

Completing the Square

Use fractions if decimals are non-terminating. Don't round!

1. $y = -2x^2 - 4x + 70$
2. $y = -12x^2 + 12x$
3. $y = x^2 - 10x + 21$
4. $y = -x^2 - 22x + 95$
5. $y = -4x^2 + 6x - 27$
6. $y = -0.5x^2 - 8x + 1$

Completing the Square - Word Problems

7. Studies show that employees on an assembly line become more efficient as their level of training goes up. In one company, the number of products, P , produced per day, at a level of training of t hours, follows the following quadratic model: $P = -0.375t^2 + 10.25t - 8$. Determine the level/hours of training that will give maximum productivity.
8. At the Mini Market, as the price of milk drops, sales increase. On an average day, a 4L bag of milk costs \$3.90, and the store sells an average of 120 bags. Studies have shown that for each \$0.10 reduction in price, sales increase by 20 bags per day. Find what price will make the revenue maximum
9. What is the smallest possible total of double a number and its square?
10. For the following word problems identify whether you must complete the square in order to solve the problem or not. Do not need to solve the problems - unless you want to practice more.
 1. **GRAVITATION** The height $h(t)$ in feet of an object t seconds after it is propelled straight up from the ground with an initial velocity of 60 feet per second is modeled by the equation $h(t) = -16t^2 + 60t$. At what times will the object be at a height of 56 feet?
 2. **STOPPING DISTANCE** The formula $d = 0.05s^2 + 1.1s$ estimates the minimum stopping distance d in feet for a car traveling s miles per hour. If a car stops in 200 feet, what is the fastest it could have been traveling when the driver applied the brakes?
 3. **BASEBALL** The height h of a baseball t seconds after being hit is given by $h(t) = -16t^2 + 80t + 3$. What is the maximum height that the baseball reaches, and when does this occur?
 4. **SCULPTURE** A modern sculpture in a park contains a parabolic arc that starts at the ground and reaches a maximum height of 10 feet after a horizontal distance of 4 feet. Write a quadratic function in vertex form that describes the shape of the outside of the arc, where y is the height of a point on the arc and x is its horizontal distance from the left-hand starting point of the arc.
 5. **FENCING Vanessa** has 180 feet of fencing that she intends to use to build a rectangular play area for her dog. She wants the play area to enclose at least 1800 square feet. What are the possible widths of the play area?
 6. **BUSINESS** A bicycle maker sold 300 bicycles last year at a profit of \$300 each. The maker wants to increase the profit margin this year, but predicts that each \$20 increase in profit will reduce the number of bicycles sold by 10. How many \$20 increases in profit can the maker add in and expect to make a total profit of at least \$100,000?

Completing the Square

Use fractions if decimals are non-terminating. Don't round!

1. $y = -2x^2 - 4x + 70$

(1.) $-2(x^2 + 2x + 1 - 1) + 70$
 $-2(x+1)^2 + 72$

2. $y = -12x^2 + 12x$

(2.) $-12(x^2 - x + \frac{1}{4} - \frac{1}{4})$

3. $y = x^2 - 10x + 21$

$-12(x - \frac{1}{2})^2 + 3$

4. $y = -x^2 - 22x + 95$

(3.) $(x^2 - 10x + 25 - 25) + 21$
 $(x-5)^2 - 4$

5. $y = -4x^2 + 6x - 27$

(4.) $-(x^2 + 22x + 121 - 121) + 95$
 $-(x+11)^2 + 216$

(5.) $-4(x^2 - \frac{3}{2}x + \frac{9}{16} - \frac{9}{16}) - 27$
 $-4(x - \frac{3}{4})^2 - \frac{99}{4}$

(6.) $-0.5(x^2 + 16x + 64 - 64) + 1$
 $-0.5(x+8)^2 + 33$

Completing the Square - Word Problems

7. Studies show that employees on an assembly line become more efficient as their level of training goes up. In one company, the number of products, P , produced per day, at a level of training of t hours, follows the following quadratic model: $P = -0.375t^2 + 10.25t - 8$. Determine the level/hours of training that will give maximum productivity.

change to fractions so no rounding error will be present
 $P = -\frac{3}{8}t^2 + \frac{41}{4}t - 8$

$-\frac{3}{8}(t^2 - \frac{82}{3}t + \frac{1681}{9} - \frac{1681}{9}) - 8$
 $-\frac{3}{8}(t - \frac{41}{3})^2 + \frac{1489}{24}$

vertex $(\frac{41}{3}, \frac{1489}{24}) \sim (13.7, 62.0)$

\therefore hours of training is 13.7 hrs for max productivity of 62 products

8. At the Mini Market, as the price of milk drops, sales increase. On an average day, a 4L bag of milk costs \$3.90, and the store sells an average of 120 bags. Studies have shown that for each \$0.10 reduction in price, sales increase by 20 bags per day. Find what price will make the revenue maximum

Rev = (price)(quantity)

$R = (3.90 - 0.10x)(120 + 20x)$

$R = -2x^2 + 66x + 468$

$x = \#$ of times reduce price by \$0.10

$R = -2(x^2 - 33x + \frac{1089}{4} - \frac{1089}{4}) + 468$

$-2(x - \frac{33}{2})^2 + \frac{2025}{2}$

vertex $(\frac{33}{2}, \frac{2025}{2}) = (16.5, 1012.5)$
 $x \quad R$

\therefore reduce price by 16.5 times to get MAX

\therefore price = $3.90 - 0.10x$
 $= 3.90 - 0.10(16.5)$
 $= \$2.25$

9. What is the smallest possible total of double a number and its square?

let $T =$ total
 $n =$ number

$T = 2n + n^2$

$T = (n^2 + 2n + 1 - 1)$

$T = (n+1)^2 - 1$

vertex $(-1, -1)$
 $n \quad T$

\therefore smallest total = -1
for number = -1

10. For the following word problems identify whether you must complete the square in order to solve the problem or not.

Do not need to solve the problems - unless you want to practice more.

$h(t)$ is just height h at time t

1. **GRAVITATION** The height $h(t)$ in feet of an object t seconds after it is propelled straight up from the ground with an initial velocity of 60 feet per second is modeled by the equation $h(t) = -16t^2 + 60t$. At what times will the object be at a height of 56 feet?
 not compl. sq.

← function notation →

sub $56 = h(t) = h$
 move to other side to get zero
 Quad. Formula

2. **STOPPING DISTANCE** The formula $d = 0.05s^2 + 1.1s$ estimates the minimum stopping distance d in feet for a car traveling s miles per hour. If a car stops in 200 feet, what is the fastest it could have been traveling when the driver applied the brakes?
 → largest value on s-axis

→ find 2nd x-int

3. **BASEBALL** The height h of a baseball t seconds after being hit is given by $h(t) = -16t^2 + 80t + 3$. What is the maximum height that the baseball reaches, and when does this occur?
 complete sq.

4. **SCULPTURE** A modern sculpture in a park contains a parabolic arc that starts at the ground and reaches a maximum height of 10 feet after a horizontal distance of 4 feet. Write a quadratic function in vertex form that describes the shape of the outside of the arc, where y is the height of a point on the arc and x is its horizontal distance from the left-hand starting point of the arc.
 not complete sq. just need equation

5. **FENCING** Vanessa has 180 feet of fencing that she intends to use to build a rectangular play area for her dog. She wants the play area to enclose at least 1800 square feet. What are the possible widths of the play area?
 not compl. sq. doesn't ask for max/min

6. **BUSINESS** A bicycle maker sold 300 bicycles last year at a profit of \$300 each. The maker wants to increase the profit margin this year, but predicts that each \$20 increase in profit will reduce the number of bicycles sold by 10. How many \$20 increases in profit can the maker add in and expect to make a total profit of at least \$100,000?
 not complete sq. doesn't ask for max/min