

# STRAND REVIEW: EXPONENTIAL

Answer each of the following questions in the space provided. Show all steps of work.

1. Evaluate. If fractional exponent or radical form appears, rewrite in the other notation.

a.  $\frac{(2^6)(2^{-4})}{(2^2)^3}$

b.  $\frac{3x^2y(3x^3y^{-2})^2}{3x^{-4}}$

c.  $\sqrt[3]{27}$

d.  $(-8)^{\frac{-4}{3}}$

e.  $\left(\frac{4}{25}\right)^{\frac{3}{2}}$

f.  $1000^{\frac{4}{3}}$

g.  $\left(-\frac{1}{8}\right)^{\frac{1}{3}}$

h.  $\left(\frac{27}{8}\right)^{\frac{2}{3}}$

2. Solve each of the following equations.

a.  $3^{2x-5} = 27$

b.  $4^{x+3} = 64$

c.  $5^{x+2} = \frac{1}{25}$

d.  $2^{5x+2} = \sqrt{2}$

3. The growth formula of a bacteria culture is  $n = 17000(8^{\frac{t}{2}})$ , where  $t$  is the number of days elapsed.
- How many bacteria are there at the start of the experiment?
  - How many times does the number of bacteria multiply every two days?
4. A bacteria culture doubles every  $\frac{1}{4}$  hour. At the start an estimate 40 000 of bacteria is made. What is the size of the culture after 1 day?
5. A bacteria culture starts with 3 000 bacteria. After 2 hours the estimated count is 48 000. What is the rate of increase of each hour?
6. The half-life of polonium is 5 minutes. At the end of 20 minutes what fraction of a sample of polonium remains?

7. At a 5% annual interest rate, a principal of \$500 earns \$30. How long was the principal invested?
  
  
  
  
  
  
  
  
  
  
8. Sarah lent \$400 to Grace at an annual simple interest rate of 12%. Grace repaid a total of \$466 to Sarah. For how long did Grace owe Sarah the money?
  
  
  
  
  
  
  
  
  
  
9. A \$500 investment earns interest at 6% compounded quarterly, for 8 years. How much interest will be earned in the final year of the 8-year term?
  
  
  
  
  
  
  
  
  
  
10. A small business loan of \$3000, at 9%, compounded monthly must be repaid after 2 years.
  - a. How much must be repaid?
  
  
  
  
  
  
  
  
  
  
  - b. How much interest will have been paid?

11. Ian has 15 monthly payments of 231.73 left on his student loan. Interest on the loan is at 9%, compounded monthly. If the bank will let Ian pay off his loan today, how much will he have to pay?
12. Julia wants to save \$2000 in 3 years. She plans to begin making regular quarterly deposits in 3 months into an account that earns interest at 4%, compounded quarterly, for 3 years. What must her regular deposits be in order to achieve her goal?
13. Ivan wants to buy a home entertainment centre priced at \$3799. He plans to buy it in 18 months, and he assumes that the price will stay the same. He will make a payment into an account at the end of every month for 18 months. The interest rate is 9% per year, compounded monthly. How much will each of Ivan's payment be?

# STRAND REVIEW: EXPONENTIAL

Answer each of the following questions in the space provided. Show all steps of work.

1. Evaluate. If fractional exponent or radical form appears, rewrite in the other notation.

$$\begin{aligned} \text{a. } & \frac{(2^6)(2^{-4})}{(2^2)^3} \\ &= \frac{2^2}{2^6} \\ &= 2^{-4} \\ &= \frac{1}{2^4} \\ &= \frac{1}{16} \end{aligned}$$

$$\begin{aligned} \text{b. } & \frac{3x^2y(3x^3y^{-2})^2}{3x^{-4}} \\ &= \frac{3x^2y3^2x^6y^{-4}}{3x^{-4}} \\ &= \frac{3^3x^8y^{-3}}{3x^{-4}} \\ &= \frac{3^2x^{12}}{y^3} = \frac{9x^{12}}{y^3} \end{aligned}$$

$$\begin{aligned} \text{c. } & \sqrt[3]{27} \\ &= 27^{1/3} \\ &= 3 \end{aligned}$$

$$\begin{aligned} \text{d. } & (-8)^{-4/3} \\ &= \frac{1}{(-8)^{4/3}} \\ &= \frac{1}{16} \end{aligned}$$

$$\left( \sqrt[3]{(-8)^4} \right)$$

$$\begin{aligned} \text{e. } & \left( \frac{4}{25} \right)^{3/2} \\ &= \frac{4^{3/2}}{25^{3/2}} \\ &= \frac{8}{125} \end{aligned}$$

$$\begin{aligned} \text{f. } & 1000^{4/3} \\ &= 10000 \\ &= \left( \sqrt[3]{1000^4} \right) \end{aligned}$$

$$\begin{aligned} \text{g. } & \left( -\frac{1}{8} \right)^{1/3} \\ &= \frac{(-1)^{1/3}}{8^{1/3}} \\ &= \frac{-1}{2} \end{aligned}$$

$$\begin{aligned} \text{h. } & \left( \frac{27}{8} \right)^{2/3} \\ &= \frac{27^{2/3}}{8^{2/3}} \\ &= \frac{9}{4} \end{aligned}$$

2. Solve each of the following equations.

$$\begin{aligned} \text{a. } & 3^{2x-5} = 27 \\ & 3^{2x-5} = 3^3 \\ & \therefore 2x-5 = 3 \\ & 2x = 8 \\ & x = 4 \end{aligned}$$

$$\begin{aligned} \text{b. } & 4^{x+3} = 64 \\ & 4^{x+3} = 4^3 \\ & \therefore x+3 = 3 \\ & x = 0 \end{aligned}$$

$$\begin{aligned} \text{c. } & 5^{x+2} = \frac{1}{25} \\ & 5^{x+2} = 5^{-2} \\ & \therefore x+2 = -2 \\ & x = -4 \end{aligned}$$

$$\begin{aligned} \text{d. } & 2^{5x+2} = \sqrt{2} \\ & 2^{5x+2} = 2^{1/2} \\ & \therefore 5x+2 = \frac{1}{2} \\ & 5x = -1.5 \\ & x = -0.3 \end{aligned}$$

3. The growth formula of a bacteria culture is  $n = 17000(8^{t/2})$ , where  $t$  is the number of days elapsed.

a. How many bacteria are there at the start of the experiment?

$$\text{initial value} = 17\,000$$

b. How many times does the number of bacteria multiply every two days?

8 times

4. A bacteria culture <sup>b=2</sup> doubles every <sup>c=0.25</sup> 1/4 hour. At the start an estimate <sup>a</sup> 40 000 of bacteria is made. What is the size of the culture after 1 day?

$$B(t) = 40\,000 (2)^{t/2}$$

$$B(24) = 40\,000 (2)^{24/2}$$

$$= 40\,000 (2)^{12}$$

$$= 163\,840\,000$$

1 day  
= 24 hrs

5. A bacteria culture starts with 3 000 bacteria. After 2 hours the estimated count is 48 000. What is the rate of increase of each hour?

$$48000 = 3000 (1+r)^2$$

$$\sqrt{16} = \sqrt{(1+r)^2}$$

$$4 = 1+r$$

$$3 = r \quad \text{or} \quad 300\%$$

6. The half-life of polonium is 5 minutes. At the end of 20 minutes what fraction of a sample of polonium remains?

$$P(m) = 1 \left(\frac{1}{2}\right)^{m/5}$$

$$P(20) = 1 \left(\frac{1}{2}\right)^{20/5}$$

$$= \left(\frac{1}{2}\right)^4$$

$$= \frac{1}{2^4}$$

$$= \frac{1}{16} \text{ fraction remains}$$

7. At a 5% annual interest rate, a principal of \$500 earns \$30. How long was the principal invested?

$$I = Prt$$

$$30 = 500(0.05)t$$

$$30 = 25t$$

$$1.2 \text{ years} = t$$

8. Sarah lent \$400 to Grace at an annual simple interest rate of 12%. Grace repaid a total of \$466 to Sarah. For how long did Grace owe Sarah the money?

$$I = 66$$

$$I = Prt$$

$$66 = 400(0.12)t$$

$$66 = 48t$$

$$1.375 \text{ years} = t$$

9. A \$500 investment earns interest at 6% compounded quarterly, for 8 years. How much interest will be earned in the final year of the 8-year term?

$$A = P(1+i)^n$$

$$= 500\left(1 + \frac{0.06}{4}\right)^{32}$$

$$= \$805.16$$

$n = 4 \times 8 = 32$

$\therefore$  interest is  $805.16 - 500$   
\$305.16

10. A small business loan of \$3000, at 9%, compounded monthly must be repaid after 2 years.
- a. How much must be repaid?

$$A = 3000\left(1 + \frac{0.09}{12}\right)^{24}$$

$$= 3000(1.196413529)$$

$$= 3589.24$$

$n = 2 \times 12 = 24$

- b. How much interest will have been paid?

$$\$589.24$$

11. Ian has  $(15)^n$  monthly payments of 231.73 left on his student loan. Interest on the loan is at 9%, compounded monthly. If the bank will let Ian pay off his loan today, how much will he have to pay? *many!*

$$PV = \frac{231.73 \left[ 1 - \left( 1 + \frac{0.09}{12} \right)^{-15} \right]}{\left( \frac{0.09}{12} \right)}$$

$$PV = \$3275.97$$

12. Julia wants to save \$2000 in 3 years. She plans to begin making regular quarterly deposits in 3 months into an account that earns interest at 4% compounded quarterly, for 3 years. What must her regular deposits be in order to achieve her goal? *FV, t, C=4*

$$FV = \frac{R \left[ (1+i)^n - 1 \right]}{i}$$

$r = 0.04$        $n = 4 \times 3 = 12$

$$2000 = \frac{R \left[ \left( 1 + \frac{0.04}{4} \right)^{12} - 1 \right]}{\left( \frac{0.04}{4} \right)}$$

$2000 = R (12.6825 \dots)$

$\$157.70 = R$

13. Ivan wants to buy a home entertainment centre priced at \$3799. He plans to buy it in 18 months, and he assumes that the price will stay the same. He will make a payment into an account at the end of every month for 18 months. The interest rate is 9% per year, compounded monthly. How much will each of Ivan's payment be? *FV, t = 18/12 yrs, r = 0.09, C = 12*

$$3799 = \frac{R \left[ \left( 1 + \frac{0.09}{12} \right)^{18} - 1 \right]}{\left( \frac{0.09}{12} \right)}$$

$n = 12 \times \frac{18}{12} = 18$

$$3799 = R (19.1947 \dots)$$

$\$197.92 = R$