NOTESallANS

Look below for ALL
answers to notes - if you find mistakes, let me know

Exponentials Unit

Tentative TEST date_______________

Reflect – previous TEST mark ___________, Overall mark now___________.
Looking back, what can you improve upon?

Big idea/Learning Goals

This unit introduces you to a new type of function – the exponential function. There are many relationships in real life that either grow or decay at a constant rate, for example: population growth, or radioactive decay. These type of relationships can be modelled with an exponential. Before you learn about properties of this function, you must review exponent laws from gr.9-10 and learn a new exponent law that allows you to work with rational (fraction) exponents. Keep in mind that this year you will not know what the inverse of the exponential is, this is something that you will learn in the gr.12 advanced functions course – but to take this course you must take the Math for College Technology MCT 4C first.

Success Criteria

- ☐ I am ready for this unit if I am confident in the following review topics
  (circle the topics you are good at & review the ones you left uncircled before you get too far behind)
  Simplifying expressions, solving equations, transformations, function notation, domain & range, exponent laws

- ☐ I understand the new topics for this unit if I can do the practice questions in the textbook/handouts
  (check off the topics for which you have finished the practice)

Date	Topics	Done?
	Working with Positive Exponents Section 7.2 p400 #5,6,7,8,9,11 & EXTRA Handout	
	Working with Integer Exponents Section 7.3 p408 #4,7,8,9,11,12 & EXTRA Handout	
	Working with Rational Exponents & Solving for x– 2 days Section 7.4 416 #7,10,11,14,15,17 & TWO EXTRA Handouts	
	If there is time – Collecting Exponential Data Handout	
	Properties of Exponentials with Graphing Calc Section 7.5 p423 #1,2,4 (use graphing technology online to help you if needed)	
	Intro to Exponential Growth & Decay Section 7.6 p430 #4,7,9,10,11	
	Practice Exponential Growth & Decay Section 7.7 p437 #2,3,6,8,11,	

- ☐ I am prepared for the test/evalutation if
  - ☐ I understand the main concepts from each lesson
    - if not, ask other students in class to help you study or visit the peer tutoring room or ask the teacher for help or get a private tutor
    - also practice "knowledge-understanding" questions from the textbook – look for questions marked by K
  - ☐ I can explain/communicate the ideas clearly
    - if not, practice explaining a solved question to someone else or complete the assigned journal questions
    - also practice "communication" questions from the textbook – look for questions marked by C
  - ☐ I can apply these concepts in word problems
    - if not, practice "application" questions from the textbook – look for questions marked by A
  - ☐ I did not just memorize steps to do for different types of questions, I understand the ideas behind each concept and therefore can do problems in new contexts
    - if not, practice "thinking-inquiry-problem-solving" questions from the textbook – look for questions marked by T
  - ☐ I can do questions independently
    - if not, try redoing an already solved example without looking at solutions
  - ☐ I can complete questions quickly and with confidence
    - if not, try timing yourself for similar type questions to see progress
  - ☐ I completed the review and/or practice test

Corrections for the textbook answers:

Working with Positive Exponents

1. Why do you think exponent notation was invented?

Used as a shortcut for $5\times5\times5 = 5^3$

2. Summarize the exponent laws you learned in grade 9 and provide examples.

LAW	GENERALIZATION	EXAMPLE
Multiplication $a^n \cdot a^m = a^{n+m}$	- keep bases that are the same - add their exponents	$3^4 \cdot 3^5 = 3^9 = 19\,683$ don't multiply 3 and 3 !!
Division $\frac{a^n}{a^m} = a^{n-m}$	- keep base that is the same - subtract exponents	$\frac{7^5}{7} = 7^4 = 2401$ don't divide 7 and 7 !!
Power of a Power $(a^n)^m = a^{nm}$	- keep the base - multiply exponents	$(3^2)^5 = 3^{10} = 59\,049$
Power of a Product $(a^x b^y)^z = a^{xz} b^{yz}$	- distribute exponent in and multiply * insert ONE on any base without exponent	$(5x^6y^2)^3 = 5^3x^{18}y^6 = 125x^{18}y^6$
Power of a Quotient $\left(\frac{a^x}{b^y}\right)^z = \frac{a^{xz}}{b^{yz}}$	- distribute exponent	$\left(\frac{2x^4}{3^2y^3}\right)^5 = \frac{2^5x^{20}}{3^{10}y^{15}} = \frac{32x^{20}}{59049y^{15}}$
Zero Exponent $a^0 = 1$	- always equals ONE	$8^0 = 1$
~~Power of Sum/Diff~~	NO such RULE !! - can't distribute exponent - MUST expand the long way	$(2^3+x)^2 = (2^3+x)(2^3+x)$ $= 2^6 + 2^3x + 2^3x + x^2$ $= 64 + 8x + 8x + x^2$ $= 64 + 16x + x^2$

Write each expression as a single power

3. $6^1 \cdot 6^2$
$= 6^3$

4. $\frac{4^2}{4^3} = 4^{-3}$

5. $(5^2)^3$
$= 5^6$

Simplify

6. $3x^3 \cdot 4x^4$

$= 3^1(4^1)x^{3+4}$

$= 12x^7$

7. $\dfrac{8x^6}{12x^4}$

$= \dfrac{8}{12}x^{6-4}$

$= \dfrac{2}{3}x^2$

8. $(3xy^3)^4$

$= 3^4x^4y^{12}$

$= 81x^4y^{12}$

Simplify first then solve for x.

9. $3^x = 243$

trial + error

$3^5 = 243$

$\therefore x = 5$

10. $6^x + 5 = 221 - 5$

$6^x = 216$

trial + error

$6^3 = 216$ $\therefore x = 3$

11. $x^2(x^3) = 1024$

$x^5 = 1025$

trial + error

$4^5 = 1025$

$\therefore x = 5$

12. $\left(\dfrac{x^{10}}{x^7}\right)^3 \div x^6 = 125$

$\left(\dfrac{x^{30}}{x^{21}}\right) \div x^6 = 125$

$x^7 \div x^6 = 125$

$x^1 = 125$ $\therefore x = 125$

Working with Integer Exponents

1. Explore what negative exponents mean by filling in the table MEAN to bring the base over division line + change to pos. exp.

DIVISION	EXPAND & DIVIDE	USE EXPONENT LAW	
$2^3 \div 2^5$	$\frac{\cancel{2}\times\cancel{2}\times\cancel{2}}{\cancel{2}\times\cancel{2}\times\cancel{2}\times 2\times 2} = \frac{1}{2^?} \; \frac{1}{2^2}$	$\frac{2^3}{2^5} = 2^{3-5} = 2^? \; 2^{-2}$	$\therefore \frac{1}{2^2} = 2^{-2}$
$3^2 \div 3^4$	$\frac{\cancel{3}\cdot\cancel{3}}{\cancel{3}\cdot\cancel{3}\cdot 3\cdot 3} = \frac{1}{3^2}$	$\frac{3^2}{3^4} = 3^{2-4} = 3^{-2}$	$\therefore \frac{1}{3^2} = 3^{-2}$
$5 \div 5^4$	$\frac{\cancel{5}}{\cancel{5}\cdot 5\cdot 5\cdot 5} = \frac{1}{5^3}$	$\frac{5}{5^4} = 5^{1-4} = 5^{-3}$	$\therefore \frac{1}{5^3} = 5^{-3}$
$10^4 \div 10^5$	$\frac{\cancel{10}\cdot\cancel{10}\cdot\cancel{10}\cdot\cancel{10}}{\cancel{10}\cdot\cancel{10}\cdot\cancel{10}\cdot\cancel{10}\cdot 10} = \frac{1}{10}$	$\frac{10^4}{10^5} = 10^{4-5} = 10^{-1}$	$\therefore \frac{1}{10} = 10^{-1}$
$x^2 \div x^5$	$\frac{\cancel{x}\cancel{x}}{xxxxx} = \frac{1}{x^3}$	$\frac{x^2}{x^5} = x^{2-5} = x^{-3}$	$\therefore \frac{1}{x^3} = x^{-3}$

State another way to write the following. (HINT: if a base has no exponent on it, place exponent ONE on it)

2. $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

3. $4^{-3} = \frac{1}{4^3} = \frac{1}{64}$

4. $\frac{1}{2^{-4}} = 2^4 = 16$

5. $2x^{-3} = 2\left(\frac{1}{x^3}\right) = \frac{2}{x^3}$

6. $\frac{3}{4^{-2}} = 3(4^2) = 3(16) = 48$ BEDMAS!!

7. $\frac{3x^{-2}}{(2y)^{-1}} = \frac{3^1x^{-2}}{2^{-1}y^{-1}} = \frac{3(2)^1y^1}{x^2} = \frac{6y}{x^2}$

8. $\frac{1}{6^{-3}} = 6^3 = 216$

9. $\left(\frac{2}{3}\right)^{-4} = \frac{2^{-4}}{3^{-4}} = \frac{3^4}{2^4} = \frac{81}{16}$

10. $\frac{4^{-3}}{9^{-2}} = \frac{9^2}{4^3} = \frac{81}{64}$

11. $\frac{8^{-2}}{3} = \frac{1}{3(8^2)} = \frac{1}{3(64)} = \frac{1}{192}$

12. $\frac{3}{5^{-2}} = 3(5^2) = 3(25) = 75$

13. $\frac{(4a)^{-1}}{5b^{-3}} = \frac{4^{-1}a^{-1}}{5^1b^{-3}} = \frac{b^3}{5^1(4^1)a^1} = \frac{b^3}{20a}$

14. Summarize the negative exponent rule

Bring just the base with the negative exponent over the division line + change to positive exponent

$a^{-n} = \frac{1}{a^n}$ or $\frac{1}{a^{-n}} = a^n$

bring down

bring up.

15. There will be several ways to simplify expressions, depending on what rule you start applying first. Final answers should still match no matter what route you take. To make things easier try to use the ___power of power___ law first and ___negative exponent___ law last.

(can do division law at the end too results go on top!!)

16. Apply the laws to the following examples as you simplify the questions. Leave everything as exact numbers, with positive exponent answers.

a. $(4x)^2 \times 4x^2$

$= 4^2x^2\,4^1x^2$

$= 4^3x^4$

$= 64x^4$

b. $(3d^{-3})^3 \times 3d^{-2}$

$= 3^3d^{-9}\,3^1d^{-2}$

$= 3^4d^{-11}$

$= \frac{3^4}{d^{11}}$

$= \frac{81}{d^{11}}$

c. $4(-2x^5y^0)^{-2} \times (2x^{-1}y^2)^{-3}$

$= 4^1(-2)^{-2}x^{-10}y^0 \times 2^{-3}x^3y^{-6}$

$= 4(-2)^{-2}x^{-7}y^{-6}$

$= \frac{4}{(-2)^2x^7y^6}$

$= \frac{4}{4x^7y^6} = \frac{1}{x^7y^6}$

d. $\frac{(-3c^4)^{-2}}{c^{-1} \times (3c^{-2})^{-2}}$

$= \frac{(-3)^{-2}c^{-8}}{c^{-1}\,3^{-2}c^4}$

$= \frac{(-3)^{-2}c^{-8}}{3^{-2}c^3}$

$= \frac{3^2}{(-3)^2c^3c^8}$

$= \frac{9}{9c^{11}}$

$= \frac{1}{c^{11}}$

e. $\left(\frac{(-2a^{-2})^3 a^3}{4a^{-4}}\right)^{-3}$

$= \left(\frac{(-2)^3a^{-6}a^3}{4^1a^{-4}}\right)^{-3}$

$= \frac{(-2)^{-9}a^{18}a^{-9}}{4^{-3}a^{12}}$

$= \frac{(-2)^{-9}a^9}{4^{-3}a^{12}}$

$= \frac{4^3a^{9-12}}{(-2)^9}$

$= \frac{64a^{-3}}{-512}$

$= \frac{64}{-512a^3}$

$= \frac{1}{-8a^3}$

f. $\frac{(-2xy^3 \times 3x^{-3}y^{-2})^3}{6x^0y^{-1}}$

$= \frac{(-2)^3x^3y^9\,(3)^3x^{-9}y^{-6}}{6x^0y^{-1}}$

$= \frac{(-2)^3(3)^3x^{-6}y^3}{6x^0y^{-1}}$

$= \frac{(-8)(27)y^3y^1}{6(1)x^6}$

$= \frac{-216y^4}{6x^6}$

$= \frac{-36y^4}{x^6}$

Working with Rational Exponents

1. What does the word rational mean? RATIONAL = FRACTION

2. Use a calculator to complete the charts

For example, to find $4^{\frac{1}{2}}$, use the sequence $\boxed{4}\boxed{y^x}\boxed{(}\boxed{1}\boxed{\div}\boxed{2}\boxed{)}\boxed{=}$.

To find $\sqrt[4]{16}$ use $\boxed{\sqrt[x]{\ }}$ button: either $\boxed{16}\boxed{\sqrt[x]{\ }}\boxed{4}$ or $\boxed{4}\boxed{\sqrt[x]{\ }}\boxed{16}$ try BOTH to see which way you need to remember

$4^{\frac{1}{2}} =$ 2	$\sqrt[2]{4} =$ 2	$16^{\frac{1}{4}} =$ 2	$\sqrt[4]{16} =$ 2
$\sqrt[3]{27} =$ 3	$27^{\frac{1}{3}} =$ 3	$\sqrt[5]{3125} =$ 5	$3125^{\frac{1}{5}} =$ 5
$216^{\frac{1}{3}} =$ 6	$\sqrt[3]{216} =$ 6	$256^{\frac{1}{4}} =$ 4	$\sqrt[4]{256} =$ 4
$\sqrt{25} =$ 5	$25^{\frac{1}{2}} =$ 5	$\sqrt[4]{81} =$ 3	$81^{\frac{1}{4}} =$ 3
$36^{\frac{1}{2}} =$ 6	$\sqrt{36} =$ 6	$1296^{\frac{1}{4}} =$ 6	$\sqrt[4]{1296} =$ 6

type in with brackets

do root 1st then exponent

a	$a^{\frac{1}{3}}$	$a^{\frac{2}{3}}$	$a^{\frac{3}{3}}$	$a^{\frac{4}{3}}$	$a^{\frac{5}{3}}$
8	$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$	$8^{\frac{2}{3}} = \sqrt[3]{8^2} = 2^2 = 4$	$8^{3/3} = 8^1 = \sqrt[3]{8^3} = 8$	$8^{4/3} = \sqrt[3]{8^4} = 2^4 = 16$	$8^{5/3} = \sqrt[3]{8^5} = 2^5 = 32$
64	$64^{1/3} = \sqrt[3]{64} = 4$	$64^{2/3} = \sqrt[3]{64^2} = 4^2 = 16$	64	256	1024
125	$125^{1/3} = \sqrt[3]{125} = 5$	25	125	625	3125

3. Summarize the rule when the exponent is in a rational form $\frac{1}{n}$ and $\frac{m}{n}$

$a^{\frac{1}{n}} = \sqrt[n]{a^1}$ and $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

- the denominator is the type of root
- the numerator is like regular exponent

4. Rewrite the following in a different notation. Simplify if possible. (HINT if brackets or exponents are not there – insert them)

remember $\sqrt[3]{125}$ = what # times itself 3 times is 125?

a. $6^{\frac{2}{7}} = \sqrt[7]{6^2}$

b. $\sqrt[4]{(2^1x^5)^1}$
$= 2^{1/4}x^{5/4}$ or $(2x^5)^{1/4}$

c. $\sqrt[3]{(3a)^5}$
$= (3a)^{5/3}$
or
$= 3^{5/3}a^{5/3}$

d. $\sqrt[2]{(16x^7)^1}$ $= (16^1x^7)^{1/2}$
or
$16^{1/2}x^{7/2}$

Two if nothing there! square root...

e. $\sqrt{64}$
$= 64^{1/2}$
$= 8$

f. $121^{\frac{1}{2}}$
$= \sqrt[2]{121}$
$= 11$

g. $\sqrt[3]{(-343)}$
$= (-343)^{1/3}$
$= -7$

h. $-125^{\frac{1}{3}}$
$= -\sqrt[3]{125}$
$= -5$

watch placement of the negative

5. Write as both versions. Evaluate using the calculator for both versions, HOWEVER if you get decimals or error, use laws of exponents to simplify things first!

a. $-49^{\frac{1}{2}}$

$= -\sqrt[2]{49}$

$= -7$

on calc:

$-49 \wedge (1 \div 2)$

OR

$-\sqrt{\ }\ 49$

For some calc must ignore negative since not in the root!

b. $(-125)^{-\frac{2}{3}}$ get error

$= \frac{1}{(-125)^{2/3}}$

$= \frac{1}{\sqrt[3]{(-125)^2}} = \frac{1}{(-5)^2} = \frac{1}{25}$

c. $\left(\frac{8}{27}\right)^{-\frac{1}{3}}$ get decimal!

$= \frac{8^{-1/3}}{27^{-1/3}} = \frac{27^{1/3}}{8^{1/3}} = \frac{\sqrt[3]{27}}{\sqrt[3]{8}} = \frac{3}{2}$

d. $\sqrt{121} =$

$= 121^{1/2}$

$= 11$

e. $\sqrt[3]{8} = 8^{1/3} = 2$

f. $\sqrt[4]{16} = 16^{1/4} = 2$

g. $8^{\frac{2}{3}} = \sqrt[3]{8^2}$

$= 2^2$

$= 4$

h. $-25^{\frac{3}{2}} = -\sqrt[2]{25}^{\,3}$

$= -5^3$

$= -125$

i. $81^{-\frac{3}{4}} = \sqrt[4]{81}^{\,-3}$

do neg at the end!!

$= 3^{-3}$

$= 27^{-1}$

$= \frac{1}{27}$

6. Solve for x.

a. $x^{\frac{1}{2}} = 7$

$(x^{\frac{1}{2}})^2 = 7^2$

$x = 49$

b. $x^{\frac{3}{2}} = 8$

$(x^{3/2})^{2/3} = 8^{2/3}$

$x = \sqrt[3]{8^2} = 2^2 = 4$

c. $x^{\frac{4}{5}} = 81$

$(x^{4/5})^{5/4} = 81^{5/4}$

$x = \sqrt[4]{81^5}$

$x = 3^5$

$x = 343$

d. $x^{\frac{4}{3}} = 625$

$(\cancel{x^{4/3}})^{3/4} = (625)^{3/4}$

$x = \sqrt[4]{625^3}$

$x = 5^3 = 125$

MORE Working with Rational Exponents & Solving for x.

1. Explain why you would get different answers for:

$\sqrt[3]{(27^1x^4)^1}$ and $(27x)^{\frac{4}{3}}$

can insert bracket to see $(27^1x^4)^{1/3}$

NOT the same outside power

2. Explain the steps in simplifying the following. Give a reason why you can't cancel x^6 or divide 512 with 4.

$\dfrac{\sqrt[3]{512^1x^6}}{\sqrt[2]{4^1x^6}}$

roots are different ∴ can't simplify division yet take care of numerator + denom. seperately

$\dfrac{512^{1/3}x^{6/3}}{4^{1/2}x^{6/2}} = \dfrac{8x^2}{2x^3} = 2x^{-1} = \dfrac{2}{x}$

3. Simplify the following. Keep answers as exact reduced fractions and and don't leave answers with negative exponents.

a. $(8^1x^6y^9)^{\frac{1}{3}}(27^1x^{-12}y^6)^{-\frac{1}{3}}$

Do negatives last!!

$= 8^{\frac{1}{3}}x^{6/3}y^{9/3}\ 27^{-1/3}x^{\frac{-12}{-3}}y^{-\frac{6}{3}}$

$= 2x^2y^3\ 3^{-1}x^4y^{-2}$

$= 2x^6y^1\ 3^{-1} = \dfrac{2x^6y}{3}$

b. $\left(\dfrac{64^1m^{15}}{343^1}\right)^{-\frac{2}{3}} = \dfrac{64^{-2/3}m^{-\frac{30}{3}}}{343^{-2/3}}$

$= \dfrac{16^{-1}m^{-10}}{49^{-1}} = \dfrac{49}{16m^{10}}$

c. $(256^1a^{12}b^{20})^{\frac{3}{4}}$

$= 256^{3/4}a^{36/4}b^{\frac{60}{4}}$

$= 64a^9b^{15}$

d. $\left(3^1a^{\frac{3}{2}}\right)\left(-7^1a^{\frac{1}{5}}\right)$

$= 3^1(-7)^1a^{3/2}a^{1/5}$ add

$\frac{3}{2}+\frac{1}{5}$

$\dfrac{15+2}{10} = \dfrac{17}{10}$

$= -21a^{17/10}$

or $= -21\sqrt[10]{a^{17}}$

e. $\left(8^1x^{\frac{3}{4}}y^2\right)^{-\frac{1}{3}}$

$= 8^{-1/3}x^{-3/12}y^{-2/3}$

$= 2^{-1}x^{-1/4}y^{-2/3}$

$= \dfrac{1}{2x^{1/4}y^{2/3}}$ or $= \dfrac{1}{2\sqrt[4]{x}\sqrt[3]{y^2}}$

f. $\dfrac{25^1x^{\frac{1}{3}}}{5^1x^{\frac{1}{4}}} = \dfrac{25}{5}x^{1/3-1/4}$

$= 5x^{1/12}$

$\frac{1}{3}-\frac{1}{4}$

$= \dfrac{4-3}{12}$

$= \dfrac{1}{12}$

4. Solve the following for x.HINT: Try to make the bases match, and combine multiple bases into one single base for each side, using laws of exponents. Then do trial and error.

a. $3^{2x-5}=1$

$3^{2x-5}=3^0$

$\therefore$ exponents are

$2x-5=0$

$2x=5$

$x=2.5$

b. $\left(\frac{1}{5}\right)^{-3x}\cdot 25^{x-1}=\frac{1}{125}$

$(5^{3x})\,5^{2(x-1)}=5^{-3}$

$5^{3x+2(x-1)}=5^{-3}$

$\therefore$ exponents are

$3x+2(x-1)=-3$

$3x+2x-2=-3+2$

$5x=-1$

$x=-\frac{1}{5}=-0.2$

c. $5^{4-x}=5^x$

$\therefore$ exponents are

$4-x=x$

$4=2x$

$2=x$

d. $4^x\cdot\frac{1}{16}=2^{3x+6}$

$(2^2)^x(2^{-4})=2^{3x+6}$

$2^{2x-4}=2^{3x+6}$

$\therefore$ exponents are

$2x-4=3x+6$

$-4-6=3x-2x$

$-10=x$

e. $7^{2x}\cdot 7^{3-x}=49^{x+5}$

$7^{2x}\,7^{3-x}=7^{2(x+5)}$

$7^{2x+3-x}=7^{2(x+5)}$

$\therefore$ exponents are

$2x+3-x=2(x+5)$

$x+3=2x+10$

$3-10=2x-x$

$-7=x$

If there is time – Collecting Exponential Data

Eli "M" ination – Radioactive Decay of Atoms

All matter is made up of atoms. Some kinds of atoms have too much energy and are unstable. These atoms are called radioactive. It is not possible to predict exactly when a radioactive atom will release its extra energy and form a different stable atom. This process is called radioactive decay. In this experiment you will model this process

Materials
M&M or skittles for groups
Paper plates or napkins
Plastic containers or cups

1. Pour a bag of M&M's or skittles onto a paper plate/napkin so that the candies are one layer thick so you can see if they have the logo in them or not. The candies represent the atoms. Count the number you have at the start and record in the table.

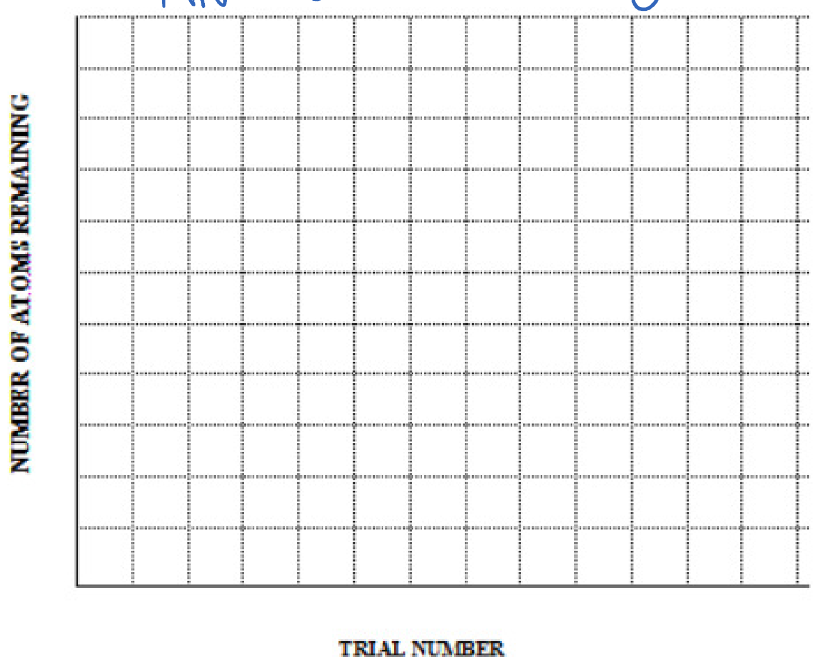
2. Remove all the M&M's with the logo showing on one side - these will represent atoms that have decayed. Count and record the number of M&M's remaining on the chart below.

3. Pour the remaining candies into a container. Shake the container and pour these M&M's back onto the plate. Again remove all the M&M's with the logo showing. Continue to repeat this process until all the M&M's are removed. Add additional trial numbers to the chart below if needed.

4. Fill in the table and sketch the relationship.

ANSWERS will vary

Trial Number	Radioactive Atoms Remaining
Start with	
1	
2	
3	
4	
5	



 11UC Date:____________ Name: ______________________

Germs! Germs! – Exponential Growth

Certain bacteria, under the right conditions, multiply themselves.
You will use strips of paper, each representing a bacterium, to model its growth.

Materials
Paper
Scissors

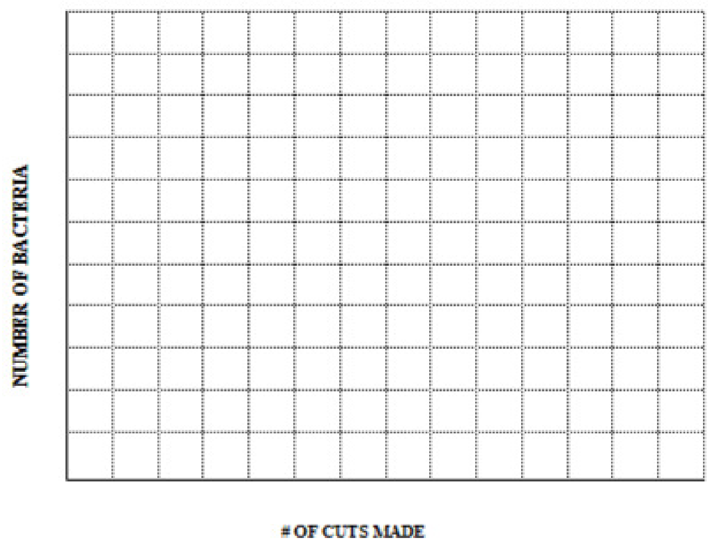
ANSWERS will vary

5. Cut your paper in the following arrangement based on group number you're given

	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6
Pieces of paper to start with	1	2	3	1	2	3
Cut each given piece into this many equal pieces each time	2	2	2	4	4	4

6. Fill in the table and sketch the relationship.

Cuts	# of Bacteria
Start with	
1	
2	
3	
4	
5	
6	



7. Clean up after your group please.

8. What is the domain and range of the Radioactive decay question? Explain.

9. What is the domain and range of the Germs question? Explain.

Properties of Exponentials with Graphing Calculators

A. Complete the tables of values for the functions $g(x)=x$, $h(x)=x^2$ and $k(x)=2^x$

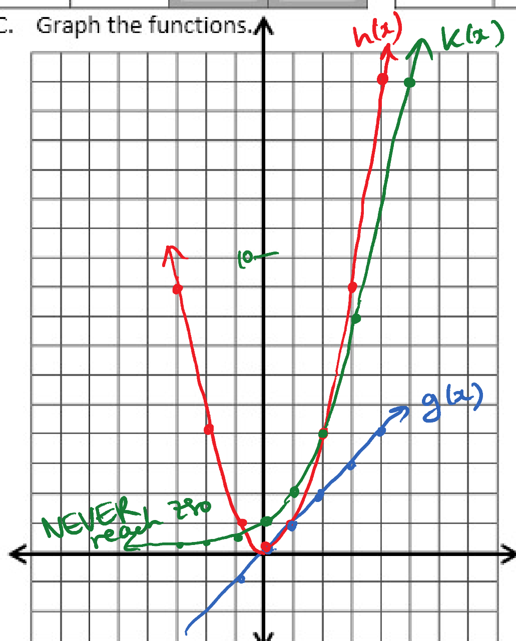
B. Calculate first and second differences. (GRAPH 1st)

x	$g(x)$	1st diff	2nd diff
−3		1	
−2		1	
−1	−1	1	
0	0	1	
1	1	1	
2	2	1	
3	3	1	
4	4	1	
5	5		

x	$h(x)$	1st diff	2nd diff
−3	9	−5	2
−2	4	−3	2
−1	1	−1	2
0	0	1	2
1	1	3	2
2	4	5	2
3	9	7	2
4	16	9	
5	25		

x	$k(x)$	1st diff	2nd diff
−3	$2^{-3}=0.125$	0.125	0.125
−2	$2^{-2}=0.25$	0.25	0.25
−1	$2^{-1}=0.5$	0.5	1
0	$2^0=1$	1	2
1	$2^1=2$	2	3
2	$2^2=4$	4	4
3	8	8	8
4	16	6	
5	32		

C. Graph the functions.



D. Complete the chart for each function.

	DOMAIN	RANGE
$g(x)$	$x\in\mathbb{R}$	$y\in\mathbb{R}$
$h(x)$	$x\in\mathbb{R}$	$y\in\mathbb{R}$, $y\geq 0$
$k(x)$	$x\in\mathbb{R}$	$y\in\mathbb{R}$, $y>0$

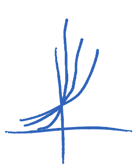
never reach zero.

E. How do the y-values change as the x-values change?

linear – y-values change by adding 1
quadratic – y-values change by squaring
exponential – y-values change by mult. by 2.

F. Graph the functions $y=2^x$, $y=5^x$ and $y=10^x$ on a graphing calculator.

G. Complete the chart for each function.

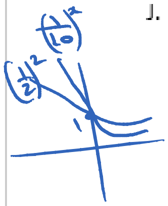


	DOMAIN	RANGE	INTERCEPTS	ASYMPTOTES
$y=2^x$	$x\in\mathbb{R}$	$y\in\mathbb{R}$, $y>0$	x-int NONE y-int=(0,1)	Horizontal y=0
$y=5^x$	"	"	"	"
$y=10^x$	"	"	"	"

H. Which curve increases faster when you trace right? Which one decreases faster when you trace left?

10^x changes faster either side!

I. Graph the functions $y=2^x$, $y=\left(\frac{1}{2}\right)^x$ and $y=\left(\frac{1}{10}\right)^x$ on a graphing calculator.



J. Complete the chart for each function.

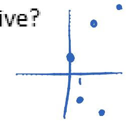
	DOMAIN	RANGE	INTERCEPTS	ASYMPTOTES
$y=\left(\frac{1}{2}\right)^x$	$x \in \mathbb{R}$	$y \in \mathbb{R},\ y>0$	x: None y: $(0,1)$	Horiz. $y=0$
$y=\left(\frac{1}{10}\right)^x$	"	"	"	"

K. How do these graphs differ from $y=2^x$?

These graphs decay from left to right base $=\frac{1}{2}<1$
2^x is growth base $=2>1$

L. What happens when the base of an exponential function is negative?

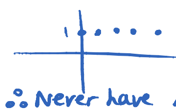
$y=(-2)^x$



bounces from neg. to pos
$\therefore$ NEVER have neg. BASE

M. What type of function is $f(x)=b^x$ when $b=1$?

$y=1^x$



doesn't grow or decay
not exponential!
$\therefore$ Never have base = ONE

N. Describe how the graph of an exponential function differs from the graph of a linear and quadratic function.

Linear constant rate of change

Quad

- dec + inc y-values
- with a turning pt.

Exp

constant growth or constant decay

O. How do the first and second differences of exponential functions differ from those of linear and quadratic functions? How can you tell that a function is exponential?

$\boxed{\text{differences = next y. minus prev. y.}}$

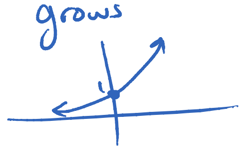
Linear – 1st differences are the same
Quadratic – 2nd differences are the same
Exponential – NONE of the differences are the same – but if you find ratios – they'll be the same $\boxed{\text{ratios = next y} \div \text{prev. y.}}$

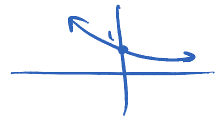
P. Investigate the graphs of the exponential function $f(x)=b^x$ for various values of b, listing all similarities and differences in their features (such as domain, range, and any intercepts and asymptotes). Generalize their features for the cases $b>1$ and $0<b<1$.

Note: An asymptote is a line that a function approaches, but never reaches.

DIFFERENCES

when $b>1$	when $0<b<1$
grows	decays





SIMILARITIES

same domain $x \in \mathbb{R}$
same range $y \in \mathbb{R}, y>0$
same intercepts x-int NONE, y-int $(0,1)$
same asymptote $y=0$

Intro to Exponential Growth & Decay

1. Most (but not all) real life word problems of growth or decay have a horizontal asymptote at y=0. State the general equation for exponentials that will most often be used for exponential word problems and explain the significance of EACH letter in the context of a word problems.
After you do question 5, come back and adjust the equation.

$y = a\,b^{x/c}$

y – final value or dependent y-value
a – initial value or y-intercept
b – growth /decay factor
to find it see →

x – any input of dependent variable usually time
c – how long it takes to grow/decay by "b"

2. Clarify the differences between growth factor and growth rate.

growth factor = b
growth rate = percent = r
ex. increase by 5% $r = 0.05$
growth factor $b = 1.05$

3. Summarize how to find the 'b' in the equation.

Look for special words: double $b=2$
half-life $b=0.5$
triple $b=3$ etc.

Look for %:
% increase $b = 1+r$
% decrease $b = 1-r$

4. Set up the models for the following word problems. If not obvious, explain what each variable represents.

a. The value of the $250 thousand cottage increases by 0.1% every month.

$V(m) = 250\,000(1+0.001)^{m/1}$
or $V = 250\,000(1.001)^m$

b. The 40 grams of radioactive matter within a mass decays at 2% every minute.

$G(m) = 40(1-0.02)^{m/1}$
$= 40(0.98)^m$

c. The 200 fruit fly population doubles every week.

$F(w) = 200(2)^{w/1}$

5. What will the equations look like if the questions were modified as follows.

a. The value of the $250 thousand cottage increases by 0.1% every 3 weeks.

$V(w) = 250000(1+0.001)^{w/3}$

b. The 40 grams of radioactive matter within a mass decays at 2% every 30 seconds.

$G(t) = 40(1-0.02)^{t/30}$
$= 40(0.98)^{t/30}$
in sec

c. The 200 fruit fly population doubles every 5 days.

$F(d) = 200(2)^{d/5}$

6. Solve the following problems:

a. A drug's effectiveness decreases as time passes. Each hour the 250mg drug loses 5% of its effectiveness. [How effective is the drug after 150 minutes?] use this info later

$E(h) = 250(1-0.05)^{h/1}$
150 min = 2.5 hr
$E(2.5) = 250(0.95)^{2.5/1}$
$= 250(0.8796\ldots) \doteq 220\text{mg}$

b. Carbon-14 (name) has a half life of 5730 years. (If no initial amount is given, assume 100% is the initial amount) [Determine the % of original carbon left after 1000 years]

$C(y) = 100(0.5)^{y/5730}$
$C(1000) = 100(0.5)^{1000/5730}$
$= 100[0.88606\ldots]$
$= 88.6\%$

c. A $1000 deposit is made at a bank that pays monthly percent, 1.5% compounded monthly. [How much will you have at the end of 10 years?]

$V(m) = 1000(1+0.015)^{m/1}$
10 yrs = 120 months
$V(120) = 1000(1.015)^{120}$
$= 1000(5.9693\ldots)$
$= \$5969.32$

7. The population of a bacteria culture is cut in half by an antibiotic every 30 minutes.

a. If the entire bacteria culture is present at 5:00 a.m., what fraction of the bacteria culture will be left at 9:30 a.m.?

not told How many → think 100% or 1

$$B(m) = 1\left(\frac{1}{2}\right)^{m/30}$$

$m = 4$ hrs + 30 min
×60
$m = 240 + 30$
$m = 270$ minutes

$$\therefore B(270) = 1\left(\frac{1}{2}\right)^{\frac{270}{30}}$$

$$= 1\left(\frac{1}{2}\right)^{9} = \frac{1}{512} \doteq 0.00195 \text{ (or } 0.195\% \text{ is left)}$$

b. At what time will the bacteria culture contain $\frac{1}{128}$ of its original population?

$$\frac{1}{128} = \left(\frac{1}{2}\right)^{\frac{m}{30}}$$

trial and error

$$\left(\frac{1}{2}\right)^{7} = \frac{1}{128}$$

$\therefore$ exponent is $7 = \frac{m}{30}$

$210 = m$
minutes since 5:00 am

$\therefore$ 3 hrs 30 min later @ 8:30 am

8. After half an hour $\frac{1}{32}$ of a sample of a radioactive material remains. What is it's half-life?

H = how long it takes to halve

$$R(t) = 1\left(\frac{1}{2}\right)^{\frac{t}{H}}$$

$$\frac{1}{32} = \left(\frac{1}{2}\right)^{\frac{0.5}{H}}$$

$$\left(\frac{1}{2}\right)^{5} = \left(\frac{1}{2}\right)^{\frac{0.5}{H}}$$

$\therefore$ exponents are

$\frac{5}{1} = \frac{0.5}{H}$

$5H = 0.5$

$H = 0.1$ of an hour
or 6 minutes

9. An ant colony quadruples its population every month. Currently, there are 13 000 in the nest. What is the monthly growth rate of the population?

$$A(m) = 13000(4)^{\frac{m}{1}}$$

$\boxed{b = 1 + r}$

$b = 4$ find $r = ?$

$4 = 1 + r$

$3 = r$ $\therefore$ 300% growth rate

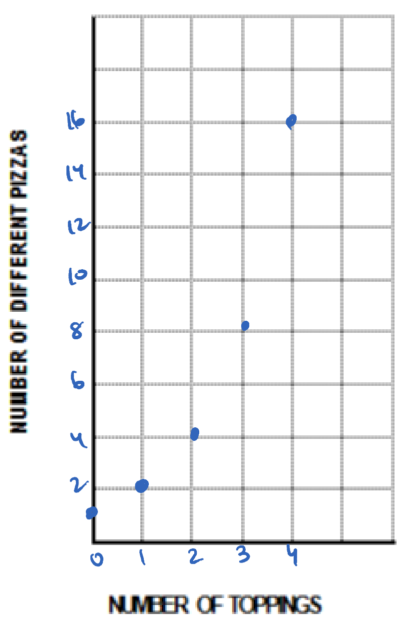
Practice Exponential Growth & Decay

Pizza toppings

1. Fill in the chart below by determining the different pizza's that can be created by choosing some or all or none of the available toppings

Toppings available	Different pizzas possible	# of different pizzas possible
None	plain crust with sause	1
Cheese	crust or crust cheese	2
Cheese, pepperoni	crust / crust cheese / crust pepperoni / crust cheese pepperoni	4
Cheese, pepperoni, mushrooms	crust / crust c / crust p / crust m / crust c p / crust c m / crust m p / crust c p m	8
Cheese, pepperoni, mushrooms, bacon	none / c / p / m / b / c p / c m / c b / m p / m b / p b / c p m / c p b / c m b / p m b / c p m b	16

2. Sketch



3. Complete the statement:
As the number of toppings increases by 1, the number of different pizza combinations increases by a factor 2.

4. Find an equation that will model this relationship.

$$P(t)=1(2)^t$$

initially

5. Use the equation to find how many different pizzas can be created if there are nine available toppings.

$$P(9)=1(2)^9$$
$$=512$$

6. If the restaurant owner would like to offer 200 different pizza combinations, what is the minimum number of available toppings she would need?

$$200=2^t$$

trial + error

$2^4=16$
$2^5=32$
$2^6=64$
$2^7=128$
$2^8=256$

$\therefore$ need at least 8 available toppings

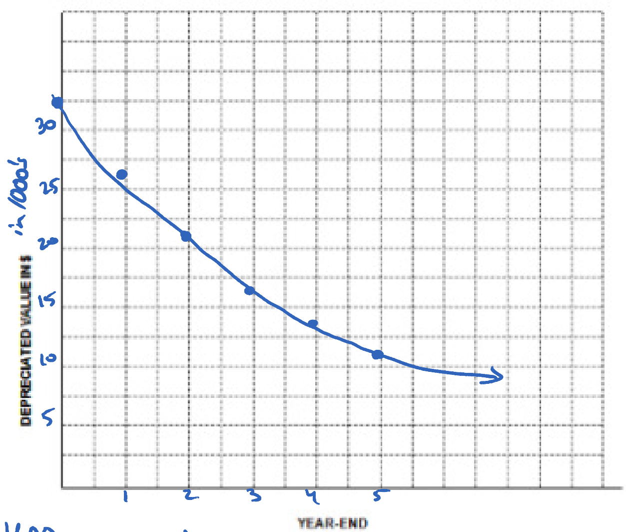
Car Depreciation

Depreciation is the decline in a car's value over the course of its useful life. It's something new-car buyers dread. Most modern domestic vehicles typically depreciate at a rate of 15%-20% per year depending on the model of the car.

1. A 2007 Ford Mustang GT convertible is valued at \$32 000 and depreciates on average at 20% per year. Fill in the table below. Hint: Since the car is depreciating by 20%, the remaining percent is $1-0.20=0.80$.

Year end	Value in \$
0	32 000
1	$32000(0.80)^1 = 25600$
2	$32000(0.80)^2 = 20480$
3	16384
4	13107.20
5	10485.76

6. Sketch



2. Find an equation that will model this relationship.

$$V(y)=32000(1-0.20)^{\frac{y}{1}}$$
$$=32000(0.80)^y$$

3. How much value does the car lose in the 1st year?

Lose \$32000 − 25600 = \$6400 in the 1st year

4. How much value does the car lose in the 5th year?

13107.20 − 10485.76 = \$2621.44

not as much of a drop

∴ if buy a new car not worth to sell it until it's much older...

5. After how many years will the value of the car be half of the original purchase price?

Half of 32000 = 16000

use the graph between 2−3 years...

Radioactive Decay

The equation $A(t)=100\left(\frac{1}{2}\right)^{\frac{t}{250}}$ was used to find the present-day radioactivity of some wooden tools at an archaeological dig.

1. What do all the letters and number represent?

A = final amount of radiation
t = time in years
100 = initial radiation (% ?)
$\frac{1}{2}$ = Decay by Half
250 = time it takes to decay by Half.

2. Find the percent of radiation left after 1000 years

$A(1000) = 100\left(\frac{1}{2}\right)^{\frac{1000}{250}}$
$= 100\,[0.0625)$
$= 6.25\%$ of radiation is left

3. Fill in the table

Years	% of Radiation
0 } Half life	100
250	50
500	25
750	12.5
1000	6.25
1250	3.125

4. Sketch