

NOTESsomeANS

Notes that are done in class will be updated online periodically

(Any questions left blank you are responsible to try yourself → get help if needed.)

Finance Unit

Tentative TEST date _____



Reflect – previous TEST mark _____, Overall mark now _____.
Looking back, what can you improve upon?



Big idea/Learning Goals

In this unit you will study the applications of linear and exponential relations within financing. You will understand the different formulas you must use for simple interest and compound interest. **Simple interest** means that the interest grows by a *constant* rate each year. **Compound interest** means that the interest grows by an *increasing* rate each year, because the interest is calculated on the amount deposited as well as on the interest already earned so far. The value of the monetary amount of many regular deposits or payments is called an **annuity**. These calculations are harder and you will learn how to use a TVM solver on the graphing calculator to solve these types of questions. To make things interesting and applicable to real life you'll be working on a LIFE PROJECT simultaneously as you learn new material. Examples of what to do appear in notes in shaded boxes, but you are to record the Life Project answers on the last few pages provided of this booklet to submit to the teacher later.



Success Criteria

- ☐ I am ready for this unit if I am confident in the following review topics
(circle the topics you are good at & review the ones you left uncircled before you get too far behind)
Simplifying expressions, solving equations, exponentials, exponent laws
- ☐ I understand the new topics for this unit if I can do the practice questions in the textbook/handouts
(check off the topics for which you have finished the practice)

Date	Topics	Done?
	Get Ready & Compare Simple and Compound Interest Get ready p450 #4,5, 7, 8 Section 8.1 p460 #7,8,9 Section 8.2 p468 #6,9 Section 8.3 p477 #6,9	
	Practice Simple & Compound Interest Section 8.4 p487 #4,6,10,11,13,14	
	Annuities Section 8.5 p499 #4,5,7,8 Section 8.6 p506 #2,4,5,7	
	Mix of Questions – What formula to use? Handout	
	Annuities with a TVM Solver Handout	
	LIFE PROJECT – marked as KU part of this unit	

- ☐ I am prepared for the test/evaluation if
 - ☐ I understand the main concepts from each lesson
 - If not, ask other students in class to help you study or visit the peer tutoring room or ask the teacher for help or get a private tutor
 - also practice "knowledge-understanding" questions from the textbook – look for questions marked by **K**
 - ☐ I can explain/communicate the ideas clearly
 - If not, practice explaining a solved question to someone else or complete the assigned journal questions
 - also practice "communication" questions from the textbook – look for questions marked by **C**
 - ☐ I can apply these concepts in word problems
 - If not, practice "application" questions from the textbook – look for questions marked by **A**
 - ☐ I did not just memorize steps to do for different types of questions, I understand the ideas behind each concept and therefore can do problems in new contexts
 - If not, practice "thinking-inquiry-problem-solving" questions from the textbook – look for questions marked by **T**
 - ☐ I can do questions independently
 - If not, try redoing an already solved example without looking at solutions
 - ☐ I can complete questions quickly and with confidence
 - If not, try timing yourself for similar type questions to see progress
 - ☐ I completed the review and/or practice test

Corrections for the textbook answers:

Get Ready

1. A **percentage** is a way of expressing a number as a fraction of 100. How do you convert a percent to a decimal?

To convert % to decimal → divide by 100
or → move decimal to the left 2 times

2. Express each percent as a decimal

a. 35% = 0.35

b. 8.5% =

c. $\frac{1}{2}\%$ =

don't write % symbol anymore

3. Convert time to years. 1 year = 12 months = 52 weeks = 365 days

a. 26 weeks

b. 8 months

c. 400 days

$\frac{52}{100}$
= 0.5 year

best to leave as fraction, rounding will give too big of an error.

**Life Project**

Randomly select a career with its corresponding salary.

- Calculate the net income using the following taxation information for Ontario.
(Net income = Gross income – deductions)
- Calculate the biweekly (every other week) gross earnings and biweekly net earnings to see how much tax is deducted from every paycheck.
- BONUS at home.** Go to www.studentcounsellor.com, login:
Investigate what are the academic requirements for your picked career, or something related to it if you cannot find an exact match, and submit your findings with pros and cons of this job and whether or not you'd choose it for yourself.

Do same thing on back page with given selection of career

Federal tax rates for 2011 are:

- 15% on the first \$41,544 of taxable income, +
- 22% on the next \$41,544 of taxable income (on the portion of taxable income between \$41,544 and \$83,088), +
- 26% on the next \$45,712 of taxable income (on the portion of taxable income between \$83,088 and \$128,800), +
- 29% of taxable income over \$128,800.

Provincial tax rates for 2011 are:

- 5.05% on the first \$37,774 of taxable income, +
- 9.15% on the next \$37,776 (on the portion of taxable income between \$37,776 and \$75,552), +
- 11.16% on the amount over \$75,552

Ex. Civil engineer's gross salary is \$75,230. Split this amount into corresponding tax brackets:

Federal: $0.15(41,544) + 0.22(33,686) = 13,642.52$

Provincial: $0.0505(37,774) + 0.0915(37,456) = 5,334.81$

Net income: $75,230 - 13,642.52 - 5,334.81 = 56,252.67$

Biweekly gross earnings = $75,230 \div 26 = 2,893.46$

Biweekly net earnings = $56,252.67 \div 26 = 2,153.56$

Taxes deducted from every paycheck = $2,893.46 - 2,153.56 = 729.90$

Compare Simple and Compound Interest

1. Suppose you put \$1000 in a bank that earns 5% **simple interest** per year.
2. Suppose you put \$1000 in a bank that earns 5% **compound interest** per year, compounded annually

Time	Simple Interest	Final Amount	Differences
0	NA	1000	
1	$1000(0.05) = 50$	$1000 + 50 = 1050$	$+50$
2	$1000(0.05) = 50$	$1050 + 50 = 1100$	$+50$
			$+50$

Time	Interest	Final Amount	Differences Ratios
0	NA	1000	
1	$1000(0.05) = 50$	$1000 + 50 = 1050$	$\times 1.05$
2	$1050(0.05) = 52.50$	$1050 + 52.50 = 1102.50$	$\times 1.05$

4. Look at the differences ^{and ratios} and determine what type of functions these are?

Simple Interest is linear since 1st differences are the same
 Compound Interest is exponential since 1st ratios are the same

5. Summarize all the simple interest formulas

P = principal or initial amount deposited, present value
 A = final amount or future value
 r = annual interest rate (% written as a decimal)
 t = time in years
 I = interest earned in \$

$$I = Prt \quad A = P + I$$

7. Summarize all the compound interest formulas

C = # of compounding periods in a year
 i = periodic interest rate (% written as decimal)
 n = total # of compounding periods

$$A = P(1+i)^n$$

$$i = \frac{r}{C}$$

$$n = Ct$$

6. How much will you have after 15 years? How much of the final amount is the interest?

$$I = 1000(0.05)(15)$$

$$= 750$$

$$\text{final amount} = 1750$$

$$\text{interest only} = 750$$

8. How much will you have after 15 years? How much of the final amount is the interest? Compare this with simple interest calculation.

$$A = 1000(1 + \frac{0.05}{1})^{15}$$

$$= 2078.93$$

$$\text{final amount} = 2078.93$$

$$\text{interest only} = 1078.93$$

compare

Practice Simple and Compound Interest



When you read a word problem, ask yourself the following questions:

- Is this simple or compound interest?
(Usually if the word "compounded" isn't there it is simple interest.)
- Given a monetary amount, is it the present value, or the future value, or the interest earned?
- Is the time given in years?
(If not, convert using the following : $1\text{ year} = 12\text{ months} = 52\text{ weeks} = 365\text{ days}$)

Finally, if the question is compounded, it will tell you the frequency of compounding. Here is what you must know:

C=1 annually

C=24 semi-monthly (twice in a month)

C=2 semi-annually (twice in a year)

C=26 bi-weekly (every other week)

C=4 quarterly

C=52 weekly

C=12 monthly

C=365 daily

1. All of the questions so far will be about a **single deposit** being made. Summarize all the formulas you will have to use for **simple and compound interest** that involves SINGLE DEPOSITS.

Simple

$$I = Prt$$

Compound

$$A = P(1+i)^n$$

$$i = \frac{r}{C} \quad n = Ct$$



Life Project Randomly select a bonus received at your job. Suppose you put your bonus aside as savings into a GIC (guaranteed investment certificate). Randomly select the GIC option and for how long you were able to keep the money invested before you had to take it out for unforeseen expenses. Then calculate the final amount and how much of that is interest.

Ex. An investment of \$4000 is invested at 5% for 130 weeks.
 What is the final amount? How much of this is interest?
 Convert time into years: $130 \div 52$ before you use the formula.
 $I = Prt$

$$I = (4000)(0.05)\left(\frac{130}{52}\right)$$

$$I = 500$$

Final amount is $4000 + 500 = \$4500$, and interest is \$500.

Life Project Randomly select a debt that you incurred because of some unforeseen circumstance and the time it takes to pay it off. Randomly select a credit card rate with which you chose to pay off the debt. Then calculate how much you'd owe in total and how much of that was interest. For simplicity assume the debt was paid in lump sum at the end – in reality, it is always best to pay off as much as possible whenever possible.

Ex. Suppose you spent \$1200 on your credit card.
 The credit card charges 19.5% compounded quarterly.
 If you didn't pay it for 6 months, how much would you owe in total? How much of that is interest?
 Convert time to years: $t = 6 \div 12 = 0.5$
 Convert rate to periodic rate: $i = r \div C = 0.195 \div 4$ (leave the calculation for later)
 Find total number of compounding periods: $n = Ct = 4(0.5) = 2$
 Now use the formula:

$$A = P(1+i)^n$$

$$A = 1200\left(1 + \frac{0.195}{4}\right)^2$$

$$A = 1319.85$$

Total debt was \$1319.85 and part of this was interest of \$119.85

2. What principal is needed to have \$100 in interest in 2 years invested at 2.5% interest?
 $r = 0.025$
 $I = Prt$
 $100 = P(0.025)(2)$
 $100 = P(0.05)$
 $\$2000 = P$
3. How much needs to be invested today to have \$25000 in 10 years, at 6% per year, compounded quarterly.
 $t = 10$ years, $r = 0.06$, $c = 4$
 $i = \frac{0.06}{4}$
 $n = 4(10) = 40$
 $A = P(1+i)^n$
 $25000 = P(1 + \frac{0.06}{4})^{40}$
 $25000 = P(1.8140184...)$
 $\$13781.56 = P$
leave undivided until later
don't round
4. How long would it take for \$2500 to grow to \$2700 at an interest of 4.5%?
5. Calculate the interest rate compounded annually a \$400 investment would make if final amount was \$600 after 2 years.
6. Calculate the simple interest of a \$675 investment at 7.25% over 2 years
7. Calculate the final amount of a \$750 investment at $4\frac{3}{4}\%$ over 3 years
8. Determine the amount of interest that was earned from an investment at 2.45% for 3 years, compounded monthly if the future value is \$3500
9. What is the final amount after 3 years, if you make 3 deposits of \$500 at the end of each year into a bank account that pays 4.5% compounded annually? (Note: when there are many same valued deposits made like in this question, the total value is considered an **annuity**.)

Annuities

When the question is not about a single deposit but about many regular deposits or payments you must use a different set of formulas. The term **annuity** describes a *series* of regular deposits or payments.

1. Summarize the annuity formulas here (give both PV and FV formulas)

PV = P = present value or discounted value (big amount of \$ in the present, without interest)
 FV = A = future value or accumulated value (big amount of \$ at a future date, with interest)
 R = regular deposit/payment (smaller amount of \$ deposited/paid many times over a period of time)
For present value – R is with interest. For future value – R is without interest
 C = # of compounding periods in a year
 r = annual interest rate (% written as decimal)
 i = periodic interest rate (% written as decimal)
 t = time in years
 n = total # of compounding periods

$$i = \frac{r}{C} \quad n = Ct$$

$$FV = \frac{R[(1+i)^n - 1]}{i}$$

$$PV = \frac{R[1 - (1+i)^{-n}]}{i}$$

2. Try to identify which of the above questions deal with present value and which with future value. What are you supposed to look for - to know when something is present value and when something is future value?

- FV** A. "Mario deposits \$600 at the end of every 6 months into a savings account paying 6% compounded semi-annually. He does this for 8 years. What is the amount of annuity and what is the total interest earned?" $C=2$
 $i = \frac{0.06}{2}$ $n = 2(8) = 16$
- PV** B. "Steve is buying a new Harley motorcycle. His monthly payments are \$650 with interest within the payments which was charged at 3% compounded monthly. What is the cash price of the motorcycle if he makes payments for 4 years?" r $C=12$

Present value - Big amount of \$ is used/needed Now - without interest
 ie. LOANS (usually)

Future Value - Big amount of \$ is used/needed in future - with interest
 ie. Investments (usually)

3. Answer problem A.

$$FV = \frac{600[(1 + \frac{0.06}{2})^{16} - 1]}{(\frac{0.06}{2})}$$

$= \$12,094.13$ total amount of annuity
 put in: $600 \times 8 \text{ years} \times 2 \text{ semiannually}$
 $= 9600$
 interest only: 2494.13

*learn to type into calc. with brackets!

4. Answer problem B

$$PV = \frac{650[1 - (1 + \frac{0.03}{12})^{-48}]}{(\frac{0.03}{12})}$$

$= 29,366.15$ total loan paid for you by bank = cash price
 paid to bank = $650 \times 4 \text{ years} \times 12 \text{ monthly}$
 $= 31,200$
 interest only = $\$18,33.85$



Life Project

Randomly select an item that you will either

Option A: save up for by putting 5% of your biweekly earnings aside every week

Option B: save up for by investing the 5% of your biweekly earnings at 3% compounded biweekly

Option C: buy now and pay off by biweekly installments at 8% compounded biweekly

- A. Suppose you can save 5% of your biweekly net income. How long will it take to save up for the item you selected? State your answer in number of biweekly periods and in years (round up).

Ex. Civil engineer's Biweekly net earnings = 2 153.56
 Save 5% biweekly = $0.05(2153.56) = 107.68$
 Item selected is media centre has value of = \$5000.
 Number of biweekly periods needed to save up for this media centre = $5000 \div 107.68 = 46$ biweekly periods
 Time in year = 1.8 years

- B. Instead of just putting the money in a regular savings account that doesn't give much back in terms of interest.

Suppose you put your biweekly savings, calculated for your salary, into an investment portfolio (every other week same amount is deposited) with a 3% rate of interest compounded biweekly. Calculate the total amount you'd have at the end, and the amount of interest you'd still have in your pocket after the purchase of your item. (use the same time you've calculated for your item in part a, even though there will be rounding error)

Ex. Regular deposit made = $R = 107.68$
 Time to save = 1.8 years
 Periodic interest rate = $i = r/C = 0.03/26$ (leave the calculation for later)
 Number of compounding periods = $n = Ct = 26(1.8) = 46.8$
 Now use the formula:

$$FV = \frac{R[(1+i)^n - 1]}{i}$$

$$FV = \frac{107.68[(1 + \frac{0.03}{26})^{46.8} - 1]}{(\frac{0.03}{26})}$$

$$FV = 5174.90$$
 Total amount at the end is \$5174.90 and \$174.90 is in your pocket for investing the money after you've bought your media centre.

- C. Suppose you didn't want to wait for the money to be saved up, but bought the item now by financing it at 8% interest charge compounded biweekly. Calculate your biweekly payments to pay off the item in the same time as you used before. Then calculate the amount of interest you ended up paying to the lending institution for the use of their money.

Ex. Item to be financed is the media centre = $PV = \$5000$
 Time to pay off the loan = 1.8 years
 Periodic interest rate = $i = r/C = 0.08/26$ (leave the calculation for later)
 Number of compounding periods = $n = Ct = 26(1.8) = 46.8$
 Now use the formula:

$$PV = \frac{R[1 - (1+i)^{-n}]}{i}$$

$$5000 = \frac{R[1 - (1 + \frac{0.08}{26})^{-46.8}]}{(\frac{0.08}{26})}$$

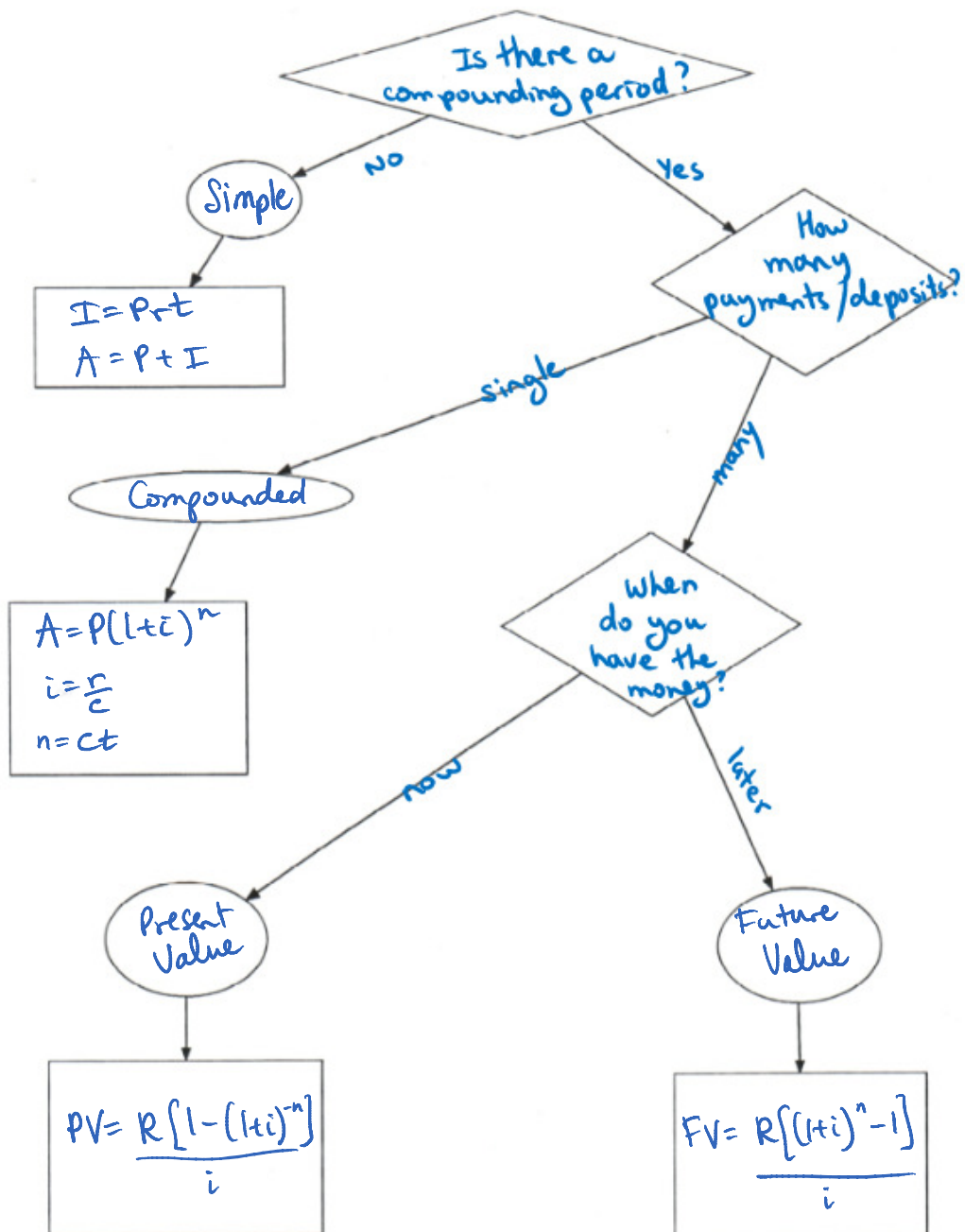
$$5000 = R(43.5242586751...)$$

$$114.88 = R$$
 Biweekly payments are \$114.88 made for 1.8 year or 46.8 biweekly payments amounts to \$5376.38. This is \$376.38 of interest that you paid to the lending institution just because you decided not to wait to save up for it yourself.



5. Jasmine wants to save money for retirement in an annuity. She plans to make equal monthly deposits, at the end of each month for 25 years in a trust account that has a guaranteed interest rate of 9% compounded monthly. She wants to have \$500 000 in the account at the end of the 25 years. What amount must be her monthly deposit? What is the interest that she earned by using the trust fund?
6. Sonia purchases a new vehicle for \$27000 at 8.2% compounded quarterly and makes payments at the end of every 3 months. Find the monthly payment. How much would Sonia pay in interest?
7. Paul is 16 years old and decides to deposit \$2000 at the end of each year that pays 9% compounded annually. Determine Paul's age when his annuity is worth \$30 000.

Calculating Interest Flowchart



Mix of Questions – what formula to use?

1. Jamail wants to have \$31000 in 4.5 years. How much should he invest now at 5.68% compounded semi-annually? $C=2$

doesn't say many deposits $\therefore A = P(1+i)^n$

$$i = \frac{0.0568}{2}$$

$$n = 2(4.5) = 9$$

$$31000 = P \left(1 + \frac{0.0568}{2} \right)^9$$

$$31000 = P(1.2866446 \dots)$$

$$\$24093.68 = P$$

2. Abbas invested a certain amount at a rate of 5% over 40 weeks. If he received \$49.96 in interest, determine his initial amount of investment.

doesn't say compounded $\therefore I = Prt$

$$49.96 = P \left(0.05 \left(\frac{40}{52} \right) \right)$$

$$49.96 = P(0.384615 \dots)$$

$$1298.96 = P$$

3. Joanna was awarded \$48000 and she has selected to invest it in an annuity which will pay her 6.8% per annum compounded quarterly for 1.5 years. How large is each payment if she is to receive a payment every 3 months? $R = ?$

$$i = \frac{0.068}{4}$$

$$n = 4(1.5) = 6$$

$$PV = \frac{R[1 - (1+i)^{-n}]}{i}$$

$$48000 = \frac{R[1 - (1 + \frac{0.068}{4})^{-6}]}{(\frac{0.068}{4})}$$

$$48000 = R(5.658585 \dots)$$

$$\$8482.69 = R$$

Annuities with a TVM Solver



Since you are dealing with money, two decimal places are appropriate.

Press the **MODE** key and find where you can select 2 decimals places – could be under **Float**
Press **ENTER** to select it
EXIT

Press the **APPS** key and find and select **Finance**. Then find and select **TVM Solver**. There may be some numbers left in the TVM solver from a previous use. Just replace these numbers with yours.

What do all the variables mean?

N = total number of payments (# of years x # of times compounded)

$I\%$ = interest rate as a percent

PV = present value or principal

PMT = amount of each payment

FV = future value

P/Y = payments per year

C/Y = compounding periods per year

PMT : **END** **BEGIN** to select when payments are made → ALWAYS USE END

```

N=
I%=7.427457867
PV=-1000
PMT=0
FV=1200
P/Y=2
C/Y=2
PMT:END BEGIN
  
```



1. Esteban and Suzanne want to take their sons on a vacation to Florida in 1 year. They estimate the trip will cost \$2500. They have an account that pays 3% interest per year, compounded monthly. Determine the amount they will need to deposit into the account at the end of each month to reach their goal.



Use the cursor keys and number keys to enter the numbers where they belong.

$n = N = 12$ ← there are 12 months in 1 year *total # of comp*

$r = I\% = 3$ ← interest rate is 3%/year

$PV = 0$ ← they have no money in the account right now

$R = PMT = \underline{\hspace{1cm}}$ ← the amount to be calculated *enter as NEGATIVE*

$FV = 2500$ ← they would like to have \$2500 at the end of the investment

$P/Y = 12$ ← there are 12 monthly payments in 1 year

$C/Y = 12$ ← since it is compounded monthly, there are 12 compounding periods per year

```

N=12.00
I%=3.00
PV=0.00
PMT=0.00
FV=2500.00
P/Y=12.00
C/Y=12.00
PMT:END BEGIN
  
```

To solve (find the payment),

- i. scroll up to PMT
- ii. press **ALPHA**, and then **ENTER**

```

N=12.00
I%=3.00
PV=0.00
PMT=-205.48
FV=2500.00
P/Y=12.00
C/Y=12.00
PMT:END BEGIN
  
```

Notice that the payment is negative.

The TVM Solver distinguishes between money received (+) and money given (-). The negative value makes sense since each payment is money that Esteban and Suzanne give up.

∴ Esteban and Suzanne need to deposit \$205.48 at the end of each month to reach their goal.



2. Jesse wants to have a party for his girlfriend's birthday in 6 weeks. He estimates it will cost him \$500 for snacks, drinks and entertainment. His savings account pays 2% interest per year, compounded monthly. How much money does he need to save weekly?

t but convert!
 $N = Ct = 52(\frac{6}{52}) = 6$
 $I\% = 2$
 $PV = 0$
 $PMT = ?$
 $FV = 500$
 $P/Y = 52$
 $C/Y = 52$
 $PMT: [END] BEGIN$

$\$83.25$ every week



3. Tatiana wants to buy a surround-sound system for her TV. She wants it in time for her vacation, in 4 months. It costs \$1100. Her account pays 1.8% interest per year, compounded monthly. How much does she need to save each month?

convert!
 $N = 12(\frac{4}{12}) = 4$
 $I\% = 1.8$
 $PV = 0$
 $PMT = ?$
 $FV = 1100$
 $P/Y = 12$
 $C/Y = 12$
 $PMT: [END] BEGIN$

$\$274.38$ every month



4. Carrie and Bill want to build a deck and landscape their backyard. They estimate it will cost \$4000. Their account pays 2.5% interest per year, compounded monthly. They can afford to save \$325 per month.

- a. How long will it take Carrie and Bill to save \$4000?

$N =$
 $I\% =$
 $PV =$
 $PMT =$
 $FV =$
 $P/Y =$
 $C/Y =$
 $PMT: [END] BEGIN$

- b. What payment would they need to make if they already had \$1000 saved?

$N =$
 $I\% =$
 $PV =$
 $PMT =$
 $FV =$
 $P/Y =$
 $C/Y =$
 $PMT: [END] BEGIN$

- c. If the interest rate is lowered to 1.4%, how long will it take?

$N =$
 $I\% =$
 $PV =$
 $PMT =$
 $FV =$
 $P/Y =$
 $C/Y =$
 $PMT: [END] BEGIN$

- d. If the interest rate is 1.4% and they only have 8 months to save, how much would they need to save each month?

$N =$
 $I\% =$
 $PV =$
 $PMT =$
 $FV =$
 $P/Y =$
 $C/Y =$
 $PMT: [END] BEGIN$

To complete homework without the graphing calculator use the following online applets:

<http://www.zenwealth.com/BusinessFinanceOnline/TVM/TVMCalculator.html>

OR

http://www.mortgage-a.com/mortgage_financial_calculators/calculator.htm

Life Project – marked as KU for this unit

As you get new information, tape/glue/staple/ write the information you get here.
Show all the calculations on this page – to be submitted for marks later.

1. Randomly selected career & salary:
 - a. Federal tax =
Provincial tax =
Net income =
 - b. Biweekly gross earnings =
Biweekly net earnings =
Taxes deducted from every paycheck =
 - c. (if you choose to do the BONUS attach that information to this page before you submit)

2. Randomly selected Bonus and GIC investment:

Final amount of investment =
Interest =

3. Randomly selected unforeseen circumstance and credit card rate:

Convert time to years, if needed: $t =$
Convert rate to periodic rate: $i = r \div C =$
Find total number of compounding periods: $n = Ct =$
Now use the formula:

Total debt =
Interest =

4. Randomly selected item to buy is:

Option A: How long will it take to save up for the item you selected?

Your biweekly net earnings =

Save 5% biweekly =

Item selected has value of =

Number of biweekly periods needed to save up for this item =

Time in years =

Option B: How much can you save if you invest your savings?

Regular deposit made = $R =$

Time to save =

Periodic interest rate = $i = r \div C =$

Number of compounding periods = $n = Ct =$

Now use the formula:

Total amount at the end is =

Money in your pocket for investing the money after you've bought your item =

Option C: How much money will you be out for buying the item with a loan?

Item to be financed is = $PV =$

Time to pay off the loan =

Periodic interest rate = $i = r \div C =$

Number of compounding periods = $n = Ct =$

Now use the formula:

Biweekly payments are =

Total amount paid to the institution =

Interest paid to the lending institution =