

NOTESsomeANS

Notes that are done in class will be updated online periodically

(Any questions left blank you are responsible
to myself → get help if needed.)

Date:

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The Algebra of Quadratics Unit



Big idea

This unit will cover expanding and factoring methods you will be required to know for all the quadratic units that will follow. You should have seen most of these methods in grade 10. (If you took applied you have not seen the complex trinomial factoring method, since only grade 10 academic course covered that.) You must understand that factoring trinomials can be taught in many different methods (decomposition/Australian/criss cross methods). The criss cross method is the one you'll concentrate on in this class since it is the most efficient of all the methods. Without practicing you may not do very well in this unit and that would carry on into the next two units. Please take the time to practice so that the next two unit marks will not suffer for it.

know all the prior concepts related to his unit.

Place a ✓ if you are confident in that section.

Place a **≈** if you are **just ok** in that section.
Leave it blank if you are **lost** in that section.

If there are gaps in any row, please see the teacher for extra help in that topic.



Tentative TEST date

Reflect – TEST mark for this unit _____, Overall mark now _____
Looking back on this unit, what should you plan to improve upon before the exam?

Corrections for the textbook answers:

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Expand & Look For Patterns - these will help you factor later on.

	A	B	C	Examine the solutions carefully. State any patterns or similarities.	Generalization
1	$(x+4)(x-4)$	$(2x+3)(2x-3)$	$(x+y)(x-y)$		$(A+B)(A-B)$
2	$(4x+1)(4x+1)$	$(x+3)(x+3)$	$(x+y)(x+y)$		$\frac{(A+B)^2}{(A+B)(A+B)}$ $= (A+B)^2$
3	$(x-2)(x-2)$	$(3x-3)(3x-3)$	$(x-y)(x-y)$		$\frac{(A-B)^2}{(A-B)(A-B)}$ $= (A-B)^2$

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Expanding Quadratic Expressions

1. Expand and simplify.

a. $3(x+4)(x-2)$ if 3 multiplied start at the end

$$= 3(x^2 - 2x + 4x - 8)$$

$$= 3x^2 - 6x + 12x - 24$$

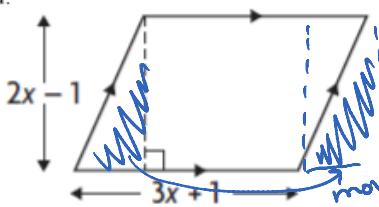
$$= 3x^2 + 6x - 24$$

b. $-2(3a+5)(a-2)$

b. $2b(b+1) - (b-1)^2$
 $= 2b^2 + 2b - (b^2 - 2b + 1)$
 $= 2b^2 + 2b - b^2 + 2b - 1$
 $= b^2 + 4b - 1$

d. $3(2x-1)^2 - 2(x-1)(x+1)$

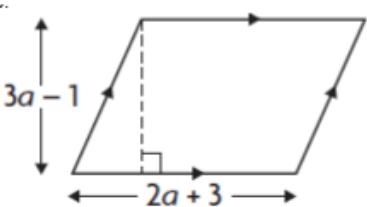
2. Write an expression for the area of each shape. Expand and simplify



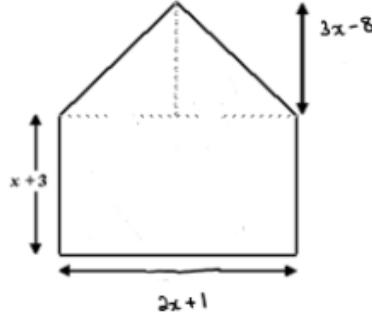
$$\begin{aligned} A &= b \times h \\ &= (3x+1)(2x-1) \\ &= 6x^2 - 3x + 2x - 1 \\ &= 6x^2 - x - 1 \end{aligned}$$

b.

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(4x-2)(2x+2) \\ &= \frac{1}{2}(8x^2 + 8x - 4x - 4) \\ &= \frac{1}{2}(8x^2 + 4x - 4) \\ &= 4x^2 + 2x - 2 \end{aligned}$$



d.



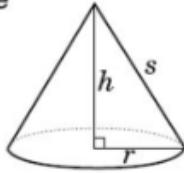
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3. Simplify the expression for the volume of the cone if $V = \frac{\pi r^2 h}{3}$, $r = 2 + x$, and $h = 2x - 3$

Cone



4. You need to understand when to expand and when distribute the exponent. Simplify the following, if possible. Summarize the rules.

a. $(5x^2y)^4$ CAN do ::

$$= (5x^2y^3)(5x^2y^3)$$

$$= 25x^4y^6$$

$$5^2 x^4 y^6$$

b. $(5x^2+y^3)^4$ not allowed because of plus sign

$$= (5x^2+y^3)(5x^2+y^3) \text{ FOIL}$$

$$\therefore \text{can distribute} \quad = 25x^4 + 5x^2y^3 + 5x^2y^3 + y^6$$

\therefore can distribute exponent for MONOMIALS $= 25x^4 + 10x^2y^3 + y^6$

$$\therefore \text{can't distribute exponent for POLYNOMIALS!}$$

c. $\sqrt{25x^2y^4}$

$$= \sqrt{25} \sqrt{x^2} \sqrt{y^4}$$

$$= 5xy^2$$

can distribute square root for MONOMIALS

d. $\sqrt{(5x^2-y^4)}$

can't do anything

$$\text{ex. } \sqrt{25-16} = \sqrt{9} = 3$$

not
the same as
 $\sqrt{25} - \sqrt{16}$
 $5 - 4$

e. $2x(3x)(4x^2)$

multiply all at once

$$= 24x^4$$

f. $(2-x)(3x+4x^2)$

distribute by FOIL

$$= 6x + 8x^2 - 3x^2 - 4x^3$$

$$= -4x^3 + 5x^2 + 6x$$

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Common Factoring

1. Summarize the steps of common factoring.

- Find G.C.F
 - divide it out of all terms
 - record the G.C.F outside of the bracket of the final answer

2. Factor the following

a. $9x^3 - 15x^2 + 3x$ G.C.F = $3x$

$$= 3x(3x^2 - 5x + 1)$$

b. $25x^2 - 100x^3$ G.C.F = $25x^2$

$$= 25x^2(1 - 4x)$$

c. $2\pi h(r+h)^2 + 4\pi(r+h)h^2$ G.C.F = $2\pi h(r+h)$

$$= 2\pi h(r+h)[(r+h) + 2h]$$

$$= 2\pi h(r+h)(r+3h)$$

d. $\underline{25x^2} - \underline{5x} - 15xy + 3y$ group!

$$= 5x(5x-1) - 3y(5x-1)$$

$$= (5x-1)(5x-3y)$$

e. $8x^4 - 4x^3 + 20x^2$

f. $54x^5 + 135x^2$

g. $3(g-h)^2 - 5(g-h)$

h. $3mp - 6m - 4p + 8$

i. $27x^3y^3 + 18x^2y^2 + 9xy$

j. $33x^3y^5 + 24x^2y^2 + 3x^2y^4$

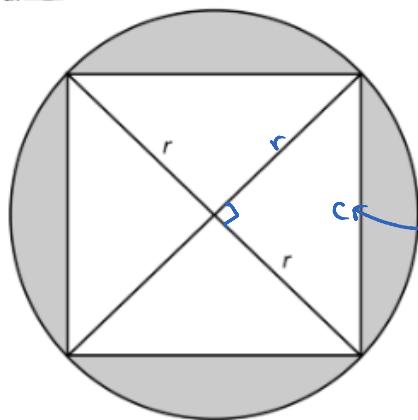
k. $2x(x+7) + 3(x+7)$

l. $2y(x-3) + 4z(3-x)$

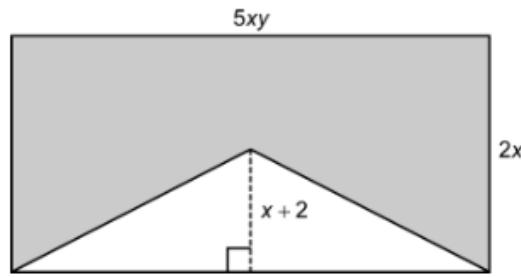
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3. Write an expression in factored form for the area of each shaded region.

a. 

$$\begin{aligned} r^2 + r^2 &= c^2 \\ 2r^2 &= c^2 \\ \sqrt{2r^2} &= c \end{aligned}$$

b. 

$$\begin{aligned} A_O - A_{\square} &= \pi r^2 - lw \\ &= \pi r^2 - \sqrt{2r}\sqrt{2r} \\ &= \pi r^2 - \sqrt{4r^2} \\ &= \pi r^2 - 2r \\ &= r(\pi r - 2) \end{aligned}$$

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Simple Trinomial Factoring

1. What type of trinomial is considered simple and what type is considered complex? Give an example of a trinomial that looks like it maybe complex but isn't.

Simple trinomial - has no "a" on x^2 term ex. $x^2 - 4x + 4$ or $a=1$
 Complex trinomial - has $a \neq 1$ ex. $2x^2 - 3x + 1$

2. Summarize how to factor simple trinomials

Not complex ex. $5x^2 + 10x + 5$ since G.C.F = 5
 $5(x^2 + 2x + 1)$

Simple trinomials are factored by Sum + Product method - find two #'s that add to middle and multiply to last

3. Factor the following

a. $x^2 + 4x - 12$ Sum Product
 $\begin{array}{r} x \\ \times \\ x \end{array}$ $\begin{array}{r} 1 & 1 & -3 & 3 & (-2) & 2 \\ \hline 12 & -12 & 4 & -4 & 6 & -6 \end{array}$ $= (x+2)(x+6)$

b. $x^2 + 8x + 12$
 $\begin{array}{r} x \\ \times \\ x \end{array}$ $\begin{array}{r} 1 & 1 & -2 & 2 \\ \hline 12 & -12 & 2 & -2 \end{array}$ $= (x+2)(x+6)$

4. Summarize how to figure out what signs (+ or -) to use in the factoring process.

5. Find two integers with the given product and sum.
 sum = 14 and product = 48

Positive Product $(+)\textcolor{red}{(+)}$

sum = 2 and product = -15

Negative Product $(+)\textcolor{red}{(-)}$

sum = -1 and product = -30

sum = -3 and product = 2

6. Factor the following, if possible

a. $x^2 - 7x - 18$

b. $x^2 - 5x + 6$

c. $c^2 + 13c - 30$

d. $x^2 + x + 1$

e. $d^2 - 12d + 35$

f. $x^2 + 15x - 16$

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7. Sometimes you must do both: common factor and trinomial factor. Show how you can do this in any order for the following example $3x^2 - 12x - 36$

8. Factor fully.

a. $-2x^2 + 2x + 4$

b. $6x^2 - 42x + 72$

c. $-3x^2 - 18x - 24$

d. $x^3 + 7x^2 + 12x$



9. Determine some values of k so that the trinomial can be factored.

a. $x^2 + kx - 12$

b. $x^2 - 9x + k$

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Complex Trinomial Factoring

1. Summarize how to factor complex trinomials - when there is an "a" value and you can't pull it out by G.C.F. do Criss-Cross method
- ① list factors of 1st and last term
 - ② criss-cross multiply to find the combination that adds to middle
 - ③ Reward $(\text{top} + \text{top})(\text{bottom} + \text{bottom})$

2. Factor the following.



a. $8x^2 + 2x - 3$

$$\begin{array}{r} 8x \\ 1x \\ \hline 2x \end{array} \quad \begin{array}{r} 4 \\ 1 \\ \hline 3 \\ 1 \\ 3 \end{array}$$
 one negative somewhere

$$= (4x+3)(2x-1) \quad \text{can check by expanding!}$$

b. $3x^2 + 14x + 8$

$$\begin{array}{r} 3x \\ 1x \\ \hline 8 \end{array} \quad \begin{array}{r} 1 \\ 8 \\ 1 \\ 4 \\ 2 \end{array}$$
 or two negatives

$$= (3x+2)(x+4)$$



c. $2x^2 + x - 10$

d. $4x^2 - 11x - 15$

e. $2x^2 + 7x + 3$

f. $6x^2 + 10x - 4$

g. $56x^2 - 9x - 2$

h. $12c^2 - 26c - 16$

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i. $6x^2 - 5xy - 4y^2$

j. $12r^2 + 7rs - 10s^2$

k. $10x^4 - 3x^2 - 18$

l. $20x^6 - 59x^3y^2 + 42y^4$

3. Summarize what the variable pattern should be for the factoring to work.

Trinomials can only factor if variable pattern is like :

first	middle	last
x^2	x	
x^2	xy	y^2
x^4	x^2y^3	y^6

even powers on 1st and last
and $\frac{1}{2}$ of these powers in
the middle.

4. Find some values of k so that the trinomial can be factored.

a. $6x^2 + kx + 10$

b. $4x^2 - 12x + k$

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Factoring Special Cases

-  1. What are the conditions for something to be classified as a difference of squares? How do you factor the difference of squares? Why does it work that way?

Difference of Squares only if
 - 2 terms
 - one negative
 - perfect squares
 $1, 4, 9, 16, \dots x^2, x^4, x^6 \dots$

2. Factor the following

 a. $4x^2 - 25$

$$\begin{aligned} &= (+)(-) \\ &= (2x+5)(2x-5) \end{aligned}$$

c. $25x^2 - 16y^2$  can't
sq. root

 e. $100r^2 + 81s^2$

g. $64a^2 - 49b^4$

i. $y^2 - 8$

 k. $50x^2 - 72$

To factor set up answer: $(+)(-)$
and square root each term

Why it works? $x^2 - 36$
when you $= (x+b)(x-b)$
expand middle $= x^2 - bx + bx - 36$
terms cancel out $= x^2 - 36$

b. $100 - (x-3)^2$
 $= [10 + (x-3)][10 - (x-3)]$
 $= [10+x-3][10-x+3]$
 $= (7+x)(13-x)$

d. $x^2 - 6$

not perfect square
(possible but don't worry about it in 3M course)

f. $121x^2 - 9y^2$

h. $(x+2)^2 - 36$

j. $81r^2 - 4t^3$

l. $16x^4 - 100y^4$

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3. What are the conditions for something to be classified as a perfect square? How do you factor a perfect square?

Perfect square only if

1st and last terms are perfect squares
and middle is twice the sq. root of each

$$\text{ex. } a^2 + 2ab + b^2 = (a+b)^2$$

To factor — use cross
since it works
for any trinomial
— just don't list
all combinations
try $\sqrt{1^2}$

4. Factor the following

g. a. $4x^2 + 32x + 64 = (2x + 8)^2$

$\downarrow \quad \downarrow$
 ~~$2x$~~ ~~8~~

b. $4x^2 - 12xy + 9y^2 = (2x - 3y)^2$

~~$2x$~~ ~~$-3y$~~
 ~~$2x$~~ ~~$-3y$~~

c. $100 - 20x + x^2$

d. $49x^2 + 70xy + 16y^2$

e. $25x^2 + 20x + 4$

f. $3x^2 - 6x + 3$

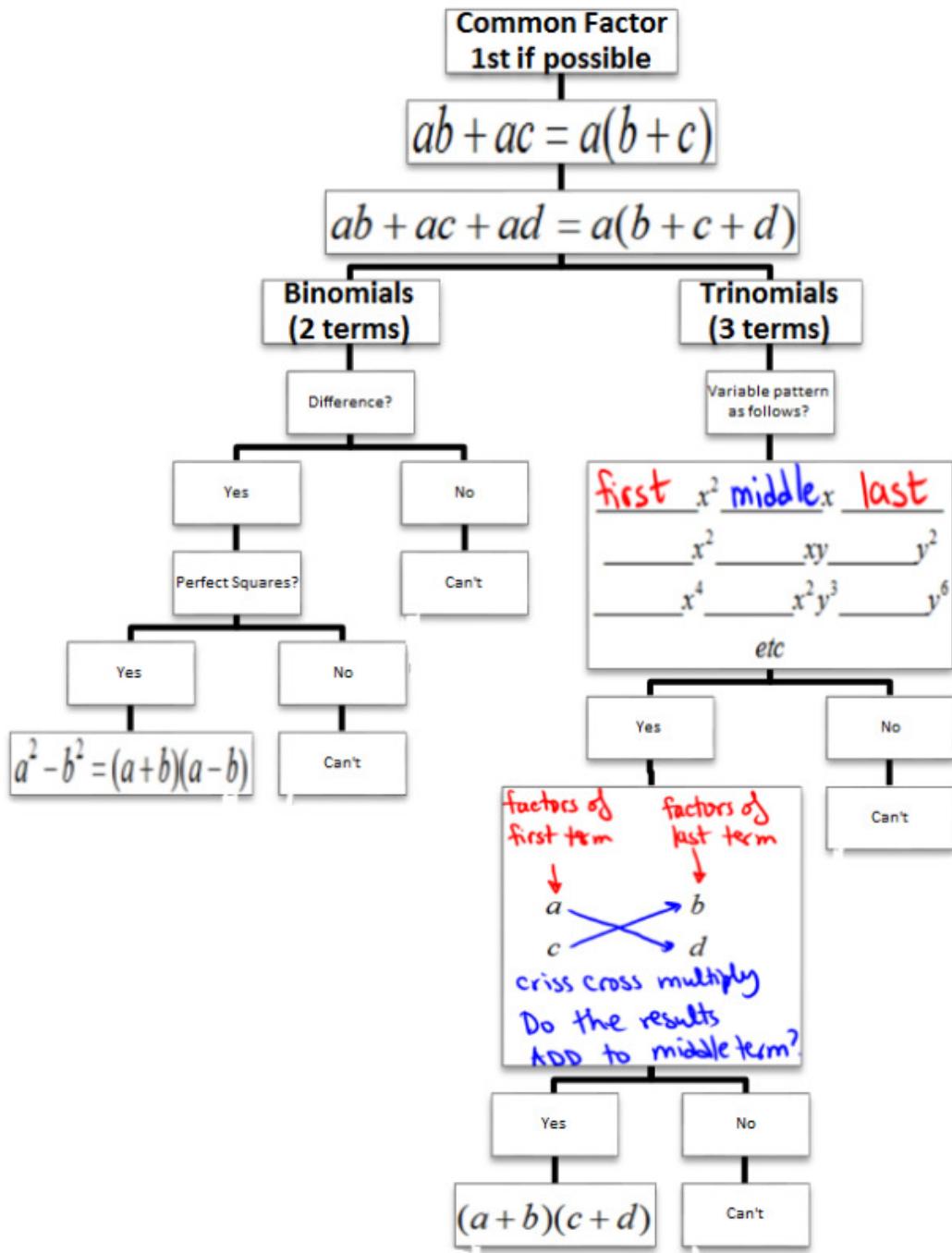
g. $25x^2y^2 - 150xyab + 225a^2b^2$

h. $4(3x-1)^2 + 28(3x-1) + 49$

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Mix of Factoring Methods



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Factor the following as much as possible.

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Ex 1. Factor the following as much as possible.

1. $18x^2y^6 + 39xy^3 - 7$

$$= (3xy^3 + 7)(6xy^3 - 1)$$

$$2. \quad 16x^2 - 49$$

$$= (+ x -)$$

$$= (4x^2 + 7)(4x^2 - 7)$$

$$\begin{aligned}3. \quad & 2y^2 - 12y + 18 \\& = 2(y^2 - 6y + 9) \\& \quad \begin{matrix} \downarrow & \end{matrix} \quad \begin{matrix} \downarrow & \\ -3 & \end{matrix} \\& = 2(y-3)^2\end{aligned}$$

$$4. \quad 8x^3 - 4x^2 \\ = 4x^2(2x - 1)$$

5. $4x^2 - 28x + 40$

$$6. \quad 7x^2 - 54xy^2 - 16y^4$$

$$7. \quad 8x^2 - 12x^4 + 24$$

$$8. \quad 30x^2y - 20x^2y^2 + 10x^3y^2$$

9. $9x^2 + 29x - 28$

$$10. \quad 2x^2 + 12x + 16$$

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11. $7x^2 - 19x - 6$

12. $4x^2 - 44x + 121$

13. $16x^2 - 81$

14. $8mn^2 - 12mn - 16m^2n$

15. $28a^2 - 7a^3$

16. $6 - 27x - 15x^2$



17. $12b^2a^4 - 75a^2$

18. $300 - 48x^4$

19. $(2c - 5)^2 - 121$

20. $(x + a)^2 + 6(x + a) + 8$