

## NOTESallANS

Look below for ALL  
answers to notes - if you find mistakes, let me know

Date: \_\_\_\_\_

Name: \_\_\_\_\_

## The Algebra of Quadratics Unit



### Big idea

This unit will cover expanding and factoring methods you will be required to know for all the quadratic units that will follow. You should have seen most of these methods in grade 10. (If you took applied you have not seen the complex trinomial factoring method, since only grade 10 academic course covered that.) You must understand that factoring trinomials can be taught in many different methods (decomposition/Australian/criss cross methods). The criss cross method is the one you'll concentrate on in this class since it is the most efficient of all the methods. Without practicing you may not do very well in this unit and that would carry on into the next two units. Please take the time to practice so that the next two unit marks will not suffer for it.

I know all the prior concepts related to this unit.  
(If not STOP & complete more review)

Place a  if you are confident in that section.

Place a  if you are just ok in that section.  
Leave it blank if you are lost in that section.

If there are gaps in any row, please see the teacher for extra help in that topic.



### Learning Goal

Finding equations of and graphing lines, finding equations of and graphing quadratics, simplifying expressions, solving equations, expanding, factoring, problem solving with lines and quadratics

Expanding Quadratic Expressions Section 2.1 #3,6,10,11,13	<i>p. 85</i>
Common Factoring Section 2.2 #3,6,7,8,9	<i>p. 93</i>
Simple Trinomial Factoring Section 2.3 #3,6,8,9,11,13	<i>p. 99</i>
Complex Trinomial Factoring Section 2.4 #4,5,7,10,12	<i>p. 110</i>
Factoring Special Cases Section 2.5 #2,3,4,6,11	<i>p. 115</i>
Mix of Factoring Methods three Handouts	
two EXTRA Assignments KU + APP/tips	

Success Criteria Assessment as Learning for Learning and of Learning									
I can understand the lesson (if not, ask clarifying questions. Be specific – "what part is unclear?")	KU	KU	KU	APP	COMM	TIPS	HW	TEST	
I can do a question with an example to follow. (if not, see the teacher for extra help.)									
I can do questions independently (if not, redo a solved example without looking at solutions.)									
I can explain/communicate this concept in my own words – JOURNAL (if not, practice explaining steps done in a solved example)									
I can apply this concept in other/new contexts/situations (This can be only attained with practice)									
I am very confident and am able to complete questions quickly (if not, time yourself to see progress)									
I completed the practice in EACH section									
I completed the practice test and the review section for this unit									

P122 - Chapter Self-Test  
P120-121 Chapter Review Questions



Tentative TEST date \_\_\_\_\_

**Reflect** – TEST mark for this unit \_\_\_\_\_, Overall mark now \_\_\_\_\_.  
Looking back on this unit, what should you plan to improve upon before the exam?

Corrections for the textbook answers:

## Expand & Look For Patterns - these will help you factor later on.

Date: \_\_\_\_\_

Name: \_\_\_\_\_

	A	B	C	Examine the solutions carefully. State any patterns or similarities.	Generalization
1	$(x+4)(x-4)$ $x^2 - 4x + 4x - 16$ $x^2 - 16$	$(2x+3)(2x-3)$ $4x^2 - 6x + 6x - 9$ $4x^2 - 9$	$(x+y)(x-y)$ $x^2 - xy + xy - y^2$ $x^2 - y^2$	- Difference of squares - pattern - middle two terms cancel	$(A+B)(A-B)$ $A^2 - B^2$
2	$(4x+1)(4x+1)$ $16x^2 + 4x + 4x + 1$ $16x^2 + 8x + 1$	$(x+3)(x+3)$ $1^2 + 3x + 3x + 9$ $1^2 + 6x + 9$	$(x+y)(x+y)$ $x^2 + xy + xy + y^2$ $x^2 + 2xy + y^2$	Perfect square monomial do F and L of FOIL and double the outer & inner term	$(A+B)^2$ $= (A+B)(A+B)$ $A^2 + 2AB + B^2$
3	$(x-2)(x-2)$ $x^2 - 2x - 2x + 4$ $x^2 - 4x + 4$	$(3x-3)(3x-3)$ $9x^2 - 9x - 9x + 9$ $9x^2 - 18x + 9$	$(x-y)(x-y)$ $x^2 - xy - xy + y^2$ $x^2 - 2xy + y^2$	Same	$(A-B)^2$ $= (A-B)(A-B)$ $A^2 - 2AB + B^2$

Date: \_\_\_\_\_

Name: \_\_\_\_\_

## Expanding Quadratic Expressions

1. Expand and simplify.

a.  $3(x+4)(x-2)$  *if 3 multiplied start at the end*

$$= 3(x^2 - 2x + 4x - 8)$$

$$= 3x^2 - 6x + 12x - 24$$

$$= 3x^2 + 6x - 24$$

b.  $-2(3a+5)(a-2)$

$$= -2(3a^2 - 6a + 5a - 10)$$

$$= -6a^2 + 12a - 10a + 20$$

$$= -6a^2 + 2a + 20$$

c.  $2b(b+1) - (b-1)^2$

$$= 2b^2 + 2b - (b-1)(b-1)$$

$$= 2b^2 + 2b - b^2 + 2b - 1$$

$$= b^2 + 4b - 1$$

d.  $3(2x-1)^2 - 2(x-1)(x+1)$

$$= 3(4x^2 - 4x + 1) - 2(x^2 + x - 1)$$

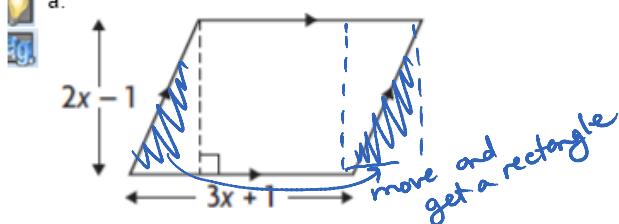
$$= 3(4x^2 - 4x + 1) - 2x^2 + 2$$

$$= 12x^2 - 12x - 6x + 3 - 2x^2 + 2$$

$$= 10x^2 - 12x + 5$$

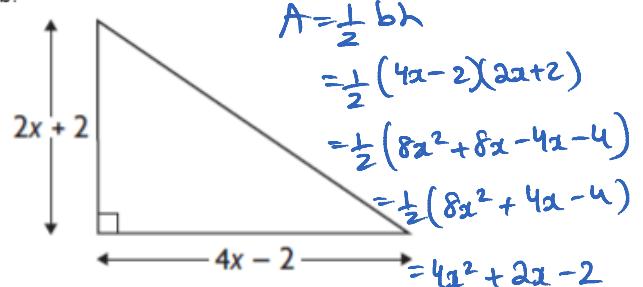
2. Write an expression for the area of each shape. Expand and simplify

- a.

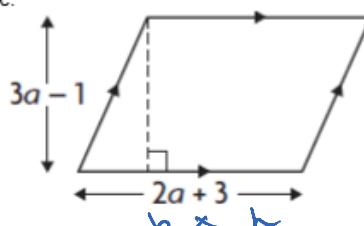


$$\begin{aligned} A &= b \times h \\ &= (3x+1)(2x-1) \\ &= 6x^2 - 3x + 2x - 1 \\ &= 6x^2 - x - 1 \end{aligned}$$

- b.

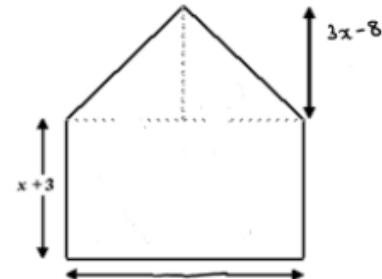


- c.



$$\begin{aligned} A &= (2a+3)(3a-1) \\ &= 6a^2 - 2a + 9a - 3 \\ &= 6a^2 + 7a - 3 \end{aligned}$$

- d.



$$\begin{aligned} A &= \frac{1}{2} b h + l w \\ &= \frac{1}{2} (2x+1)(3x-8) + (x+3)(2x+1) \\ &= \frac{1}{2} (6x^2 - 16x + 3x - 8) + 2x^2 + x + 6x + 3 \\ &= 3x^2 - \frac{13}{2}x - 4 + 2x^2 + 7x + 3 \\ &= 5x^2 + \frac{1}{2}x - 1 \end{aligned}$$

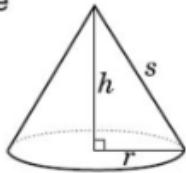
Date: \_\_\_\_\_

Name: \_\_\_\_\_



3. Simplify the expression for the volume of the cone if  $V = \frac{\pi r^2 h}{3}$ ,  $r = 2 + x$ , and  $h = 2x - 3$

Cone



$$\begin{aligned}
 &= \frac{\pi}{3} (2+x)^2 (2x-3) \\
 &= \frac{\pi}{3} (4+4x+x^2)(2x-3) \\
 &= \frac{\pi}{3} [8x^2 - 12 + 8x^3 - 12x^2 + 2x^4 - 3x^3] \\
 &= \frac{\pi}{3} [2x^4 + 5x^3 - 4x^2 - 12]
 \end{aligned}$$



4. You need to understand when to expand and when distribute the exponent. Simplify the following, if possible. Summarize the rules.

a.  $(5x^2y^3)^2$  CAN do :

$$= (5x^2y^3)(5x^2y^3)$$

$$= 25x^4y^6$$

$$5^2 x^4 y^6$$

b.  $(5x^2+y^3)^2$  not allowed because of plus sign

$$= (5x^2+y^3)(5x^2+y^3) \text{ FOIL}$$

$$= 25x^4 + 5x^2y^3 + 5x^2y^3 + y^6$$

$$= 25x^4 + 10x^2y^3 + y^6$$

 $\therefore \text{CAN'T distribute exponent for POLYNOMIALS!}$ 

$$c. \sqrt{25x^2y^4}$$

$$= \sqrt{25} \sqrt{x^2} \sqrt{y^4}$$

$$= 5xy^2 \text{ can distribute square root for MONOMIALS}$$

$$d. \sqrt{25x^2-y^4}$$

can't do anything

$$\text{ex. } \sqrt{25-16} = \sqrt{9} = 3$$

$$\text{not the same as } \frac{\sqrt{25}-\sqrt{16}}{5-4}$$

e.  $2x(3x)(4x^2)$  don't distribute

multiply all at once

$$= 24x^4$$

$$f. (2-x)(3x+4x^2)$$

distribute by FOIL

$$= 6x + 8x^2 - 3x^2 - 4x^3$$

$$= -4x^3 + 5x^2 + 6x$$

Date: \_\_\_\_\_

Name: \_\_\_\_\_

## Common Factoring

1. Summarize the steps of common factoring.

- Find G.C.F  
 - divide it out of all terms  
 - record the G.C.F outside of the bracket of the final answer

2. Factor the following

a.  $9x^3 - 15x^2 + 3x$  G.C.F =  $3x$

$$= 3x(3x^2 - 5x + 1)$$

b.  $25x^2 - 100x^3$  G.C.F =  $25x^2$

$$= 25x^2(1 - 4x)$$

c.  $2\pi h(r+h)^2 + 4\pi(r+h)h^2$  G.C.F =  $2\pi h(r+h)$

$$= 2\pi h(r+h)[(r+h) + 2h]$$

$$= 2\pi h(r+h)(r+3h)$$

d.  $\underline{25x^2} - \underline{5x} - 15xy + 3y$  group!

$$= 5x(5x-1) - 3y(5x-1)$$

$$= (5x-1)(5x-3y)$$

$\begin{array}{r} 135 \\ 5 \overline{)27} \\ \underline{-25} \\ 2 \end{array}$   
 $\begin{array}{r} 54 \\ 2 \overline{)27} \\ \underline{-24} \\ 3 \end{array}$

e.  $8x^4 - 4x^3 + 20x^2$

$$= 4x^2(2x^2 - x + 5)$$

f.  $54x^5 + 135x^2$

$$= 27x^2(2x^3 + 5)$$

$\begin{array}{r} 54 \\ 2 \overline{)27} \\ \underline{-24} \\ 3 \end{array}$

g.  $3(g-h)^2 - 5(g-h)$

$$= (g-h)[3(g-h) - 5]$$

$$= (g-h)[3g - 3h - 5]$$

h.  $\underline{3mp} - \underline{6m} - 4p + 8$

$$= 3m(p-2) - 4p(p-2)$$

$$= (p-2)(3m-4p)$$

i.  $27x^3y^3 + 18x^2y^2 + 9xy$

$$= 9xy(3x^2y^2 + 2xy + 1)$$

j.  $33x^3y^5 + 24x^2y^2 + 3x^2y^4$

$$= 3x^2y^2[11xy^3 + 8 + y^2]$$

k.  $2x(x+7) + 3(x+7)$

$$= (x+7)(2x+3)$$

l.  $2y(\underline{x-3}) + 4z(\underline{3-x})$

not the same

$$= 2y(x-3) + 4z(-x+3)$$

$$= 2y(x-3) - 4z(x-3)$$

common 2

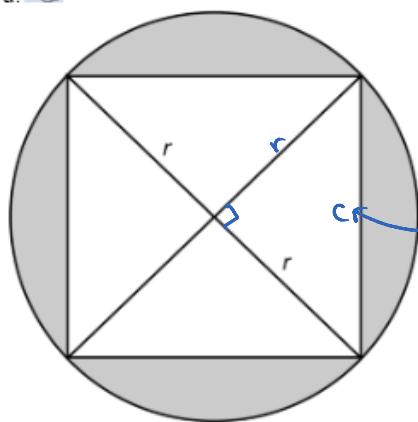
$$= 2(x-3)(y-2z)$$

Date: \_\_\_\_\_

Name: \_\_\_\_\_

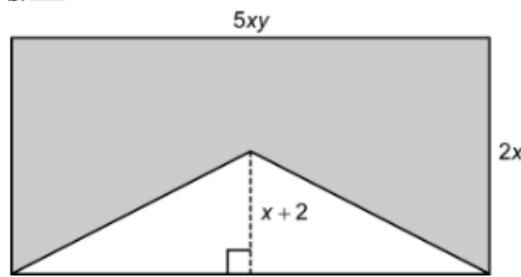
3. Write an expression in factored form for the area of each shaded region.

a.



$$\begin{aligned} r^2 + r^2 &= c^2 \\ 2r^2 &= c^2 \\ \sqrt{2r^2} &= c \end{aligned}$$

b.



$$\begin{aligned} A_O - A_{\square} \\ &= \pi r^2 - lw \\ &= \pi r^2 - \sqrt{2r} \sqrt{2r} \\ &= \pi r^2 - \sqrt{4r^2} \\ &= \pi r^2 - 2r \\ &= r(\pi r - 2) \end{aligned}$$

$$\begin{aligned} A_{\square} - A_{\triangle} \\ &= lw - \frac{1}{2}bh \\ &= (5xy)(2x) - \frac{1}{2}(5xy)(x+2) \\ &= (10x^2y) - \frac{5}{2}xy(x+2) \\ &= 5xy[2x - \frac{1}{2}(x+2)] \\ &= 5xy[2x - \frac{x}{2} - 1] \\ &= 5xy[\frac{3}{2}x - 1] \end{aligned}$$

Date: \_\_\_\_\_

Name: \_\_\_\_\_

## Simple Trinomial Factoring

1. What type of trinomial is considered simple and what type is considered complex? Give an example of a trinomial that looks like it maybe complex but isn't.

Simple trinomial - has no "a" on  $x^2$  term ex.  $x^2 - 4x + 4$  or  $a=1$   
 Complex trinomial - has  $a \neq 1$  ex.  $2x^2 - 3x + 1$

2. Summarize how to factor simple trinomials

Simple trinomials are factored  
 by Sum + Product method

- find two #'s that add to middle  
 and multiply to last

3. Factor the following

a.  $x^2 + 4x - 12$  =  $(x+2)(x+6)$

Sum Product  
 $\begin{array}{r} x \\ x \\ \hline -12 \end{array}$     $\begin{array}{r} 1 \\ -1 \\ \hline -12 \end{array}$     $\begin{array}{r} 3 \\ -3 \\ \hline -12 \end{array}$     $\begin{array}{r} -2 \\ -4 \\ \hline -6 \end{array}$

b.  $x^2 + 8x + 12$  =  $(x+2)(x+6)$

Sum Product  
 $\begin{array}{r} x \\ x \\ \hline 12 \end{array}$     $\begin{array}{r} 1 \\ -1 \\ \hline 12 \end{array}$     $\begin{array}{r} 2 \\ 2 \\ \hline 12 \end{array}$     $\begin{array}{r} -2 \\ -6 \\ \hline 12 \end{array}$

4. Summarize how to figure out what signs (+ or -) to use in the factoring process.

5. Find two integers with the given product and sum.

sum = 14 and product = 48  
 $\begin{array}{r} 1 \\ 2 \\ 3 \\ 4 \\ \hline 16 \\ 12 \\ 8 \end{array}$  or two neg.

Positive Product  $(+)\textcolor{red}{(+)}$

sum = 2 and product = -15

$\begin{array}{r} 1 \\ -3 \\ \hline 15 \\ 5 \end{array}$  one neg

sum = -1 and product = -30

$\begin{array}{r} 1 \\ 3 \\ \hline 30 \\ 10 \\ -6 \\ 15 \end{array}$  2 one neg

sum = -3 and product = 2

$\begin{array}{r} -2 \\ -1 \end{array}$  or two neg

6. Factor the following, if possible

a.  $x^2 - 7x - 18$

$\begin{array}{r} x \\ x \\ \hline 18 \\ 6-9 \end{array}$  one neg

$= (x+2)(x-9)$

b.  $x^2 - 5x + 6$

$\begin{array}{r} x \\ x \\ \hline 6 \\ 2-3 \end{array}$  or two neg

$= (x-2)(x-3)$

c.  $c^2 + 13c - 30$

$\begin{array}{r} c \\ c \\ \hline 30 \\ 15 \\ 10 \\ 6 \end{array}$  or one neg

$= (c-2)(c+15)$

d.  $x^2 + x + 1$

$\begin{array}{r} x \\ x \\ \hline 1 \\ 1 \end{array}$  or two negatives!  
 can't get 1 ever!

e.  $d^2 - 12d + 35$

$\begin{array}{r} d \\ d \\ \hline 35 \\ 25-7 \end{array}$  or two neg

$= (d-5)(d-7)$

f.  $x^2 + 15x - 16$

$\begin{array}{r} x \\ x \\ \hline 16 \\ 1 \\ 8 \\ 4 \end{array}$  or one neg

$= (x-1)(x+16)$

Date: \_\_\_\_\_

Name: \_\_\_\_\_

-  7. Sometimes you must do both: common factor and trinomial factor. Show how you can do this in any order for the following example  $3x^2 - 12x - 36$

$$\begin{aligned} &= 3(x^2 - 4x - 12) \\ \text{carry down} &\quad \downarrow \quad \begin{array}{r} x \\ x \\ \hline 12 \end{array} \quad \begin{array}{r} 1 \\ 12 \\ \hline -6 \end{array}^3 \quad \text{one neg} \\ &= 3(x+2)(x-6) \end{aligned}$$

8. Factor fully.

a.  $-2x^2 + 2x + 4$

$$\begin{aligned} &= -2(x^2 - x - 2) \\ &\quad \begin{array}{r} x \\ x \\ \hline -2 \\ 1 \end{array} \quad \text{one neg} \\ &= -2(x-2)(x+1) \end{aligned}$$

b.  $6x^2 - 42x + 72$

$$\begin{aligned} &= 6(x^2 - 7x + 12) \\ &\quad \begin{array}{r} x \\ x \\ \hline 12 \\ 6 \end{array}^2 \quad \begin{array}{r} 3 \\ -4 \end{array} \quad \text{or two negatives} \\ &= 6(x-3)(x-4) \end{aligned}$$

c.  $-3x^2 - 18x - 24$

$$\begin{aligned} &= -3(x^2 + 6x + 8) \\ &\quad \begin{array}{r} x \\ x \\ \hline 8 \\ 4 \end{array}^1 \quad \text{or two neg} \\ &= -3(x+2)(x+4) \end{aligned}$$

d.  $x^3 + 7x^2 + 12x$

$$\begin{aligned} &= x(x^2 + 7x + 12) \\ &\quad \begin{array}{r} x \\ x \\ \hline 12 \\ 6 \end{array}^2 \quad \begin{array}{r} 3 \\ 4 \end{array} \\ &= x(x+3)(x+4) \end{aligned}$$



9. Determine some values of k so that the trinomial can be factored.

a.  $x^2 + kx - 12$

$$\begin{array}{r} 1 \\ 12 \\ -2 \\ 6 \\ -4 \end{array} \quad \begin{array}{r} 3 \\ -4 \\ \hline -1 \\ 1 \end{array} \quad \text{one neg}$$

$\downarrow$        $\downarrow$

$k=4$        $k=-1$

etc

b.  $x^2 - 9x + k$

$$\text{add to } -9 \quad \begin{array}{r} -4 \\ -5 \\ \hline -9 \end{array} \quad \text{or } \begin{array}{r} -10 \\ -1 \\ \hline -9 \end{array}$$

$$\begin{array}{ll} \text{multiply} & \text{multiply} \\ k=20 & k=-10 \end{array}$$

Date: \_\_\_\_\_

Name: \_\_\_\_\_

## Complex Trinomial Factoring

1. Summarize how to factor complex trinomials - when there is an "a" value and you can't pull it out by G.C.F. do Criss-Cross method
- ① list factors of 1st and last term
  - ② criss-cross multiply to find the combination that adds to middle
  - ③ Reward  $(\text{top} + \text{top})(\text{bottom} + \text{bottom})$

2. Factor the following.

a.  $8x^2 + 2x - 3$

$$\begin{array}{c} 8x \\ 1x \\ \hline 2x \end{array} \quad \begin{array}{c} 4x \\ 1 \\ \hline 3 \\ 1 \\ 3 \end{array}$$

one negative somewhere

$$= (4x+3)(2x-1) \quad \text{can check by expanding!}$$

b.  $3x^2 + 14x + 8$

$$\begin{array}{c} 3x \\ 1x \\ \hline 2x \end{array} \quad \begin{array}{c} 1 \\ 8 \\ 1 \\ 4 \\ 2 \end{array}$$

or two negatives

$$= (3x+2)(x+4)$$

c.  $2x^2 + x - 10$

$$\begin{array}{c} 2x \\ 2 \\ \hline x \end{array} \quad \begin{array}{c} 1 \\ 10 \\ 1 \\ 5 \\ -2 \end{array}$$

one neg

$$= (2x+5)(x-2)$$

d.  $4x^2 - 11x - 15$

$$\begin{array}{c} 4x \\ 2x \\ 2x \\ \hline 2x \end{array} \quad \begin{array}{c} 1 \\ 15 \\ 1 \\ 5 \\ 3 \\ 3 \end{array}$$

one neg

$$= (4x-15)(x+1)$$

e.  $2x^2 + 7x + 3$

$$\begin{array}{c} 2x \\ 2 \\ \hline x \end{array} \quad \begin{array}{c} 3 \\ 1 \\ 3 \end{array}$$

or two neg.

$$= (2x+1)(x+3)$$

f.  $6x^2 + 10x - 4$

$$\begin{array}{c} 6x \\ 2x \\ 2x \\ \hline 3x \end{array} \quad \begin{array}{c} 4 \\ -1 \\ 4 \\ 2 \\ 2 \end{array}$$

one neg

$$= (2x+4)(3x-1)$$

g.  $56x^2 - 9x - 2$

$$\begin{array}{c} 1 \\ 56 \\ 2 \\ 28 \\ 14 \\ 8 \end{array} \quad \begin{array}{c} 7 \\ 2 \\ 1 \\ 1 \\ 2 \end{array}$$

one neg

don't forget  $x^2$ 's

$$= (7x-2)(8x+1)$$

h.  $12c^2 - 26c - 16$  Common 1st!!

$$= 2(6c^2 - 13c - 8)$$

$$\begin{array}{c} 6c \\ c \\ 3c \end{array} \quad \begin{array}{c} 1 \\ -8 \\ 1 \\ 4 \\ 2 \end{array}$$

one neg

$$= 2(2c+1)(3c-8)$$

Date: \_\_\_\_\_

i.  $6x^2 - 5xy - 4y^2$

$$\begin{array}{r} 6x^2 \\ \cancel{2} \quad \cancel{3x} \\ \cancel{2} \quad \cancel{4y} \end{array}$$

$$\begin{array}{r} 4y \\ \cancel{1y} \quad \cancel{4y} \\ \cancel{1y} \quad \cancel{2y} \end{array}$$

one neg

$$= (2x+y)(3x-4y)$$

Name: \_\_\_\_\_

j.  $12r^2 + 7rs - 10s^2$

$$\begin{array}{r} 12r^2 \\ \cancel{2r} \quad \cancel{6r} \\ \cancel{2r} \quad \cancel{4r} \end{array}$$

$$\begin{array}{r} 10s \\ \cancel{10s} \quad \cancel{5s} \\ \cancel{5s} \quad \cancel{2s} \end{array}$$

one neg

$$= (3r-2s)(4r+5s)$$

k.  $10x^4 - 3x^2 - 18$

$$\begin{array}{r} 10x^2 \\ \cancel{x^2} \quad \cancel{5x^2} \\ \cancel{10} \quad \cancel{5} \end{array}$$

$$\begin{array}{r} 18 \quad 1 \quad 2 \quad 9 \quad -3 \\ \cancel{1} \quad \cancel{18} \quad \cancel{9} \quad \cancel{2} \quad \cancel{6} \\ \cancel{1} \quad \cancel{3} \end{array}$$

one neg

$$(2x^2 - 3)(5x^2 + 6)$$

l.  $20x^6 - 59x^3y^2 + 42y^4$

$$\begin{array}{r} x^3 \\ \cancel{2x^2} \quad \cancel{4} \\ \cancel{1} \quad \cancel{10} \end{array}$$

$$\begin{array}{r} y^2 \\ \cancel{42} \quad \cancel{21} \quad \cancel{14} \quad \cancel{3} \quad 6 \\ \cancel{42} \quad \cancel{21} \quad \cancel{14} \quad \cancel{3} \quad \cancel{7} \end{array}$$

or  
two neg

$$(4x^3 - 7y^2)(5x^3 - 6y^2)$$

3. Summarize what the variable pattern should be for the factoring to work.

Trinomials can only factor if variable pattern is like :

first	middle	last
$x^2$	$x$	
$x^2$	$xy$	$y^2$
$x^4$	$x^2y^3$	$y^6$

even powers on 1st and last  
and  $\frac{1}{2}$  of these powers in  
the middle.

4. Find some values of k so that the trinomial can be factored.

a.  $6x^2 + kx + 10$

$$\begin{array}{r} 6x^2 \\ \cancel{2} \quad \cancel{3x} \\ \cancel{2} \quad \cancel{5} \end{array}$$

$$\begin{array}{r} 10 \quad 1 \quad 2 \quad 5 \\ \cancel{1} \quad \cancel{10} \quad \cancel{5} \quad \cancel{2} \end{array}$$

$\therefore k = 32$

$\therefore k = 16$

b.  $4x^2 - 12x + k$

$$\begin{array}{r} 4x^2 \\ \cancel{2} \quad \cancel{2x} \\ \cancel{2} \quad \cancel{-3} \end{array}$$

$\therefore k = 9$

$$\begin{array}{r} 4x \\ \cancel{x} \quad \cancel{-1} \\ \cancel{x} \quad \cancel{-8} \end{array}$$

$\therefore k = 8$

Date: \_\_\_\_\_

Name: \_\_\_\_\_

## Factoring Special Cases

1. What are the conditions for something to be classified as a difference of squares? How do you factor the difference of squares? Why does it work that way?



Difference of Squares only if

- 2 terms
- one negative
- perfect squares  
1, 4, 9, 16, ...  $x^2, x^4, x^6 \dots$

2. Factor the following



a.  $4x^2 - 25$

$$\begin{aligned} &= ( + )( - ) \\ &= (2x+5)(2x-5) \end{aligned}$$

c.  $25x^2 - 16y^2$  (Can't sq. root)



e.  $100r^2 + 81s^2$

not a difference

g.  $64a^2 - 49b^4$

$$= (8a-7b)(8a+7b)$$

i.  $y^2 - 8$  (Not perfect square.)



k.  $50x^2 - 72$  common factor 1st

$$= 2(25x^2 - 36)$$

$$= 2(5x+6)(5x-6)$$

To factor set up answer:  $( + )( - )$  and square root each term

Why it works?  $x^2 - 36$   
 when you  $= (x+6)(x-6)$   
 expand middle  $= x^2 - 6x + 6x - 36$   
 terms cancel out  $= x^2 - 36$

b.  $100 - (x-3)^2$

$$\begin{aligned} &= [10 + (x-3)][10 - (x-3)] \\ &= [10+x-3][10-x+3] \\ &= (7+x)(13-x) \end{aligned}$$

d.  $x^2 - 6$

not perfect square  
 (possible but don't worry about it in 3M course)

f.  $121x^2 - 9y^2$

$$= (11x+3y)(11x-3y)$$

h.  $(x+2)^2 - 36$

$$\begin{aligned} &= [(x+2) - 6][(x+2)+6] \\ &= (x-4)(x+8) \end{aligned}$$

j.  $81r^2 - 4t^3$

not perfect square

l.  $16x^4 - 100y^4$

$$= 4(4x^4 - 25y^4)$$

$$= 4(2x^2 - 5y^2)(2x^2 + 5y^2)$$

Date:

Name:



3. What are the conditions for something to be classified as a perfect square? How do you factor a perfect square?

Perfect square only if

1<sup>st</sup> and last terms are perfect squares  
and middle is twice the sq. root of each  
ex.  $a^2 + 2ab + b^2 = (a+b)^2$

To factor - use cross  
since it works  
for any trinomial  
- just don't list  
all combinations  
try  $\sqrt{1^2}$

4. Factor the following

g. a.  $4x^2 + 32x + 64 = (2x+8)^2$

$$\begin{array}{cc} 2x & \cancel{8} \\ \cancel{2x} & 8 \end{array}$$

b.  $4x^2 - 12xy + 9y^2 = (2x-3y)^2$

$$\begin{array}{cc} 2x & \cancel{-3y} \\ \cancel{2x} & -3y \end{array}$$

c.  $100 - 20x + x^2$

$$\begin{array}{cc} 10 & -x \\ 10 & -x \end{array}$$

two neg

$$= (10-x)^2$$

e.  $25x^2 + 20x + 4$

$$\begin{array}{cc} 5x & 2 \\ 5x & 2 \end{array}$$

$$= (5x+2)^2$$

g.  $25x^2y^2 - 150xyab + 225a^2b^2$

$$= 25(x^2y^2 - 6xyab + 9a^2b^2)$$

$$\begin{array}{cc} xy & 3ab \\ xy & 3ab \end{array}$$

$$= 25(xy-3ab)^2$$

d.  $49x^2 + 70xy + 16y^2$

$$\begin{array}{cc} 7x & \cancel{4y} \\ \cancel{7x} & 4y \end{array}$$

$$\begin{array}{c} 56 \\ \text{not} \\ 70 \end{array}$$

$\therefore$  not perfect square

f.  $3x^2 - 6x + 3$

$$= 3(x^2 - 2x + 1)$$

$$\begin{array}{cc} x & 1 \\ x & 1 \end{array}$$

two neg

$$= 3(x-1)^2$$

h.  $4(3x-1)^2 + 28(3x-1) + 49$

$$\begin{array}{cc} 2(3x-1) & + \\ 2(3x-1) & + \end{array}$$

$$= [2(3x-1) + 7]^2$$

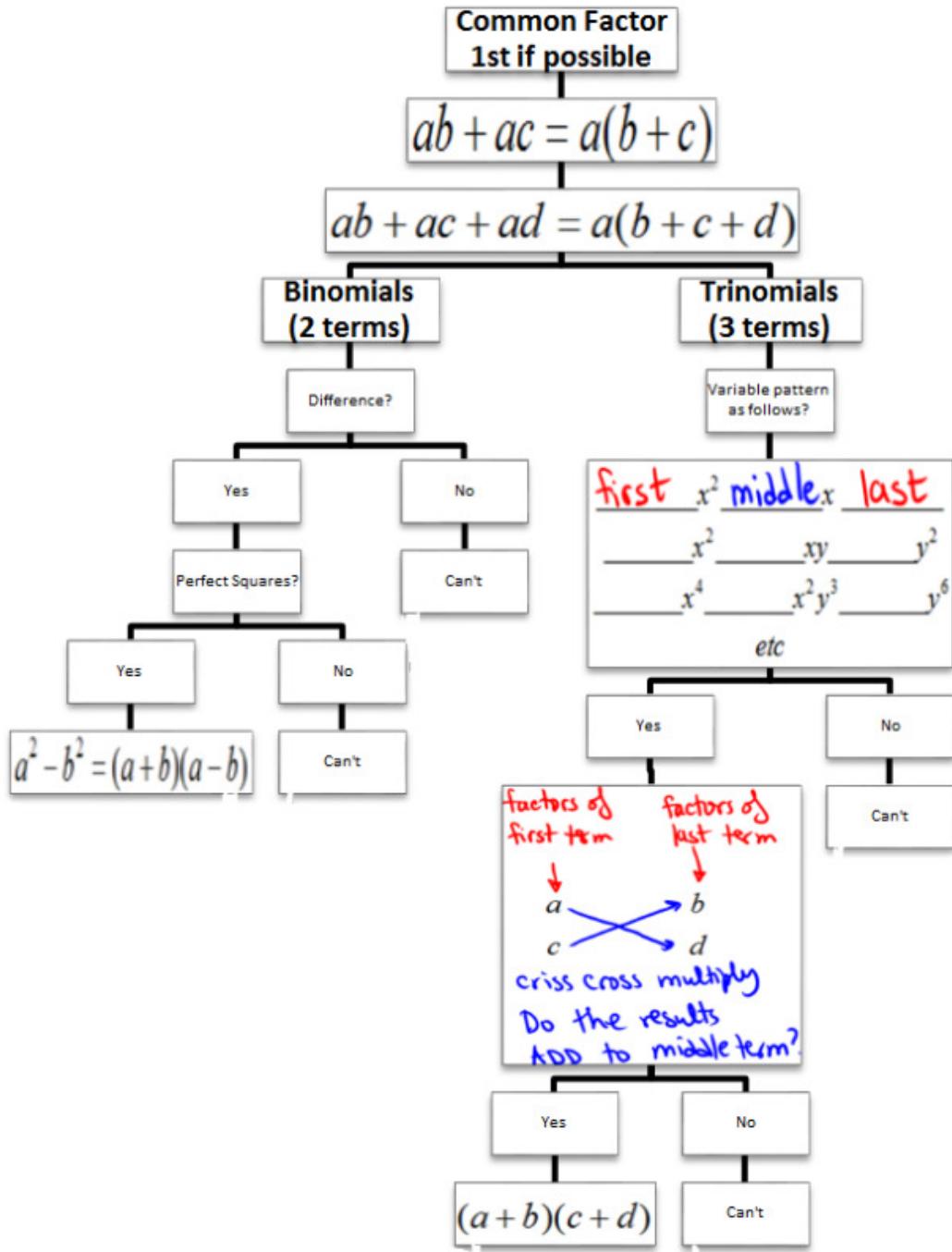
$$= (6x-2+7)^2$$

$$= (6x+5)^2$$

Date: \_\_\_\_\_

Name: \_\_\_\_\_

## Mix of Factoring Methods



Date: \_\_\_\_\_

Factor the following as much as possible.

Name: \_\_\_\_\_



1.  $18x^2y^6 + 39xy^3 - 7$

$$\begin{array}{r} 18xy^3 \quad 2xy^3 \quad (3xy^3) \\ \cancel{1} \cancel{xy} \quad \cancel{9} \cancel{xy}^3 \quad \cancel{6} \cancel{xy}^3 \\ \cancel{1} \cancel{8} \cancel{x}^2 \end{array} \rightarrow \begin{pmatrix} 7 \\ 1 \end{pmatrix} \text{ one neg}$$

$$= (3xy^3 + 7)(6xy^3 - 1)$$

2.  $16x^2 - 49$

$$\begin{pmatrix} 1 & + & x & - \end{pmatrix}$$

$$= (4x^2 + 7)(4x^2 - 7)$$

3.  $2y^2 - 12y + 18$

$$\begin{array}{r} 2(y^2 - 6y + 9) \\ \quad \quad \quad \begin{pmatrix} y \\ y \end{pmatrix} \quad \begin{pmatrix} -3 \\ -3 \end{pmatrix} \end{array}$$

$$= 2(y-3)^2$$

4.  $8x^3 - 4x^2$

$$= 4x^2(2x - 1)$$



5.  $4x^2 - 28x + 40$

$$\begin{array}{r} 4(x^2 - 7x + 10) \\ \quad \quad \quad \begin{pmatrix} x \\ x \end{pmatrix} \quad \begin{pmatrix} 10 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 2 \\ -5 \end{pmatrix} \end{array} \text{ or two neg}$$

$$= 4(x-2)(x-5)$$

6.  $7x^2 - 54xy^2 - 16y^4$

$$\begin{array}{r} 7x^2 \quad 4y^2 \quad 8 \\ \quad \quad \quad 4y^2 \quad 2 \end{array} \begin{pmatrix} 2 \\ -8 \end{pmatrix} \begin{pmatrix} 16 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 16 \end{pmatrix} \text{ one neg}$$

$$= (7x + 2y^2)(x - 8y^2)$$

7.  $8x^2 - 12x^4 + 24$  wrong order

$$\begin{array}{r} -12x^4 + 8x^2 + 24 \\ = -4(3x^4 - 2x^2 - 8) \\ \quad \quad \quad \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 8 \end{pmatrix} \quad \begin{pmatrix} 4 \\ 4-2 \end{pmatrix} \text{ one neg} \end{array}$$

$$= -4(3x^2 + 4)(x^2 - 2)$$

8.  $30x^2y - 20x^2y^2 + 10x^3y^2$

$$= 10x^2y(3 - 2y + xy)$$

can't do more  
since variable  
pattern d.n. work.

9.  $9x^2 + 29x - 28$

$$\begin{array}{r} 9x^2 \quad 4x^2 \quad 14 \quad 28 \\ \quad \quad \quad 7 \quad 14 \quad 2 \quad 1 \quad 28 \end{array} \text{ one neg}$$

$$\begin{array}{r} 3x \\ 2 \\ 3x \\ \hline 3x \end{array}$$

$$= (9x+7)(x+4)$$

10.  $2x^2 + 12x + 16$

$$\begin{array}{r} 2(x^2 + 6x + 8) \\ \quad \quad \quad \begin{pmatrix} x \\ x \end{pmatrix} \quad \begin{pmatrix} 8 \\ 1 \end{pmatrix} \end{array}$$

$$= 2(x+2)(x+4)$$

Date: \_\_\_\_\_

Name: \_\_\_\_\_

11.  $7x^2 - 19x - 6$

$$\begin{array}{r} (7x) \\ \times x \\ \hline 7x \end{array}$$

$$= (7x+2)(x-3)$$

13.  $16x^2 - 81$

$$= (4x+9)(4x-9)$$

15.  $28a^2 - 7a^3$

$$= 7a^2(4-a)$$



17.  $12b^2a^4 - 75a^2$

$$\begin{aligned} &= 3a^2(4b^2a^2 - 25) \\ &= 3a^2(2ba+5)(2ba-5) \end{aligned}$$

19.  $(2c-5)^2 - 121$

$$\begin{aligned} &= [(2c-5) - 11][(2c-5) + 11] \\ &= (2c-16)(2c+6) \\ &= 2(c-8)(2)(c+3) \\ &= 4(c-8)(c+3) \end{aligned}$$

12.  $4x^2 - 44x + 121$

$$\begin{array}{r} (4x) \\ \times x \\ \hline 4x \end{array}$$

$$= (2x-11)^2$$

14.  $8mn^2 - 12mn - 16m^2n$

$$= 4mn(2n-3-4m)$$

16.  $6 - 27x - 15x^2$

$$\begin{array}{r} (2) \\ \times x \\ \hline 6 \end{array}$$

$$= 3(2+3x)(1-5x)$$

18.  $300 - 48x^4$

$$\begin{aligned} &= 12(25 - 4x^4) \\ &= 12(5 + 2x^2)(5 - 2x^2) \end{aligned}$$

20.  $(x+a)^2 + 6(x+a) + 8$

$$\begin{array}{r} (x+a) \\ \times (x+a) \\ \hline (x+a) \end{array}$$

$$\begin{aligned} &= ((x+a)+2)((x+a)+4) \\ &= (x+a+2)(x+a+4) \end{aligned}$$