

NOTESsomeANS

Notes that are done in class will be updated online periodically

(Any questions left blank you are responsible to try yourself → get help if needed.)

Date: _____

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Introduction to a Quadratic Function Unit



Big idea

The first half of this course – the first four units – are review of quadratics you've learned in grade 10. If you took grade 10 applied, you will learn a lot of new things in addition to what was done last year. If you took academic course you will find that there is not a lot of new material to learn. Whatever was done before, you must learn to be very proficient with working with quadratic equations as well as learn some new concepts that relate to quadratics you have never seen before. In this unit you will concentrate on the following topics that are outlined in the table below. (The review topics in the table are from BOTH academic and applied course.)

I know all the prior concepts related to this unit. (If not STOP & complete more review)		Success Criteria Assessment as Learning for Learning and of Learning							
		I can understand the lesson (If not, ask clarifying questions. Be specific – "what part is unclear?")	I can do a question with an example to follow. (If not, see the teacher for extra help)	I can do questions independently (If not, redo a solved example without looking at solutions)	I can explain/communicate this concept in my own words – JOURNAL (If not, practice explaining steps done in a solved example)	I can apply this concept in other/new contexts/situations (This can be only attained with practice)	I am very confident and am able to complete questions quickly (If not, time yourself to see progress)	I completed the practice in EACH section	I completed the practice test and the review section for this unit.
KU	Learning Goal	KU	KU	APP	COMM	TIPS		HW	TEST
Finding equations of and graphing lines, finding equations of and graphing quadratics, simplifying expressions, solving equations, expanding, factoring, problem solving with lines and quadratics	Characteristics of Functions Section 1.1 #2, 4, 7, 10, 12, 13								P70 - Chapter Self-Test P68-69 Chapter Review Questions
	Lines and Quadratics & Function Notation Section 1.2 #1, 2, 5, 6, 8 & EXTRA Handout								
	Working with Function Notation Section 1.3 #3, 4, 5, 6, 13, 15								
	Transformations of Quadratics – INVESTIGATION with Graphing Calculators Section 1.5 #1, 2 i, ii, 3, 5, 6, 9, 10 & EXTRA Handout								
	Sketching Quadratics using Transformations Section 1.6 #5, 7 def, 10, 11 & EXTRA Handout								
	Domain and Range Section 1.7 #2, 3, 6, 7, 8, 9								
	EXTRA – domain & range activity								



Tentative TEST date _____

Reflect – TEST mark for this unit _____, Overall mark now _____

Looking back on this unit, what should you plan to improve upon before the exam?

Corrections for wrong textbook answers:

Date: _____

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Characteristics of Functions

1. Mathematics is a study of relationships. These relationships when written with variables are called **relations**. Sometimes a problem may relate variables that have interdependence. We usually call x variable as independent and the y variable the dependent. The independent variable can also be called the input and dependent variable the output. This terminology is often used when the relations are **functions**.

2. What is a function? (Explain using the input/output terminology, as well as explain how to determine if it is a function from an equation and from a graph.)

Functions have only one output (y) for every input (x).
 Graphs pass a vertical line test (touch line only once)
 Equations don't have an even power on output y .

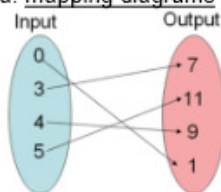
3. Define the terms domain and range.

Domain is a set of all defined inputs x (Think of shadow on x -axis)
 Range is a set of all resulting outputs from defined inputs
 Symbols: $\{ \}$ set of, \in element of, \mathbb{R} - real #'s, \mathbb{I} - integer #'s

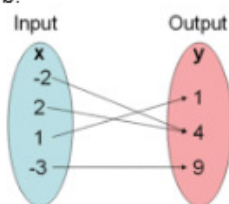
4. Find the domain and range then determine if the following are functions or not.



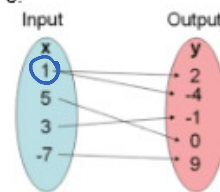
a. mapping diagrams



b.



c.

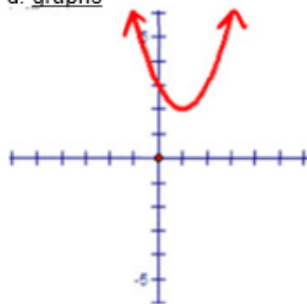


$$D = \{x = 1, 5, 3, -7\}$$

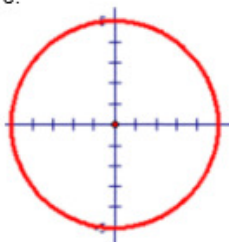
$$R = \{y = 2, -4, -1, 0, 9\}$$

Not a function
 input $x=1$ has two outputs $y=2$
 $y=-4$

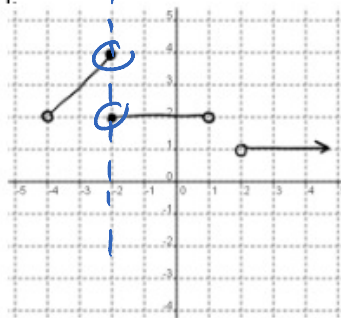
d. graphs



e.



f.



$$D = \{x \in \mathbb{R}, -4 < x < -2, 2 < x\}$$

$$R = \{y \in \mathbb{R}, y = 1, 2 \leq y < 4\}$$

Not a function
 Doesn't pass vertical
 line test at
 $x = -2$

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g. points

$\{(-3, 9), (-2, 8), (-1, 2), (-2, -4)\}$

h.

$\{(2, 5), (3, 5), (4, 10), (5, 0)\}$

k. equations

$y = 3 - 6x$



n. descriptions

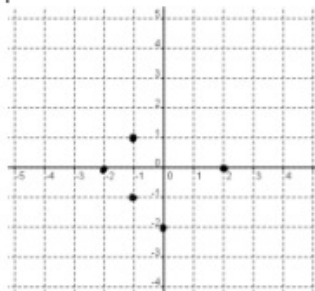
A vending machine produces pop, gum, chocolate bars, etc. depending on the button pressed.



For which pair of related quantities would time be the independent variable?

- | | |
|---------------------------------|-----------------------------------|
| a. grade, time spent on project | c. flight time, rainfall |
| b. length of race, finish time | d. distance to work, commute time |

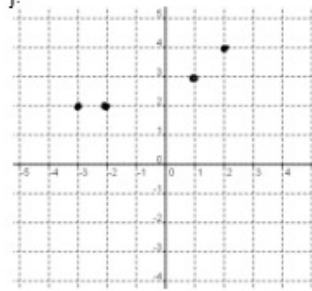
i.



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j.



$D = \{x = -3, -2, 1, 2\}$

$R = \{y = 2, 3, 4\}$

Function.

m.

$$x^2 + y^2 = 4$$

$$y^2 = 4 - x^2$$

$$y = \pm \sqrt{4 - x^2}$$

when isolate output get 2 answers!

Not a function

$D = \{-2 \leq x \leq 2\}$

$R = \{-2 \leq y \leq 2\}$

Circle
 $r = 2$



p.

The forensic office analyzing DNA, is trying to determine the identity of a person it belongs to.

DNA $\begin{cases} \rightarrow \text{twin \#1} \\ \rightarrow \text{twin \#2} \end{cases}$
if identical
(fingerprints are different)

Not a function if
twins are analyzed
(otherwise it is.)

$D = \{\text{DNA}\}$

$R = \{\text{person's identity}\}$

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Lines and Quadratics & Function Notation



1. What is function notation?

name \rightarrow $f(x)$ \rightarrow input
output

instead of
output y
write $f(x)$

2. What could function notation be confused with?

Brackets in function
notation do NOT mean to
multiply



3. You have seen function notation for specific functions, but you just didn't know it. Indicate what is the name of the function, the input and the output of each of the following

a. $\cos(\theta)$

name = cosine
input = θ angle
output = $\cos \theta$ ratio

b. $\sin^{-1}(0.5)$

c. $\sqrt[3]{(9)}$

d. $(2x)^2$

4. Explain why it is incorrect to write the following, give corrected versions.

a. $\cos = \frac{1}{2}$

b.

$\tan x = 0.5234$

$x = 0.5234 \tan^{-1}$



5. What do differences of the dependent variable tell you about the relation given?

If 1st differences are the same \rightarrow Linear
If 2nd differences are the same \rightarrow Quadratic
* x values must go up by the same interval
otherwise it won't work

6. Determine what type of functions are these? Specify what variable is the function of what other variable and record the output in function notation.



a.

x	y
0	-9
2	-10
4	-7
6	0
8	11
10	26

-1 $+4$
 $+3$ $+4$
 $+7$ $+4$
 $+11$ $+4$
 $+15$ $+4$
not
linear

\therefore quadratic
 y is a function of x
 $y(x)$

b.

n	P
1	-8
7	2
13	12
19	22
22	27
25	32

$+10$
 $+10$
 $+10$
 $+10$
 $+10$

\therefore linear
 P is a function
of n
 $P(n)$

ignore not up by 6 on x 's
 $+10$



c.

Time (years)	Radiation level
1	17
2	9
3	5
4	3
5	2
6	1.5

d.

Length	Area of figure
1	2
2	5
3	10
4	17
5	26
6	37

e.

# of items	Cost
10	7
12	9
14	11
16	13
18	15
20	17

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7. What is a degree of a function? How does it help you determine what type of function is given?

Degree is the highest power of a term
 Linear functions have degree ONE (ie. no exponents)
 Quad. functions have degree TWO (ie. x^2)

8. Decide what type of function is given by expanding and simplifying then determining the degree of the function.



a. *expand to see*

$$f(x) = (2-x)(x+3)$$

$$= 2x + 6 - x^2 - 3x$$

$$= -x^2 - x + 6$$

quadratic



b. $g(x) = 5 + 4(x-2) - 8x$

c.

$$h(x) = 2x - (x-2)^2$$

d.

$$i(x) = 3 + x - 5(x - 7(x+1))$$



9. The helium balloon is launched from a height of 2 meters and it rises at a rate of 0.5 m/s.

- a. Write an equation in function notation for the height, h , of the balloon at time, t , seconds.

$$y = mx + b$$

$$y = 0.5x + 2$$

$$\therefore h(t) = 0.5t + 2$$

rate of change = m
 initial value = b

- b. What is the domain and range of this function, assuming that the balloon pops at a height of 1 km.

$$\text{sub } h = 1000$$

$$1000 = 0.5t + 2$$

$$998 = 0.5t$$

$$1996 \text{ sec} = t$$

$$D = \{ 0 \leq t \leq 1996 \}$$

$$R = \{ 2 \leq h \leq 1000 \}$$



10. Video games cost on average \$8.00 to rent. The player costs \$300. The total cost of playing video games can be represented by a function $C(v)$.

- a. Write an equation in function notation for the cost of v , video games.

- b. State the degree of this function and whether it is linear or quadratic.

- c. Use your equation to calculate the cost of renting 20 video games.

- d. What is the domain and range of this function, assuming that you have \$780 to spend?

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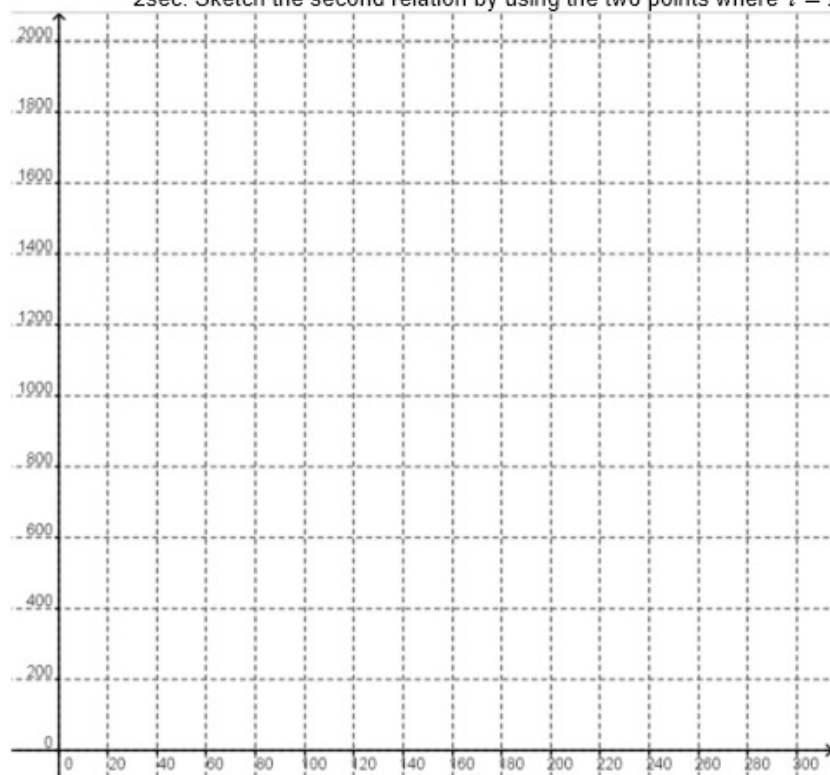
11. A skydiver's height is modelled by $h(t) = 2000 - 4.9t^2$ for $\{t \in \mathbb{R}, 0 \leq t \leq 14\}$, where t is the time elapsed, in seconds, and h is the height in meters. After 14 seconds the skydiver opens up the parachute and his descent is modelled by $h(t) = 1039.6 - 3.5(t - 14)$.

a. Use the degrees to determine what type of relations the equations are.

b. At what height was the parachute released?

c. At what time did the skydiver land on the ground?

d. Sketch the first relation on the domain of $\{t \in \mathbb{R}, 0 \leq t \leq 14\}$ using a table of values with time increments of 2sec. Sketch the second relation by using the two points where $t = 14$ and $t =$ answer from c.



e. Show that the differences in the table for the first relation match with your answer about what type of relation it is in question a.

f. Use vertical line test to determine if the relation is a function.

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Function Notation

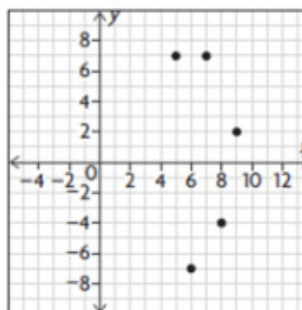
1. Determine
- $f(7)$
- of each of the following

$$f = \{(-9, 7), (-7, 6), (0, -7), (7, -9), (9, 7)\}$$

$$f(7) = -9$$

x	0	1	3	5	7
f(x)	-7	-5	-3	-1	0

$$f(7) = 0$$

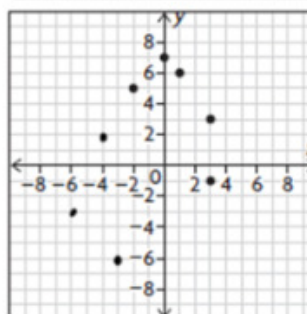


Is this graph a function? Explain.

Yes since it passes vertical line test or for every input there's only one output

2. Determine
- $f(-6)$
- of each of the following
-
- $f = \{(-10, -6), (-9, -7), (-8, -8), (-7, -9), (-6, -10)\}$

x	-6	-4	-2	0	2
f(x)	-6	-6	-6	-6	-6



Is this graph a function? Explain.

3. For the following function determine
- $f(-2)$
- ,
- $f(0)$
- ,
- $f(3)$
- ,
- $f(4x)$

$$f(x) = x^2 + 7x + 12$$

$$\begin{aligned} f(-2) &= (-2)^2 + 7(-2) + 12 \\ &= 4 - 14 + 12 \\ &= 2 \end{aligned}$$

$$\begin{aligned} f(0) &= 0^2 + 7(0) + 12 \\ &= 12 \end{aligned}$$

$$\begin{aligned} f(3) &= 3^2 + 7(3) + 12 \\ &= 9 + 21 + 12 \\ &= 42 \end{aligned}$$

$$\begin{aligned} f(4x) &= (4x)^2 + 7(4x) + 12 \\ &= 16x^2 + 28x + 12 \end{aligned}$$

4. For the following function determine
- $f(0)$
- ,
- $f(-1)$
- ,
- $f(2)$
- ,
- $f(5x)$

$$f(x) = 7x^2 - 25x + 12$$

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5. Three consecutive even integers numbers are picked from the domain $-10 \leq x \leq 10$. The equation that models the sum of their squares is $S(x) = x^2 + (x+2)^2 + (x+4)^2$

- a. What does each part of the equation represent?

$S(x)$ = sum $x, x+2, x+4$ are the 3 consecutive even #'s.

- b. Evaluate $S(-10)$, what does it represent?

$S(-10) = (-10)^2 + (-8)^2 + (-6)^2 = 100 + 64 + 36 = 200$ sum of 3 consecutive even squares from -10

- c. Evaluate the function for all valid values of the domain. Organize results in a table.

x	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	
S	200	155	116	83	56	35	20	11	8	11	20

- d. Find which three numbers would give the minimum sum?

$x = -2$ \therefore the 3 numbers are -2, 0, 2 that give a Minimum Sum



6. A pebble falls straight to the ground from a cliff that is 1102.5 m tall. The function representing the distance the pebble has fallen in meters is $d(t) = 4.9t^2$, where t is the time the pebble has been falling in seconds.

- a. Evaluate $d(3)$. What does it represent?

- b. Explain what does the following tell you? $d(5) = 122.5$

- c. When did the pebble land on the ground?

- d. What is the domain and range of this situation?



7. Explain what each part of the following tells you $m(p) = p^2 + p$



- a. Evaluate and simplify the expression for $m(1-x)$

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Transformations of Quadratics - INVESTIGATION

In this investigation you will graph different parabolas and determine the link between the equation in "vertex form" $y=a(x-h)^2+k$ and the transformations from the basic parabola $y=x^2$.

TECHNOLOGY OPTION

To help you graph and plot the parabolas, enter the equation in the $y =$

Use x, T, Θ, n to type a variable

Remember to use GREY $(-)$ when negative appears FIRST and BLUE $-$ otherwise

Press $\boxed{\text{graph}}$ to see the graph

Press $\boxed{2^{\text{nd}}}$ $\boxed{\text{graph}}$ to see a table of values for the parabola

Parabola Investigation #1

Basic Equation		$y = x^2$																				
Vertex Form $y = a(x - h)^2 + k$																						
Change values for a keep $h=0$ and $k=0$ for now																						
values	Equations	Colour																				
$a=2$																						
$a=0.2$																						
$a=-1$																						
$a=-2$																						
$a=-0.2$																						

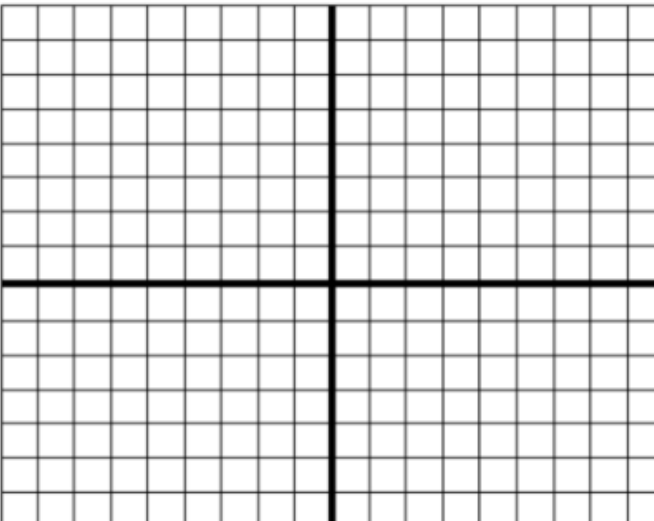
Summarize what effect does a have on the graph of $y = x^2$

- State the transformations performed on $y = x^2$ in each of the following quadratics
 - $y = \frac{1}{3}x^2$
 - $y = -5x^2$
 - $y = -0.001x^2$

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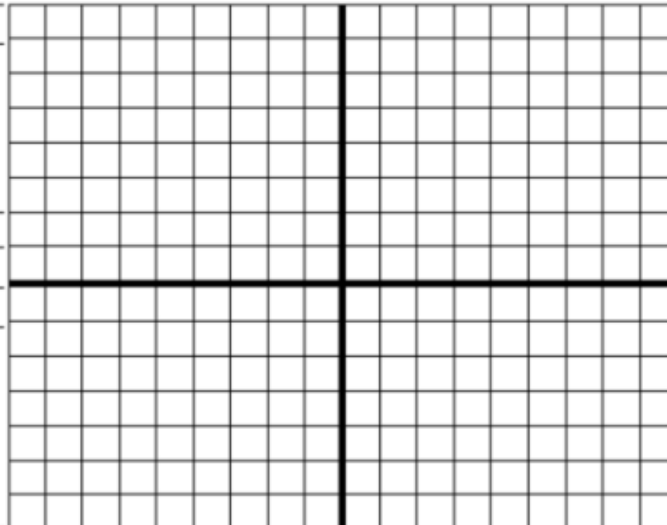
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Parabola Investigation #2

Basic Equation	$y = x^2$		
Vertex Form $y = a(x - h)^2 + k$			
Change values for k keep $a=1$ and $h=0$ for now			
values	Equations		Colour
$k=4$			
$k=-6$			
What effect does changing k have on the graph of $y = x^2$?			

2. State the transformations performed on $y = x^2$ in each of the following quadratics
- a. $y = 2x^2 - 9$ b. $y = -0.5x^2 - 16$ c. $y = -3x^2 + 9$

Parabola Investigation #3

Basic Equation	$y = x^2$		
Vertex Form $y = a(x - h)^2 + k$			
Change values for h keep $a=1$ and $k=0$ for now			
values	Equations		Colour
$h=3$			
$h=-5$			
What effect does changing h have on the graph of $y = x^2$?			

3. State the transformations performed on $y = x^2$ in each of the following quadratics
- a. $y = (x+2)^2$ b. $y = (x-4)^2 - 7$ c. $y = -(x+4)^2 + 3$ d. $y = 2(x-1)^2$

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Sketching Quadratics using Transformations

1. Summarize what the letters of $y = a(x-h)^2 + k$ stand for. Make a note on the order that transformations should be applied in.

a → neg = reflect in x -axis
 → vertical stretch or compression
 ex. $a = 2$ or -2 $a = \frac{1}{2}$ or $-\frac{1}{2}$...

h → shift left/right (* switch sign for h)

k → shift up down

ORDER

x	$y = \text{parent}$	a	$y \cdot a$	$x \pm h$	$y \pm k$

2. Write an equation of a parabola that satisfies each set of conditions

a. Opens down, congruent with $y = \frac{1}{4}x^2$, vertex $(5, -4)$
 h k

$a = \text{neg}$
 same shape
 $\therefore a = -\frac{1}{4}$

$$\therefore y = -\frac{1}{4}(x-5)^2 - 4$$

b. Vertex $(-3, -8)$, x -intercepts of 1 and -7 .
 h k

$$y = a(x+3)^2 - 8 \quad \text{sub pt } (1, 0)$$

$$0 = a(1+3)^2 - 8 \quad \text{or } (-7, 0)$$

$$8 = a(4)^2$$

$$8 = 16a$$

$$\frac{1}{2} = a$$

$$\therefore y = \frac{1}{2}(x+3)^2 - 8$$

c. Vertex $(2, 7)$, y -intercept -3 .

d. Vertex $(0, -4)$, passes through $(-3, 2)$

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3. Describe the transformations (in the correct order) applied to the graph of $y = x^2$ to obtain the graphs of the following quadratic relations. Sketch the graph by hand. Start with the graph of $y = x^2$ and show all transformations with different colours.

a. 

$$y = -3(x+2)^2 + 8$$

$a = -3$ reflect in x-axis
vertical stretch

$h = -2$ left
 $k = 8$ up

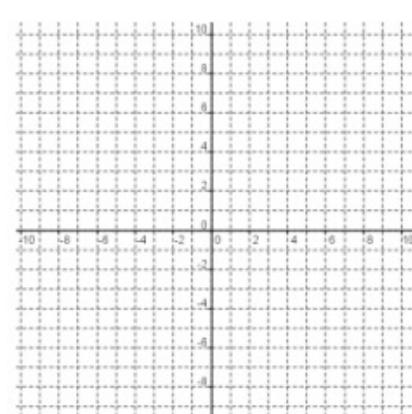
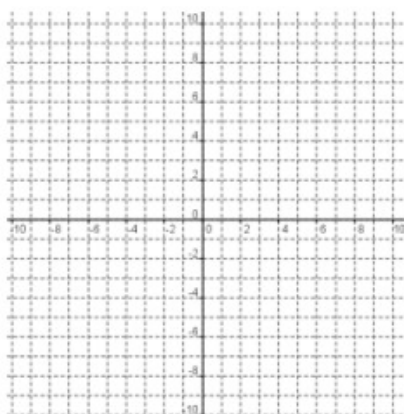
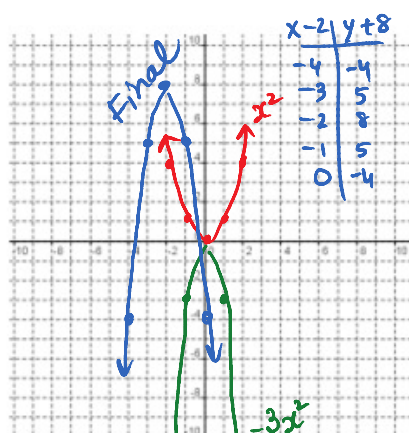
x	y	x	y · a
-2	4	-2	-12
-1	1	-1	-3
0	0	0	0
1	1	1	-3
2	4	2	-12

b. 

$$y = 0.5x^2 - 9$$

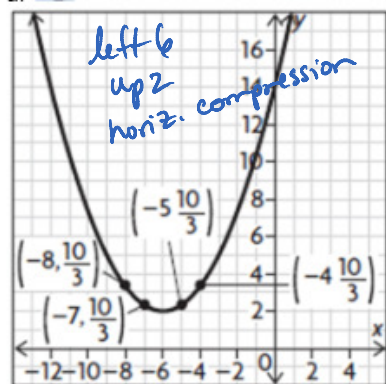
c.

$$y = -1.5(x-5)^2$$



4. Determine the combination of transformations that would result in the following graphs, then determine a possible equation for the graph.

a. 



$$y = a(x+6)^2 + 2$$

sub $(-4, \frac{10}{3})$

$$\frac{10}{3} = a(-4+6)^2 + 2$$

$$\frac{10}{3} - 2 = a(2)^2$$

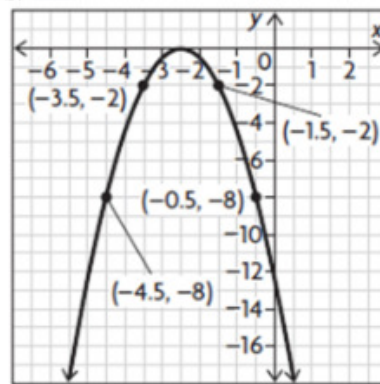
$$\frac{10-6}{3} = 4a$$

$$\frac{4}{3} = 4a$$

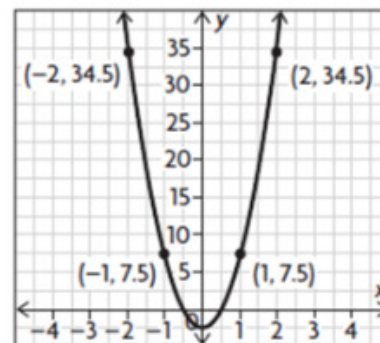
$$\frac{1}{3} = a$$

$$y = \frac{1}{3}(x+6)^2 + 2$$

b. 



c.



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Domain and Range

- Some of the questions in the textbook require you to graph with technology. There are lots of applets you can use online, or you can download a free program to use on your computer offline.

Online Graphing Calculator

http://my.hrw.com/math06_07/nsmedia/tools/Graph_Calculator/graphCalc.html

Download GeoGebra (offline and online)

<http://www.geogebra.org/cms/en/download>

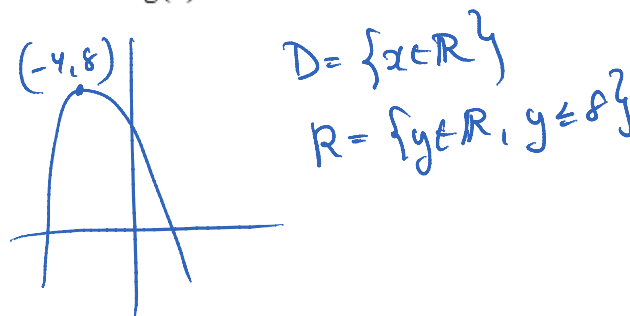
select **webstart** for offline

← select **appletstart** for online

1. Use graphing software to graph each function and then use the graph to state the domain and range.

a. $f(x) = 2(x+3)^2 - 8$

b. $g(x) = 4 - 2x - 0.25x^2$



2. Notice that range is visible from ONE of the versions of quadratic equation above. What must the equation look like for range to be visible and how can you determine the range from it?

To see range the quadratic should be in vertex form. The range is $y \leq k$ if \cap or $y \geq k$ if \cup

3. What is the domain and range for lines? (make sure you explain all types of possible lines)

sloped line

$D = \{x \in \mathbb{R}\}$
 $R = \{y \in \mathbb{R}\}$

vertical $D = \{x = \# \}$
 $R = \{y \in \mathbb{R}\}$

horizontal $D = \{x \in \mathbb{R}\}$
 $R = \{y = \# \}$

4. Find the domain and range for each of the following relations.



a. $f(x) = -3x^2 + 5$

$D = \{x \in \mathbb{R}\}$
 $R = \{y \in \mathbb{R}, y \leq 5\}$

b. $g(x) = 5$

$D = \{x \in \mathbb{R}\}$
 $R = \{y = 5\}$



c. $x = -6$

d. $h(x) = 4(x+3)^2$

e. $f(x) = 7x - 3$

f. $g(x) = -(x-5)^2 + 8$

g. $f(x) = x^2 + 5x - 3$

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5. While on vacation, Talisha won a lot of tickets at two arcades on the boardwalk she was visiting. The first arcade charges \$1 to cash in and gives you 12 cents back on each ticket won, the second arcade gives you 10 cents back on each ticket and no fee to cash in.

a. Determine the equations for the money you can leave with as functions of winning tickets for both arcades.

let W = winnings
 t = tickets

first
 $W(t) = 0.12t - 1$

second
 $W(t) = 0.10t$

b. Determine the domain and range for both arcades.

for BOTH: $D = \{t \in \mathbb{R}, t \geq 0\}$ can't have negative # of tickets

1st $R = \{W \in \mathbb{R}, W \geq -1\}$

2nd $R = \{W \in \mathbb{R}, W \geq 0\}$

6. A rocket is launched and its height in feet as a function of time in seconds is given by $1600t - 16t^2$. Find the domain and range that will make sense in the context of this real life situation. (use a table of values)

t	0	20	40	60	80	100
h	0	25600	38400	38400	25600	0

$0 \rightarrow$ can't go below zero

vertex in middle
 $t = 50$
 $h = 40000$

$D = \{t \in \mathbb{R}, 0 \leq t \leq 100\}$
 $R = \{h \in \mathbb{R}, 0 \leq h \leq 40000\}$



7. Oberon Cell Phone Company advertises service for 3 cents per minute plus a monthly cost of \$29.95.

a. Determine the equation for the cost as a function of minutes.

b. Determine the domain and range for this real life situation.



8. Two people are playing golf. The height above the ground in meters is given by $h_1(t) = -5t^2 + 40t$ for a person hitting the golf ball from ground level and $h_2(t) = -5t^2 + 40$ for a person hitting the golf ball from a roof of a building.

a. For which player must you create a table of values to see domain and range? Create it.

(if you took grade 10 academic course, you may: for the RANGE - complete the square to see vertex, and for the DOMAIN - common factor to see zeros instead of doing the table of values - if you remember how)

b. Find the domain and range for both players.