x-1)$

x-int $0 = -(2x+1)(x-1)$

$2x+1=0$, $2x=-1$, $x=-\frac{1}{2}$ $\qquad x-1=0$, $x=1$

$\therefore$ zeros = x-int = $(-0.5, 0)$ and $(1, 0)$

$a.of s = \frac{-0.5+1}{2} = \frac{0.5}{2} = 0.25$

opt.val $= -(2(0.25)+1)(0.25-1)$

MAX

$= -(0.5+1)(-0.75)$

$= -(1.5)(-0.75)$

$= 1.125$

sketch below

$\therefore$ vertex = $(0.25, 1.125)$

y-int $y = -(2(0)+1)(0-1)$

$y = -(1)(-1)$

$y = 1$ $\qquad \therefore$ y-int $(0,1)$

Standard given

b. $f(x) = -4x^2 - 16x + 9$

y-int $y = -4(0)^2 - 16(0) + 9$

$y = 9$ $\qquad \therefore$ y-int = $(0, 9)$

x-int factor $0 = -4x^2 - 16x + 9$

-4: $-2, 2$ / $1, 2$ $\qquad 9$: $1, 9$ / $3, 3$ or two neg.

$0 = (-2x+1)(2x+9)$

$-2x+1=0$, $1=2x$, $0.5 = \frac{1}{2} = x$ $\qquad 2x+9=0$, $2x=-9$, $x=-\frac{9}{2}=-4.5$

$\therefore$ x-int = $(0.5, 0)$ and $(-4.5, 0)$

$a.of s = \frac{0.5 + -4.5}{2} = \frac{-4}{2} = -2$

opt.val $= -4(-2)^2 - 16(-2) + 9$

MAX

$= -4(4) + 32 + 9$

$= -16 + 32 + 9$

$= 25$ $\qquad \therefore$ vertex $(-2, 25)$

c. $f(x) = 6x(x+5)$

$0 = 6(x-0)(x+5)$

x-int $(0,0)$ and $(-5,0)$

$a.of s. = \frac{0 + -5}{2} = -2.5$

opt.val $= 6(-2.5)(-2.5+5)$

MIN

$= 6(-2.5)(2.5)$

$= -37.5$

$\therefore$ vertex $(-2.5, -37.5)$

y-int $y = 6(0)(0+5)$

$= 6(0)(5)$

$= 0$ $\qquad \therefore$ y-int $(0,0)$

d. $f(x) = x^2 - 25$

$0 = x^2 - 25$

$0 = (x+5)(x-5)$

OR

$0 = x^2 - 25$

$25 = x^2$

$\pm\sqrt{25} = x$

$\pm 5 = x$

$\therefore$ x-int $(5, 0)$ and $(-5, 0)$

don't need to find a.of s and opt.val since it is in vertex form $y = (x-0)^2 - 25$

vertex = $(0, -25)$

same

y-int $y = 0^2 - 25$

$y = -25$ $\qquad \therefore$ y-int = $(0, -25)$

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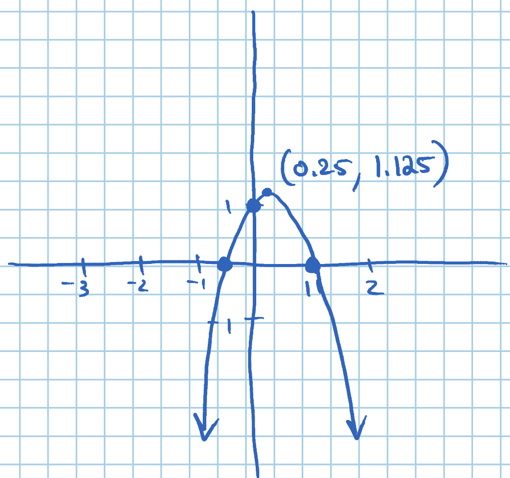
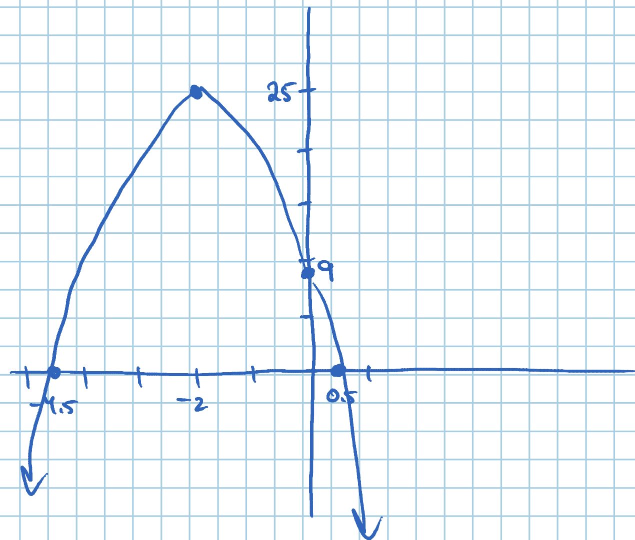
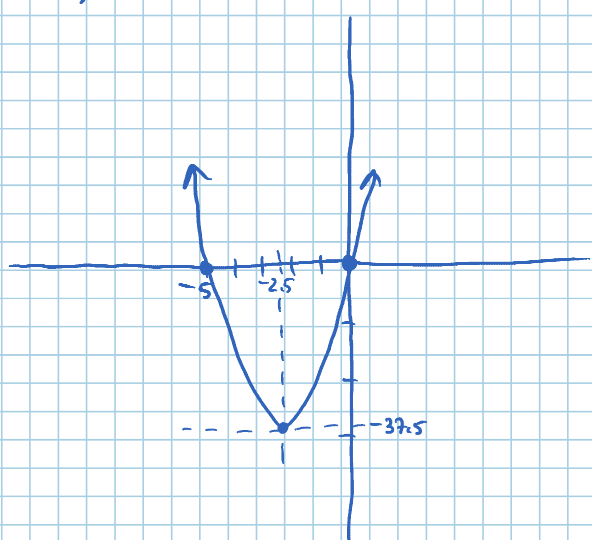
1ⓐ $f(x) = -(2x+1)(x-1)$



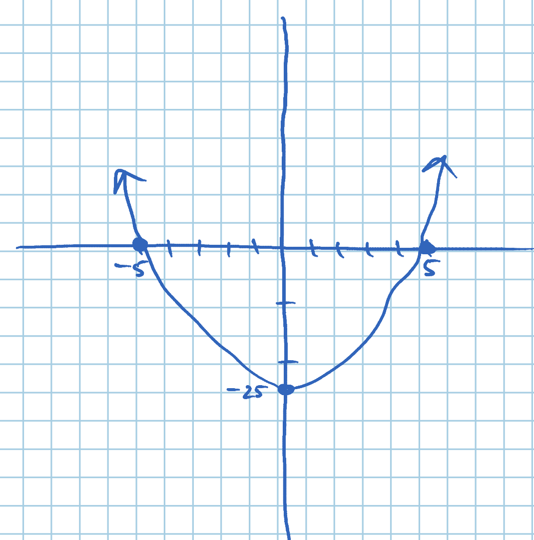
b) $f(x) = -4x^2 - 16x + 9$



c) $f(x) = 6x(x+5)$



d) $f(x) = x^2 - 25$



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e. $f(x) = (3x+8)(3x-8)$

f. $f(x) = x^2 + 18x - 40$

g. $f(x) = (4-x)(4x+5)$

h. $f(x) = -6x^2 + 13x + 5$

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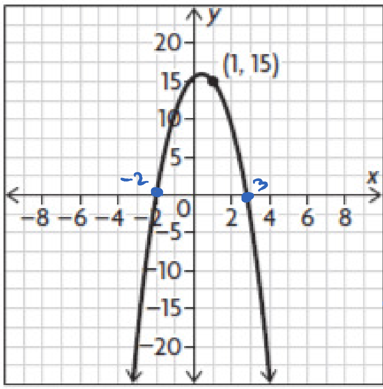
2. Find the equation in factored and standard forms from either graph or description.

a.



x-int $(-2,0)$ r

$(3,0)$ t

random pt

$(1,15)$

$y=a(x-r)(x-t)$

$y=a(x--2)(x-3)$

$15=a(1+2)(1-3)$

$15=a(3)(-2)$

$15=-6a$

$-2.5=a$

$\therefore y=-2.5(x+2)(x-3)$

b. A function has zeros of 2 and -4 and a y-intercept of 16

r: 2, t: -4

pt$(0,16)$

$y=a(x-2)(x--4)$

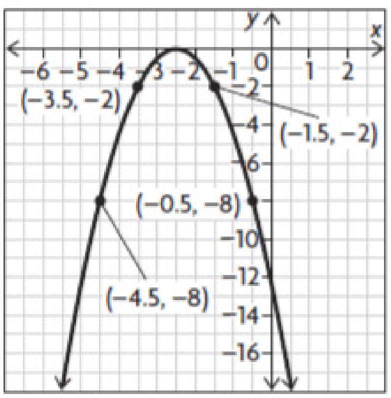
$16=a(0-2)(0+4)$

$16=-8a$

$-2=a$

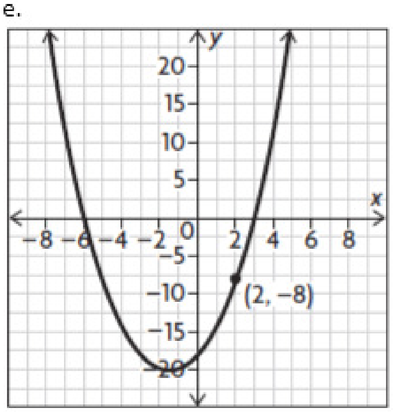
$\therefore y=-2(x-2)(x+4)$

c.



d. A function has zeros of 3 and -1 and passes through the point (5, 36)

e.



f. A function has zeros of ½ and -3 and vertex $\left(-\frac{5}{4}, 10\right)$

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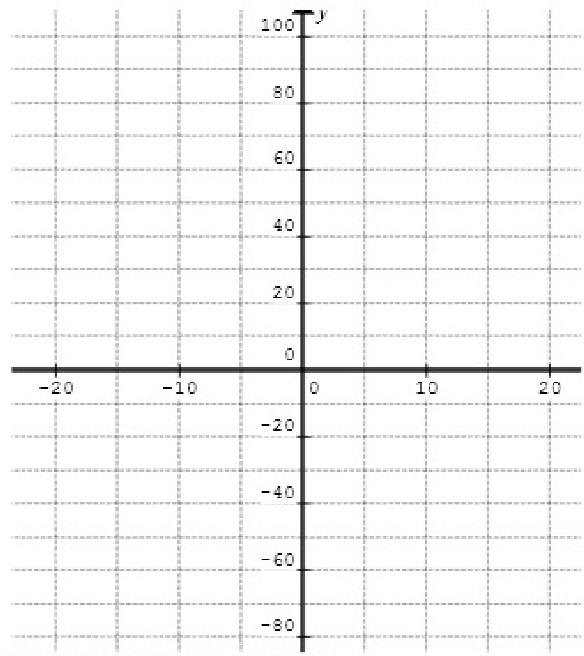
3. A DC electrical circuit is represented by the formula $P = IV - I^2 R$.
the relationship between the power used by a device P (in watts, W)
the electric potential difference (voltage), V (in volts, V)
the current, I (in amperes, A)
the resistance, R (in ohms, Ω)

a. Represent graphically and algebraically the relationship between the power and the current when the electric potential difference is 24 V and the resistance is 1.5 Ω.



b. Determine the current needed in order for the device to use the maximum amount of power.

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Solving Quadratics by Graphing

Some of the questions in the textbook require you to graph with technology. There are lots of applets you can use online, or you can download a free program to use on your computer offline.

Online Graphing Calculator
http://my.hrw.com/math06_07/nsmedia/tools/Graph_Calculator/graphCalc.html

Download GeoGebra (offline and online)
http://www.geogebra.org/cms/en/download, select **webstart**, for offline
select **appletstart**, for online

1. Summarize the following terms, use examples in the explanation

QUADRATIC FUNCTION – has both input and output variables

$f(x) = x^2 - 7x$
or $y = x^2 - 7x$

QUADRATIC EQUATION – has only one variable specifically the input.

ex. $0 = x^2 - 7x$
or $10 = x - 4x^2 + 7$

ZEROS/x-INTERCEPTS

values of x where the function meets the x-axis
*write as coordinate pt

ROOTS

values of x that make the equation true ie. solutions $x = ?$

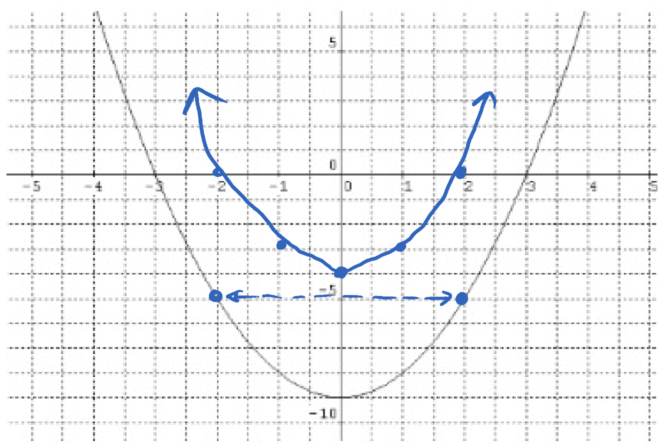
Note: if equation $= 0$ then roots = zeros

2. Here is a quadratic function $f(x) = x^2 - 9$.

Solve for x if y=-5

a. using the graph

if output is $y = -5$
then input is $x = -2$ or $x = 2$



b. using the equation

$-5 = x^2 - 9$
$+9 - 5 = x^2$
$4 = x^2$
$\pm\sqrt{4} = x$
$\pm 2 = x$

c. graph $g(x) = x^2 - 4$

vertex $(0, -4)$
step $\frac{1}{1}, \frac{3}{1}, \frac{5}{1}$

d. how is finding the zeros of $g(x) = x^2 - 4$ quadratic function relate to finding the roots of $-5 = x^2 - 9$ equation?

if you sub $y = -5$ into function $f(x) = x^2 - 9$
it becomes the equation $-5 = x^2 - 9$
$0 = x^2 - 4$

and roots/solutions to that equation are the zeros of $g(x) = x^2 - 4$ function

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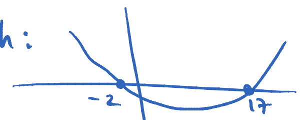
3. What is the corresponding function that has same zeros as the roots to the following equations? Using technology graph the corresponding functions to determine the roots of the equations.

a. $x^2 - 15x = 34$

$x^2 - 15x - 34 = 0$

$\therefore f(x) = x^2 - 15x - 34$ will have zeros that are solutions to the original question

Using graph tech:



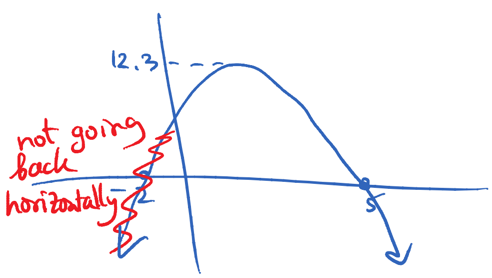
$\therefore$ zeros of $f(x)$ are $(-2, 0)$ and $(17, 0)$

$\therefore$ roots (solutions) of original are $x = -2$ and $x = 17$

b. $-2x^2 - 9x + 4 = 14$

4. A diver jumps off a platform that is 10 meters above the water below, following a path described by $f(x) = -x^2 + 3x + 10$, where x is the horizontal position and f(x) is the corresponding vertical position.

   a. Sketch with the help of technology



   b. How much horizontal distance does the diver cover before hitting the water?

   5 meters

   c. What is the maximum height above the water that the diver attains?

   about 12.3 meters

5. The population of a Canadian city is modelled by $P(t) = 12t^2 + 800t + 40000$ where t is the time in years since the year 2000.

   a. Sketch with the help of technology

   b. According to the model, what will the population be in 2020?

   c. In what year is the population predicted to be 300 000?

If you do not have access to a computer/internet at home to do your homework, take the time in school to sketch the required graphs for the problems assigned. It is also possible to solve all the problems algebraically, but you will practice that later on in this unit.

Date: ______________________ move terms to one side so that the question becomes like finding zeros. Name: ______________________

Solving Quadratics by Factoring

1. Solve by factoring. Verify your solutions.

a. $4x^2+2=x^2-8x+2$

$4x^2+2-x^2+8x-2=0$

$3x^2+8x=0$ common factor

$(x)(3x+8)=0$

$\therefore x=0$ or $3x+8=0$

$3x=-8$

$x=-\frac{8}{3}$

Use your calculator to check:

LS	RS
$4\left(-\frac{8}{3}\right)^2+2$	$\left(-\frac{8}{3}\right)^2-8\left(-\frac{8}{3}\right)+2$
$4\left(\frac{64}{9}\right)+\frac{2\times 9}{1\times 9}$	$\frac{64}{9}+\frac{64\times 3}{3\times 3}+\frac{2\times 9}{1\times 9}$
$\frac{256+18}{9}=\frac{274}{9}$	$\frac{64+192+18}{9}=\frac{274}{9}$ $\therefore LS=RS$

b. $(5x+7)(x-1)=(x-1)(x-2)$ expand 1st then move over

$5x^2-5x+7x-7=x^2-2x-1x+2$

$5x^2+2x-7-x^2+3x-2=0$

$4x^2+5x-9=0$ criss-cross factor

$\begin{pmatrix}4\\1\end{pmatrix}\begin{matrix}2\\2\end{matrix}$ $\begin{pmatrix}9\\-1\end{pmatrix}\begin{matrix}1\ 1\\9\end{matrix}\ \begin{matrix}3\\3\end{matrix}$ one neg

$(4x+9)(x-1)=0$

$\therefore 4x+9=0$ or $x-1=0$

$4x=-9$ $\quad x=1$

$x=-\frac{9}{4}$

check on calc. if LS=RS

c. $7x^2+3x+2=3x^2+3x+3$

d. $(2x+7)(x-3)=3(x+1)(2x-5)$

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2. A farmer wishing to fence in a rectangular area determines that the area enclosed is given by the equation $A(w) = -w^2 + 64w$, where A(w) is the area of the enclosure in square meteres and w is its width in metres. Use factoring to answer the following questions.
   a. What widths will result in the area of 0 square metres?
   b. What widths will result in an area of 183 square metres? What are the lengths of the rectangles that correspond to these widths?
   c. What is the maximum possible area of the enclosure? What are the dimensions of the rectangle that will provide this area?

(a) sub $A=0$ or $A(w)=0$

$0 = -w^2 + 64w$ common factor

$0 = -w(w-64)$

$\therefore w = 0$ or $w - 64 = 0$, $w = 64$

∴ widths of 0m and 64m will give no area

(b) $183 = -w^2 + 64w$

$0 = -w^2 + 64w - 183$

$\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 3 \\ -61 \end{pmatrix}$ one neg

$0 = (-1w + 3)(w - 61)$

$\therefore -w + 3 = 0$, $3 = w$ $\quad w - 61 = 0$, $w = 61$

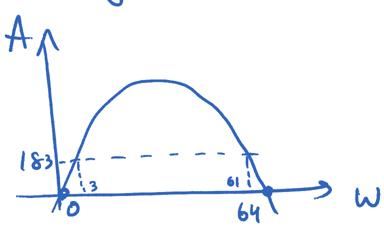
∴ widths of 3m and 61m will give you area 183

$A = lw$, $183 = l(3)$, $61 = l$

$A = lw$, $183 = l(61)$, $3 = l$

∴ when you solve you get both dimensions since it doesn't matter if you call it length or width

(c)



How do you find MAX? need to find the vertex.

$a.o.s = \frac{0+64}{2} = 32$

$opt.val = -(32)^2 + 64(32)$

$= -1024 + 2048$

$= 1024$

∴ vertex $(32, 1024)$
x y
w A

∴ max Area is $1024\,m^2$ for $w = 32$

and $A = lw$

$1024 = l(32)$

$32 = l$

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3. The revenue function for a company that makes board games is $R(x)=10x$, where x and R(x) are both in thousands. The cost function for producing the board games is $C(x)=2x^2-19x+50$, where x and C(x) are also both in thousands. Use factoring to answer the following questions.
   a. Write the profit function for this company
   b. What are the board games to be produced to make a positive profit?
   c. How many board games should the company produce to make the maximum possible profit? What will the maximum profit be?

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Problem Solving

There are 3 methods of solving quadratic word problems

- Using a table of values
- Using a graph
- Using algebra – factoring methods

1. A quarterback who is 2 meters tall throws a football. Its height over time is modelled by the equation $h(t) = -5t^2 + 9t + 2$. Use all three methods to answer the following questions.
   a. When does the ball hit the ground?
   b. What is the maximum height that the ball reaches?

$\boxed{\text{Table of values}}$

t	0	1	2	3
h	2	6	0	-1.6

one zero at $t = 2$
but can't see vertex
choose smaller t values

* a lot of tedious work

t	0	0.2	0.4	0.6	0.8	1	1.2
h	2	3.6	4.8	5.6	6	6	5.6

→ will repeat

t	0.9
h	6.05

∴ vertex right in middle of 0.8 and 1

∴ $\frac{0.8+1}{2} = 0.9$

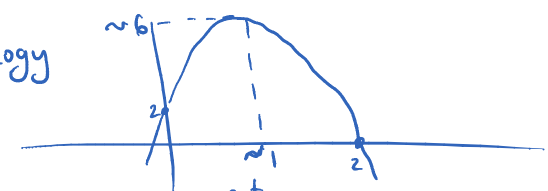
∴ vertex is $(0.9, 6.05)$

∴ ⓐ lands on the ground at $t = 2$ sec

ⓑ MAX height is 6.05 meters

$\boxed{\text{GRAPH}}$ using technology

*not always exact



not clear exactly what # that is

$\boxed{\text{ALGEBRAIC}}$

*Best method :)

$h = -5t^2 + 9t + 2$ criss cross factor

5, −1 2, 1, 1, 2 or two neg.

$h = (5t+1)(-t+2)$

zeros

$5t + 1 = 0$
$5t = -1$
$t = \frac{-1}{5} = \cancel{-0.2}$

time can't be negative

$-t + 2 = 0$
$2 = t$

ⓐ ∴ ball lands on the ground at $t = 2$ sec.

a.o.s $= \frac{2 + -0.2}{2} = \frac{1.8}{2} = 0.9$

opt. val $= -5(0.9)^2 + 9(0.9) + 2$
$= -5(0.81) + 8.1 + 2$
$= -4.05 + 10.1$
$= 6.05$

ⓑ ∴ MAX height is 6.05 meters.

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2. The population of a city is modelled by $P(t) = 0.5t^2 - 9.65t + 100$, where P(t) is the population in thousands and t=0 corresponds to the year 2000. Use graphing technology to answer the questions
   a. In what year did the population reach its minimum value? How low was the population at this time?
   b. When will the population reach 200 000.
   c. Why is graphing technology the best method to use for this question?

3. A company selling CDs models its profits with the equation $P(x) = -3x^2 + 29x - 18$, where x and P(x) are both in thousands. Use factoring methods to answer the following questions.
   a. For what number of CDs produced will the company make a positive profit?
   b. How many CDs should the company produce to make the maximum possible profit?

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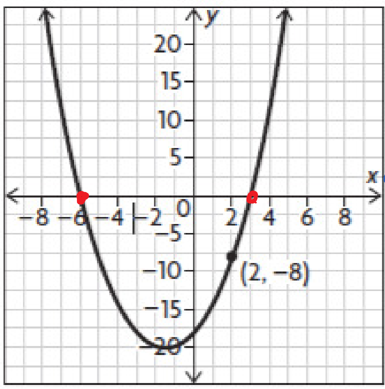
Creating Quadratic Models

Find the equations for each of the following

1.



x-int $(3,0)$ and $(-6,0)$
r t

$y=a(x-r)(x-t)$

$y=a(x-3)(x--6)$

sub pt $(2,-8)$

$-8=a(2-3)(2+6)$

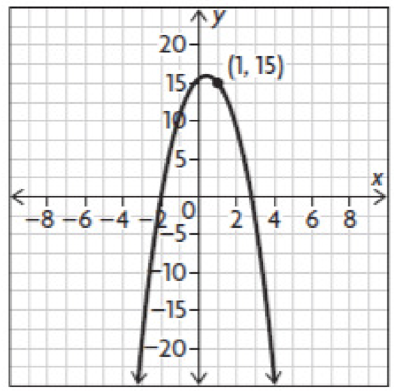
$-8=a(-1)(8)$

$-8=-8a$

$1=a$

$\therefore y=1(x-3)(x+6)$

2.



Use symmetry and/or zeros to find the equations

3.

x	$f(x)$
-4	-36
-3	-20
-2	-8
-1	0
0	4
1	4
2	0

x-int at $(-1,0)$ and $(2,0)$
r t

$y=a(x-r)(x-t)$

$y=a(x--1)(x-2)$

sub any pt.
$(0,4)$

$4=a(0+1)(0-2)$

$4=-2a$

$-2=a$

$\therefore y=-2(x+1)(x-2)$

4.

x	$f(x)$
-1	15
0	24
1	27
2	24
3	15
4	0

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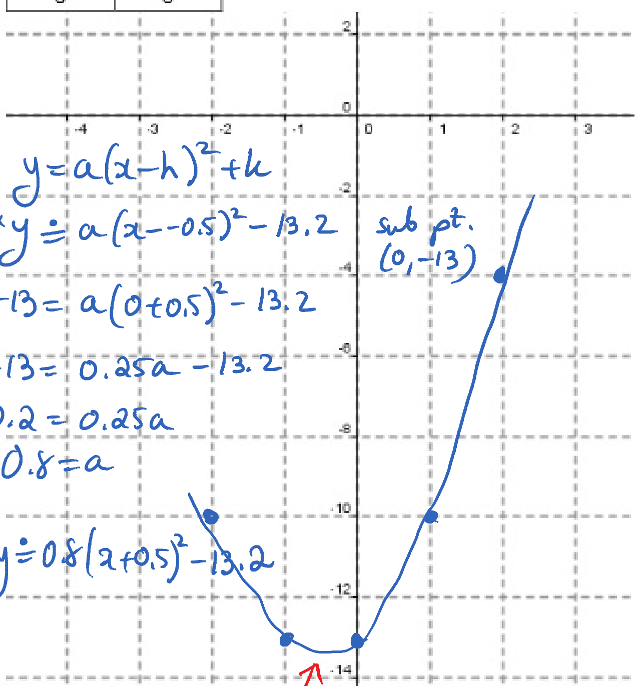
Use graphs to find the equations

5.

x	$f(x)$
-2	-10
-1	-13
0	-13
1	-10
2	-4
3	5



$y = a(x-h)^2 + k$

approx $y \doteq a(x - -0.5)^2 - 13.2$

sub pt. $(0, -13)$

$-13 = a(0+0.5)^2 - 13.2$

$-13 = 0.25a - 13.2$

$0.2 = 0.25a$

$0.8 = a$

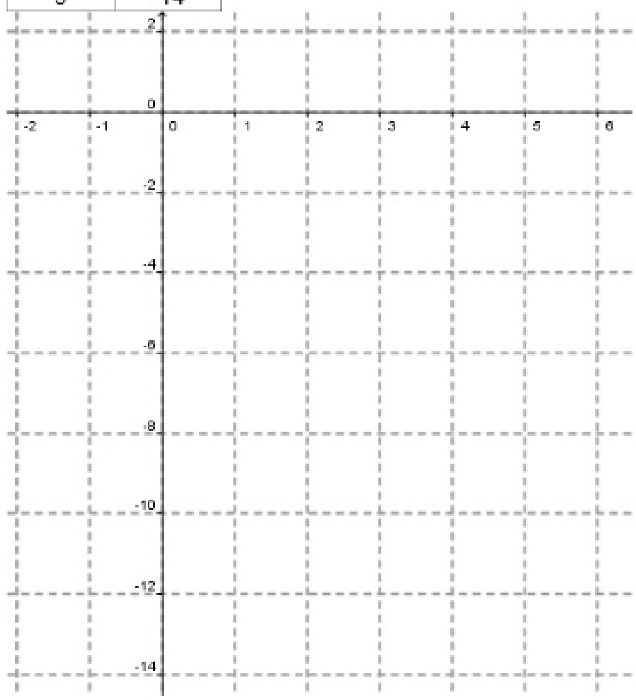
$\therefore y \doteq 0.8(x+0.5)^2 - 13.2$

vertex at $(-0.5, ?)$

can't be clear how low it goes

6.

x	$f(x)$
0	-9
1	-2
2	1
3	0
4	-5
5	-14



7. One zero at $x = -\frac{3}{4}$ and vertex at $\left(\frac{5}{8}, -121\right)$

8. Symmetric about the y-axis, zero at x=3, and y-intercept at y=27.