

Trigonometry

Recall:

Trigonometric Identities

The Pythagorean identity: $\sin^2 \theta + \cos^2 \theta = 1$

The quotient identity: $\tan \theta = \frac{\sin \theta}{\cos \theta}$, $\cos \theta \neq 0$

Sine, Cosine, and Tangent of Special Angles

θ	$0^\circ, 0$	$30^\circ, \frac{\pi}{6}$	$45^\circ, \frac{\pi}{4}$	$60^\circ, \frac{\pi}{3}$	$90^\circ, \frac{\pi}{2}$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined

- Predict whether each value will be positive or negative. Sketch each angle on a coordinate grid.
 - $\tan 167^\circ$
 - $\sin 99^\circ$
 - $\cos 132^\circ$
- Given that $0^\circ \leq \theta \leq 180^\circ$, determine the value(s) of θ to 1 decimal place.
 - $\cos \theta = 0.4772$
 - $\tan \theta = -0.2272$
 - $\sin \theta = 0.5476$
 - $\tan \theta = 1.6191$
 - $\sin \theta = 0.3486$
 - $\cos \theta = -0.5577$
- Angle θ is obtuse.
 - $\tan \theta = -0.4452$; calculate $\sin \theta$ to 4 decimal places.
 - $\sin \theta = 0.9707$; calculate $\cos \theta$ to 4 decimal places.
- Solve each triangle.
 - $\triangle ABC$ in which $BC = 62.5$ cm, $\angle A = 112^\circ$, and $\angle C = 42^\circ$
 - $\triangle PQR$ in which $QR = 42.2$ cm, $PQ = 21.2$ cm, and $\angle P = 100.5^\circ$
 - $\triangle XYZ$ in which $XY = 31$ mm, $XZ = 52$ mm, and $\angle X = 33^\circ$
- Triangle ABC has area 30 cm^2 , $AC = 9$ cm, and $BC = 7$ cm. Calculate the measure of $\angle ACB$ and the largest possible length of AB.
- Sketch each angle θ in standard position, then write a coterminal angle.
 - $\theta = 170^\circ$
 - $\theta = 293^\circ$
 - $\theta = -30^\circ$
 - $\theta = -320^\circ$
 - $\theta = 450^\circ$
 - $\theta = 600^\circ$
 - $\theta = -370^\circ$
 - $\theta = 200^\circ$

9. Determine two angles between 0° and 360° that have each trigonometric function value. Write the angle to the nearest degree.

a) $\sin \theta = 0.42$ b) $\cos \theta = -0.31$ c) $\tan \theta = 3.46$

10. The point $P(4, -15)$ lies on the terminal arm of an angle θ in standard position. Determine each trigonometric function value to 3 decimal places.

a) $\sin \theta$ b) $\cos \theta$ c) $\tan \theta$

11. The terminal arm of an angle θ lies in Quadrant II on the line with equation $4x + 3y = 0$. Determine each trigonometric function value.

a) $\sin \theta$ b) $\cos \theta$ c) $\tan \theta$

12. State each exact value. Do not use a calculator.

a) $\cos 135^\circ$ b) $\tan 225^\circ$ c) $\sin 210^\circ$ d) $\frac{1}{\tan 60^\circ}$

13. Simplify each expression. Do not use a calculator.

a) $\sin 30^\circ + \cos 60^\circ$ b) $\tan 45^\circ + \tan 225^\circ$ c) $\sin 240^\circ + \cos 300^\circ$

22. Solve each equation for $0^\circ \leq \theta \leq 360^\circ$. (Thinking)

a) $\sin \theta - 2 \sin^2 \theta = 0$ b) $2 \cos^2 \theta - 3 \cos \theta - 2 = 0$
c) $3 \tan^2 \theta - \tan \theta = 0$ d) $8 \sin^2 \theta - 6 \sin \theta + 1 = 0$

23. Solve each equation for $0^\circ \leq \theta \leq 360^\circ$. (Thinking)

a) $2 \cos^2 \theta = 1 - \sin \theta$ b) $\cos \theta + 1 - 2 \sin^2 \theta = 0$
c) $5 \cos^2 \theta - 12 \sin \theta + 6 = 0$ d) $5 - 6 \sin^2 \theta - \cos \theta = 0$

24. Prove each identity.

a) $(\sin \theta + \cos \theta)^2 = 1 + 2 \sin \theta \cos \theta$ b) $\frac{1}{\cos \theta} - \tan \theta \sin \theta = \cos \theta$
c) $\sin^2 \theta \left(1 + \frac{1}{\tan^2 \theta}\right) = 1$ d) $\frac{\cos \theta}{1 + \sin \theta} - \frac{1}{\cos \theta} = -\tan \theta$

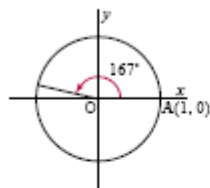
25. Prove each identity.

a) $\frac{\tan \theta - \sin \theta}{\sin^3 \theta} = \frac{1}{\cos \theta(1 + \cos \theta)}$ b) $\tan^2 \theta \cos^2 \theta + \frac{\sin^2 \theta}{\tan^2 \theta} = 1$

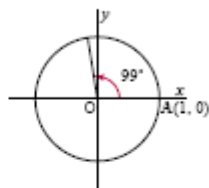
Number 24
and 25 is
Monday's
work

Chapter 5 Review Exercises, page 316

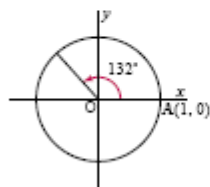
1. a) Negative



b) Positive



c) Negative



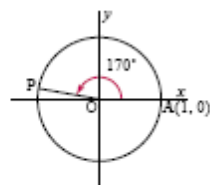
2. a) 61.5° b) 167.2° c) $33.2^\circ, 146^\circ$
 d) 58.3° e) $20.4^\circ, 159.6^\circ$ f) 123.9°
3. a) 0.4067 b) -0.2403

6. a) $\angle B = 26^\circ$; $AC \doteq 29.5$ cm; $AB \doteq 45.1$ cm
 b) $\angle R \doteq 29.6^\circ$; $\angle Q \doteq 49.9^\circ$; $PR \doteq 32.8$ cm
 c) $YZ \doteq 31$ mm; $\angle Z = 33^\circ$; $\angle Y = 114^\circ$

7. 107.8° ; 13.0 cm

8. Coterminal angles may vary.

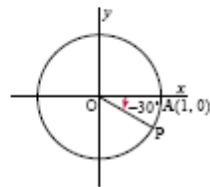
a) 530°



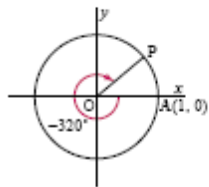
b) 653°



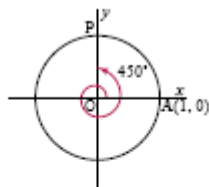
c) 330°



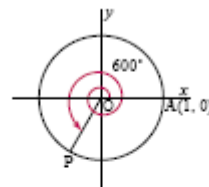
d) 40°



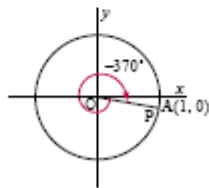
e) 90°



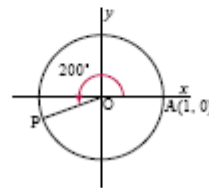
f) 240°



g) 350°



h) 560°



9. a) $25^\circ, 155^\circ$ b) $108^\circ, 252^\circ$ c) $74^\circ, 254^\circ$
 10. a) -0.966 b) 0.258 c) -3.750
 11. a) $\frac{4}{5}$ b) $\frac{3}{5}$ c) $\frac{4}{3}$
 12. a) $-\frac{1}{\sqrt{2}}$ b) 1 c) $-\frac{1}{2}$ d) $\frac{1}{\sqrt{3}}$
 13. a) 1 b) 2 c) $\frac{1-\sqrt{3}}{2}$

22. a. $\theta = 0, 180, 30, 150$
 b. $\theta = 120, 240$
 c. $\theta = 0, 180, 360, 18.43, 198.43$
 d. $\theta = 30, 150, 14.47, 165.53$

23. a. $\theta = 90, 210, 330$
 b. $\theta = 180, 120, 300$
 c. $\theta = 45, 135$
 d. $\theta = 60, 300, 70.5, 289.47$