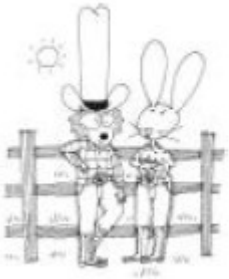
Date: _____ Name: _____

Practice Quadratic Problem Solving

1. The Starks have 24 m of fencing with which to make a rectangular dog run. If they use a side of the house as one side of the run, what dimensions will give the maximum area?

2. A rectangle has a perimeter of 30 meters and an area of 54 square meters. Find its dimensions.



5. Cathy has 28 m of fencing to make a rectangular pen for some rabbits. What dimensions will give a maximum area?

Date: _____ Name: _____

13. A bridge is 100m long and has a maximum height of 40m. Sketch a graph of the bridge and find the quadratic equation that represents this bridge. If a boat that is 30m high sails underneath the bridge and is 20m from shore, will the boat hit the bridge or coast through?

14. At football games they sometimes shoot the teams logo t-shirts out of a big projectile gun into the bleachers. A shirt travels 20m and reaches a maximum height of 60m, if the gun is shot from 3m above the ground. If a person on the bleachers is 40m from the ground and catches the t-shirt, how far is he from the gun (on the horizontal)?

Identify what is missing
Why can't this be solved in this class?

15. A garden measuring 12 meters by 16 meters is to have a pedestrian pathway installed all around increasing the total area to 285 square meters. What will be the width of the pathway?

Date: _____ Name: _____

16. A square garden is to be increased in size by extending the length by $12m$ and the width by $10m$. The area of the new rectangular garden is $675m^2$. What are the dimensions of the garden?

19. A rectangular skating rink measuring $30m$ by $20m$ is doubled in area by adding a strip at one end of the rink, and a strip of the same width along one side of the rink. Find the width of the strips if the enlarged rink is still rectangular.

20. A chemical plant is rectangular and has a length of $100m$ and a width of $60m$. A safety zone of uniform width surrounds the plant. If the area of the safety zone equals the area of the plant, what is the width of the safety zone?

21. When 15m is added to the length of two opposite sides of a square and 5 m is added to the length of the other sides. The area of the resulting rectangle is $441m^2$. Find the length of the sides of the original square.

Date: _____ Name: _____

22. The underside of a bridge has the shape of a parabolic arch. It has a maximum height of 30m and a width of 100m. Can a boat with a height of 20m and a width of 30m pass under the bridge?

23. A man working on a bridge has a life support rope tied to his waist. The bridge is 60m long with a maximum height of 20m from the water below. If the man is working 5m from the centre of the bridge, what would be the maximum length the rope can be so that the man would not hit the water if he fell?

29. Deep within a cave (shaped like a parabolic arch) a bat, 25cm in length, is hanging upside down on the underside of the cave, 1.75m from the highest point in the arch. If the cave is 8m wide and has a maximum height of 3m, how far is the bat from the floor of the cave?

31. A playground, which measures 50m by 35m, is to be doubled in area by adding a strip of uniform width around the outside of the existing area. What is the width of the new strip around the playground?

Date: _____ Name: _____

Practice Quadratic Problem Solving

ANSWERS

1. The Starks have 24 m of fencing with which to make a rectangular dog run. If they use a side of the house as one side of the run, what dimensions will give the maximum area?

$24 = 2w + l$

$24 - 2w = l$

$A = lw$

$A = (24 - 2w)w$

$A = -2w^2 + 24w$ find vertex

$A = -2(w^2 - 12w\ \ )$

$\left(\frac{-12}{2}\right)^2 = (-6)^2 = 36$

$A = -2(w^2 - 12w + 36 - 36)$

$A = -2(w^2 - 12w + 36) - 36(-2)$

$A = -2(w-6)(w-6) + 72$

$A = -2(w-6)^2 + 72$

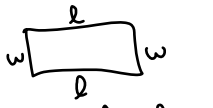
$\therefore$ vertex $(6, 72)$

x (width), y (Area)

$\therefore$ dimensions are $w = 6m$

$l = 24 - 2w = 24 - 2(6) = 12m$

2. A rectangle has a perimeter of 30 meters and an area of 54 square meters. Find its dimensions.



$30 = 2l + 2w$

$15 = l + w$

$15 - w = l$

$A = lw$

$54 = (15 - w)w$ not asked for MAX/min just solve! make $= 0$ first

$0 = -w^2 + 15w - 54$

$-1 \quad 6$

$1 \quad -9$

$0 = (-w+6)(w-9)$

$-w + 6 = 0$ or $w - 9 = 0$

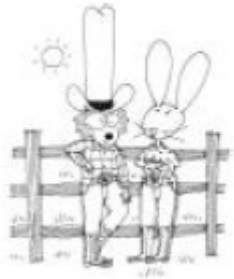
$6 = w$ $\quad w = 9$

$\therefore$ dimensions can be

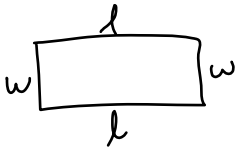
$w = 6m \quad l = 15 - w = 15 - 6 = 9m$

or

$w = 9m \quad l = 15 - w = 15 - 9 = 6m$



5. Cathy has 28 m of fencing to make a rectangular pen for some rabbits. What dimensions will give a maximum area?



$28 = 2l + 2w$

$14 = l + w$

$14 - w = l$

$A = lw$

$A = (14 - w)w$

$A = -w^2 + 14w$ find MAX at vertex

$A = -(w^2 - 14w\ \ )$

$\left(\frac{-14}{2}\right)^2 = (-7)^2 = 49$

$A = -(w^2 - 14w + 49 - 49)$

$A = -(w^2 - 14w + 49) - 49(-1)$

$1 \quad -7$

$1 \quad -7$

$A = -(w-7)(w-7) + 49$

$A = -(w-7)^2 + 49$

vertex $(7, 49)$

x (width), y (Area)

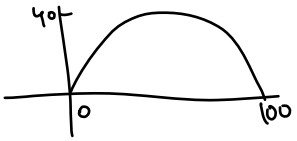
dimensions are

$w = 7m$

$l = 14 - 7 = 7m$

Date: _____ Name: _____

13. A bridge is 100m long and has a maximum height of 40m. Sketch a graph of the bridge and find the quadratic equation that represents this bridge. If a boat that is 30m high sails underneath the bridge and is 20m from shore, will the boat hit the bridge or coast through?



sub pt. (50, 40)

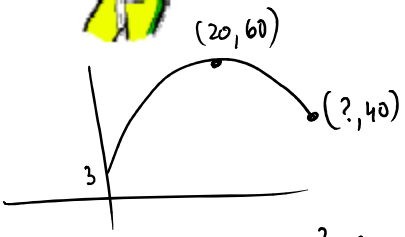
$y = a(x-r)(x-t)$
$y = a(x-0)(x-100)$
$40 = a(50-0)(50-100)$
$40 = a(50)(-50)$
$40 = -2500a$
$-0.016 = a$
$\therefore y = -0.016(x-0)(x-100)$

check if boat < bridge ?

30 | $-0.016(20-0)(20-100)$
$-0.016(20)(-80)$
25.6

No boat is higher it will hit.

14. At football games they sometimes shoot the teams logo t-shirts out of a big projectile gun into the bleachers. A shirt travels 20m and reaches a maximum height of 60m, if the gun is shot from 3m above the ground. If a person on the bleachers is 40m from the ground and catches the t-shirt, how far is he from the gun (on the horizontal)?



$y = a(x-20)^2 + 60$
sub $(0, 3)$

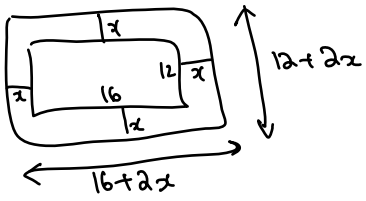
$3 = a(-20)^2 + 60$
$\frac{-57}{400} = a$
$\therefore y = \frac{-57}{400}(x-20)^2 + 60$

sub $y = 40$ find x
$40 = \frac{-57}{400}(x-20)^2 + 60$
$-20\left(\frac{400}{-57}\right) = (x-20)^2$
$\pm\sqrt{\frac{8000}{57}} = x-20$

31.8 or $8.2 = x$
↑ before MAX

$\therefore$ Person catching is 31.8 m horizontally from the gun

15. A garden measuring 12 meters by 16 meters is to have a pedestrian pathway installed all around increasing the total area to 285 square meters. What will be the width of the pathway?



$A = lw$
$285 = (12+2x)(16+2x)$
$285 = 192 + 24x + 32x + 4x^2$
$0 = 4x^2 + 56x - 93$

$4 \cdot 2$, $1 \cdot 2$ $\quad$ $31 \cdot -3$, $3 \cdot 31$

$0 = (2x-3)(2x+31)$

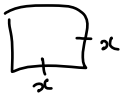
$2x - 3 = 0$
$2x = 3$
$x = 1.5$

$2x + 31 = 0$
$2x = -31$
$x = -15.5$
can't have negative length of sidewalk.

$\therefore$ pathway is 1.5 m wide

Date: _____ Name: _____

16. A square garden is to be increased in size by extending the length by $12m$ and the width by $10m$. The area of the new rectangular garden is $675m^2$. What are the dimensions of the garden?



$A = (x+12)(x+10)$

$675 = x^2 + 12x + 10x + 120$

$0 = x^2 + 22x - 555$

$1 \quad -15$

$1 \quad 37$

$0 = (x-15)(x+37)$

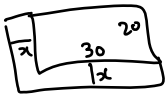
$x - 15 = 0$ or $x + 37 = 0$

$x = 15$ $\quad x = -37$ can't have negative side length

∴ garden was 15m by 15m

Now its 25m by 27m

19. A rectangular skating rink measuring $30m$ by $20m$ is doubled in area by adding a strip at one end of the rink, and a strip of the same width along one side of the rink. Find the width of the strips if the enlarged rink is still rectangular.



$A = 600$ → doubled

$A = 1200 = (20+x)(30+x)$

$1200 = 600 + 20x + 30x + x^2$

$0 = x^2 + 50x - 600$

$1 \quad -10$

$1 \quad +60$

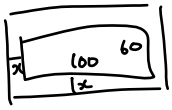
$0 = (x-10)(x+60)$

$x - 10 = 0$ or $x + 60 = 0$

$x = 10$ $\quad x = -60$ not possible

∴ width added is 10m long on the two sides

20. A chemical plant is rectangular and has a length of $100m$ and a width of $60m$. A safety zone of uniform width surrounds the plant. If the area of the safety zone equals the area of the plant, what is the width of the safety zone?



$A = 60 \times 100 = 6000$

$6000 = \boxed{\text{Big}} - \boxed{\text{Small}}$

$6000 = (100+2x)(60+2x) - 6000$

$0 = 6000 + 200x + 120x + 4x^2 - 6000 - 6000$

$0 = 4x^2 + 320x - 6000$

$0 = 4(x^2 + 80x - 1500)$ won't factor ∴ quad formula

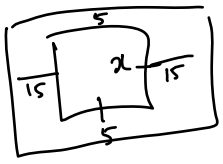
$x = \dfrac{-80 \pm \sqrt{80^2 - 4(1)(-1500)}}{2(1)}$

$x = \dfrac{-80 \pm 111.36}{2}$

$x \doteq 15.6$ or $x \doteq -95.68$

∴ safety zone width is 15.6m

21. When 15m is added to the length of two opposite sides of a square and 5 m is added to the length of the other sides. The area of the resulting rectangle is $441m^2$. Find the length of the sides of the original square.



$A = lw$

$441 = (x+15+15)(x+5+5)$

$441 = (x+30)(x+10)$

$0 = x^2 + 30x + 10x + 300 - 441$

$0 = x^2 + 40x - 141$

won't factor

$x = \dfrac{-40 \pm \sqrt{40^2 - 4(1)(-141)}}{2(1)}$

$x = \dfrac{-40 \pm 46.52}{2}$

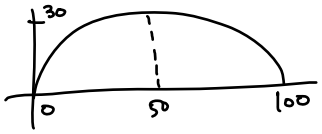
$x \doteq 3.3$

$x \doteq -43.26$

∴ original square was 3.3m by 3.3m

Date: _____ Name: _____

22. The underside of a bridge has the shape of a parabolic arch. It has a maximum height of 30m and a width of 100m. Can a boat with a height of 20m and a width of 30m pass under the bridge?



$y = a(x-0)(x-100)$ sub pt. $(50, 30)$

$30 = a(50-0)(50-100)$

$30 = a(50)(-50)$

$30 = -2500a$ $\therefore a = -0.012$ $\therefore y = -0.012(x-0)(x-100)$

boat < bridge?

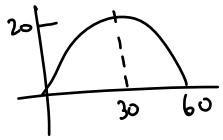
20 | $-0.012(30-0)(30-100)$

$-0.012(30)(-70)$

25.2

∴ yes it will pass through

23. A man working on a bridge has a life support rope tied to his waist. The bridge is 60m long with a maximum height of 20m from the water below. If the man is working 5m from the centre of the bridge, what would be the maximum length the rope can be so that the man would not hit the water if he fell?



$y = a(x-0)(x-60)$ sub pt. $(30, 20)$

$20 = a(30-0)(30-60)$

$20 = a(30)(-30)$

$20 = -900a$

$-0.02\bar{2} = a$

$\therefore y = -0.02(x-0)(x-60)$

5m from centre

$x = 25$ or $x = 35$m

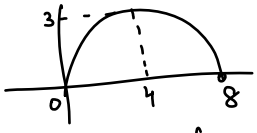
sub $x = 25$ or $x = 35$ to see height at that pt.

$y = -0.02(25-0)(25-60)$

$y = -0.02(25)(-35)$

$y = 17.5$ ∴ He needs a rope shorter than 17m.

29. Deep within a cave (shaped like a parabolic arch) a bat, 25cm in length, is hanging upside down on the underside of the cave, 1.75m from the highest point in the arch. If the cave is 8m wide and has a maximum height of 3m, how far is the bat from the floor of the cave?



$y = a(x-0)(x-8)$ sub pt. $(4, 3)$

$3 = a(4-0)(4-8)$

$3 = a(4)(-4)$

$3 = -16a$

$-0.1875 = a$

$\therefore y = -0.1875(x-0)(x-8)$

1.75 from centre

$x = 5.75$ or $x = 2.25$

sub $x = 2.25$ or $x = 5.75$ to find height

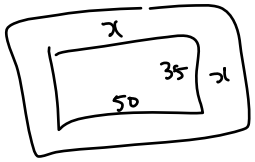
$y = -0.1875(2.25-0)(2.25-8)$

$y = -0.1875(2.25)(-5.75)$

$y \doteq 2.4256\text{m} = 242.6\text{cm}$ − bat length

∴ bat is 218 cm from floor

31. A playground, which measures 50*m* by 35*m*, is to be doubled in area by adding a strip of uniform width around the outside of the existing area. What is the width of the new strip around the playground?



$A = 35 \times 50$

1750 → double

$3500 = l\,w$

$3500 = (50+2x)(35+2x)$

$3500 = 1750 + 100x + 70x + 4x^2$

$0 = 4x^2 + 170x - 1750$

$x = \dfrac{-170 \pm \sqrt{170^2 - 4(4)(-1750)}}{2(4)} = \dfrac{-170 \pm 238.5}{8}$

$x \doteq 8.6$ or ~~$x \doteq -51.1$~~

∴ width of new strip is 8.6m