Trigonometric Ratios Unit 5

Tentative TEST date_______________

Big idea/Learning Goals

In this unit you will extend your knowledge of SOH CAH TOA to work with obtuse and reflex angles. This extension will involve the unit circle which will allow you to understand that inverse sine or inverse cosine or inverse tangent actually give more than one solution. You will learn how to find the exact ratios for special angles without using a calculator. You will then learn the differences between a trigonometric equation and a trigonometric identity. You will practice proving different trigonometric identities. Finally you will review Sine and Cosine laws that you've studied in grade 10 and see that the calculator doesn't always give you the solution that you need. You will learn how to interpret the results from the calculations and decide on the proper answer.

Corrections for the textbook answers:
Sec 5.6 #8 4138.9 m #9 97.6
Sect 5.8 #9 7°

Success Criteria

- ☐ I understand the new topics for this unit if I can do the practice questions in the textbook/handouts

Date	pages	Topics	# of quest. done? You may be asked to show them	**Questions I had difficulty with** ask teacher before test!
	2-3	Primary and Secondary Trig Ratios Section 5.1 & Handout		
	4-6	Obtuse & Reflex Angles on the Unit Circle – 1.5 days Section 5.3 & Handout		
	7-9	Using the Unit circle – 1.5 days Section 5.4 & Handout		
	10-12	Exact Values using Special Angles Section 5.2 & two Handouts		
		QUIZ no calculators		
	13-14	Trigonometric Identities & Proofs Section 5.5 & Handout		
	15-16	Sine Law Review & Ambiguous Case Section 5.6		
	17-18	Cosine Law Review & Solve 3D Problems Section 5.7 & 5.8		
		REVIEW – 2 days?		

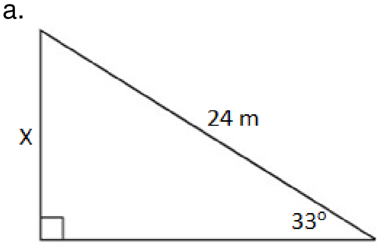
Reflect – previous TEST mark __________, Overall mark now__________.

Primary and Secondary Trig Ratios

1. Since you will be extending your trigonometry knowledge from grade 10 to be able to solve obtuse and reflex angles, it is a good idea to review the **primary trigonometric ratios,** or SOH CAH TOA, and Pythagorean Theorem. Summarize what you should know:

2. Solve the following for X.

a.



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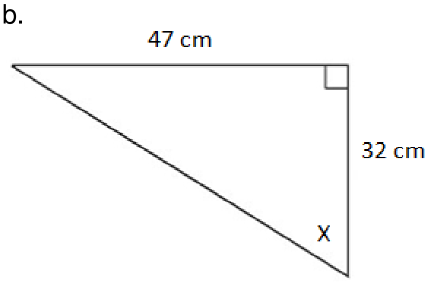
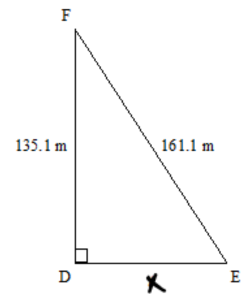


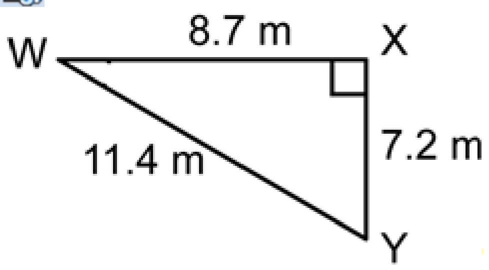
c.



3. From the top of a building, the angle of elevation of the top of a nearby building is 28° and the angle of depression of the bottom of the nearby building is 48°. The distance between the two buildings is 50 m. What is the height of the taller building?

4. You've been using the primary trigonometric ratios, there are also secondary ones.
Summarize what the **secondary trigonometric ratios** are:

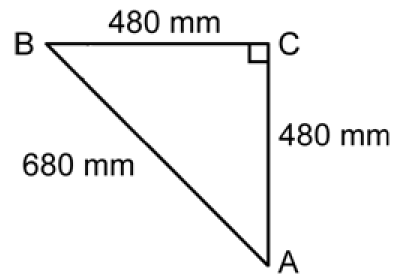
5. Find the following ratios (DON'T find angle unless asked)



a. $\cos Y$

b. $\cot W$

6. Find the following ratios



a. $\csc A$

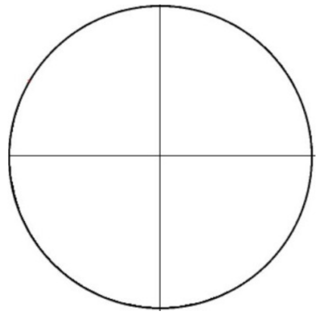
b. $\tan B$

7. If $\sin\theta = \frac{1}{4}$ then show how to find the values of $\csc\theta$ and $\sec\theta$

8. If $\cot\theta = 0.4732$ then what is θ?

Obtuse & Reflex Angles on the Unit Circle

1. To understand the way obtuse and reflex angles relate to the unit circle, you need to learn some new definitions. Define(or show on a diagram) the following terms:



terminal arm

initial position

positive angle rotation

negative angle rotation

co-terminal angles

standard position

principle angle

related acute angle (or reference angle)

quadrants

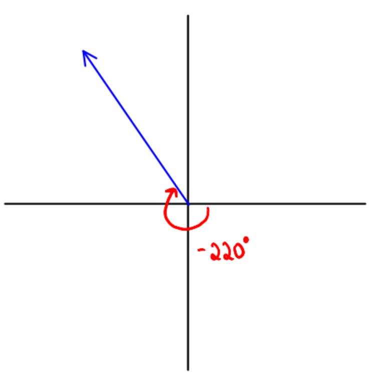
acute angles

obtuse angles

reflex angles

2. State the principal angle and the related acute angle, then state two more co-terminal angles.

a.



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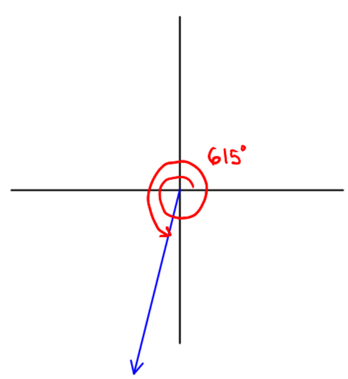
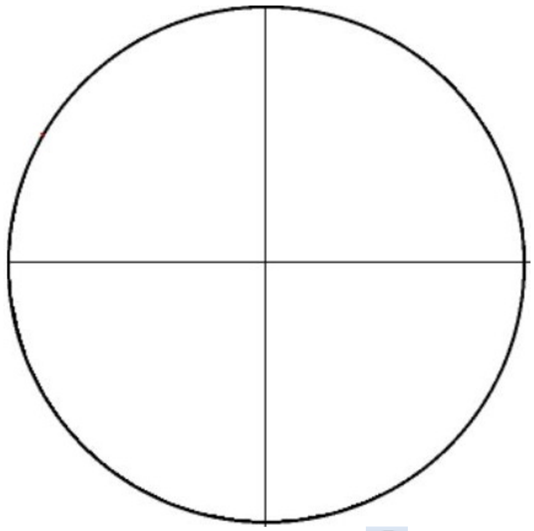


3. You must understand why sometimes ratios are negative and sometimes positive. Answer the following questions to see what the pattern is.

   a. Draw the related acute angle 20^o in each quadrant, and state the principal angles each forms with the initial arm.

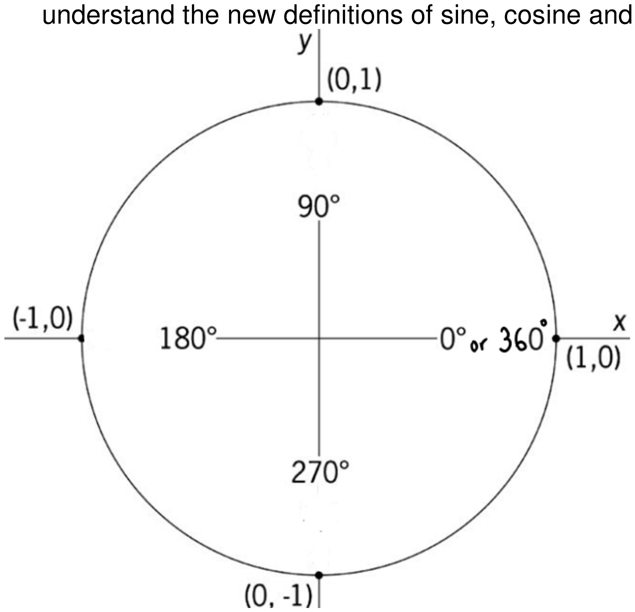
   b. In each quadrant, use the calculator to find all three primary trig ratios for the principal angle in that quadrant.



   c. What do you notice?

   d. What does the acronym CAST stand for?

4. The acronym CAST is not always useful. Angles can fall onto the x or y axes. Answer the following questions to really understand the new definitions of sine, cosine and tangent.

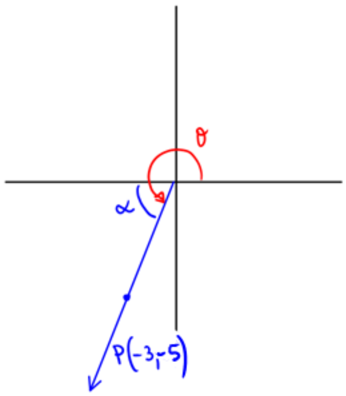


   a. Use the calculator to find all three primary trig ratios of all the angles that fall onto the axes

   b. Compare your answers to the coordinate points that the answers correspond to, what do you notice?

   c. What are the new definitions of sine, cosine and tangent? (Keep in mind the cicle doesn't have to have a radius of 1.) Also, what does the pythagorean theorem remind you of here?

5. You CANNOT rely on the calculator to give you the answers that you need anymore, to see this, answer the following questions.



a. For the point P(–3, – 5) find the following ratios:

$\sin\theta$

$\cos\theta$

$\tan\theta$

b. Now use the inverse buttons for EACH ratio to find the angle θ

c. What do you notice with answers the calculator gives you for θ?

d. What must be done when you use inverse buttons when dealing with obtuse or reflex angles?

6. For the angle -170^o

a. Find the related acute or the reference angle

b. Predict the signs of all the primary trig ratios. Explain your choice using CAST **and** using the new definitions of the trig ratios.

7. For $\tan(-140^o)$

a. Sketch the given angle

b. State the principal angle and the related acute angle

c. Find a few equivalent expressions to $\tan(-140^o)$ that give the same answer for the ratio.

Using the Unit Circle

1. Predict whether each value will be positive or negative. Explain the MEANING of each ratio.

a. $\tan 195^o$

b. $\sin(-115^o)$

c. $\cos 670^o$

2. For all of the above state an equivalent trigonometric expressions with same value of the ratio.

3. Find the following ratios without using the calculator.

a. $\cos(-90^o)$

b. $\cos 180^o$

c. $\tan 270^o$

d. $\sin 360^o$

4. Find the angles without using the calculator.

a. $\cos\theta = -1$

b. $\cos\theta = 0$

c. $\sin\theta = 1$

d. $\tan\theta = undefined$

5. For the ratio $\sin\theta = -\frac{2}{5}$, the angle θ is in standard position i.e. $0^o \le \theta \le 360^o$.
   a. How many answers for θ are there?
   b. Is θ acute, obtuse or reflex in Quadrant III or reflex in Quadrant IV?
   c. Find all possible measures of θ in the given domain.

6. For the point $P(-2,6)$
   a. Sketch the angle, θ, in standard position
   b. Find $\cot\theta$
   c. Find the angle θ.

7. For the ratio $\sec\theta = -\frac{4}{3}$, the angle θ is in standard position $0^o \le \theta \le 360^o$.
   a. Find all 5 other trig ratios for θ
   b. Find all possible measures of θ in the given domain

8. For the point $P(5,-7)$
   a. Find $\csc\theta$
   b. Find the angle θ.

9. When trig ratios are 0, and sine/cosine are ± 1 then:

10. If trig ratios are something else:

11. Solve for angle if $0^o \leq angle \leq 360^o$

a. $2 = -5\tan\beta$

b. $-4\sin\theta - 3 = 0$

c. $\sin\beta = 0.5$

g. $2\cos t + 1 = 0$

h. $\tan\theta = 0$

i. $\cos\alpha = 0$

d. $\cot\omega = 5.64$

e. $\sec\lambda = -1$

f. $\csc\varphi = -4$

Exact Values using Special Angles

1. What are special angles?

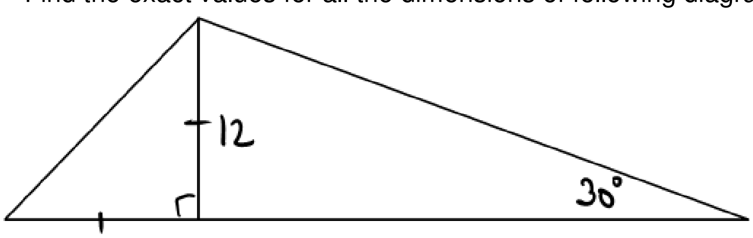
2. What answer is better to record of the two below and why?

$\cos 30^o = 0.866025403...$ or $\cos 30^o = \frac{\sqrt{3}}{2}$

3. Almost everytime trig functions are used there is rounding error. However, it is possible to find exact values for some special angles. Draw two special triangles and explain where the side lengths come from.

4. Does it matter what is the size of the triangle used when dealing with ratios? Draw different sized triangles and label the dimensions. Show that ratios are still the same.

5. Find the exact values for all the dimensions of following diagram



6. Find the exact values of each of the following, without using the calculator.

a. $\csc 330^o$

b. $\cos 720^o$

c. $\sin 315^o$

d. $\tan(-120^o)$

e. $\sec 225^0$

f. $\cot 120^0$

g. $\sin 270^o \cos 45^o - \cot 60^o \sec 150^o$

h. $2\csc 90^o - 3\tan 135^o \cos 210^o$

7. Without using the calculator explain how you can find the solutions for angles to the following

a. $\cos\theta = -\frac{\sqrt{3}}{2}$

b. $\tan\theta = \frac{\sqrt{3}}{3}$

c. $\sin\theta = -\frac{\sqrt{3}}{2}$

d. $\tan\theta = -1$

8. For the point $P(-2,-8)$ find the exact values of the three primary trig ratios for the principal angle that is made with the terminal arm with point P on it.

9. The angle θ, is in Quadrant III, and $\sin\theta = -\frac{\sqrt{3}}{2}$. Point P lies on the terminal arm. Determine θ, and state at least two possible coordinates for point P.

10. The terminal arm of θ is in quadrant III and on the line $\sqrt{3}y - 3x = 0$. Determine the angle θ in standard position

Trigonometric Identities & Proofs

1. Before you begin proving identities, it is important to understand the differences between the following words. Explain with the use of examples what each term means.

EXPRESSION EQUATION IDENTITY

2. Show that the following identities are true for any angle

a. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

b. $\sin^2\theta + \cos^2\theta = 1$

3. The proofs to identities are not unique, hence it is important to write out what you do in each step. List some strategies to try when doing proofs.

Things that you will lose marks for are:

- Not explaning steps or skipping steps
- For writting terms incorrectly eg. $\cos$, $\sin$, $\tan$ without θ, or $\sin\theta^2$ when you mean $\sin^2\theta$
- For incorrectly canceling or simplifying.
- Moving terms over the equals sign. This is not wrong to do, however the proofs in grade 11 are not particularly hard and if terms are moved proofs can become too simple.

4. Prove the following

a. $\sin\theta + \dfrac{\cos\theta}{\tan\theta} = \dfrac{1}{\cos\theta\tan\theta}$

b. $1 - \sin^2\theta = \dfrac{\sin^2\theta}{\tan^2\theta}$

c. $\sin^2 x + 2\cos^2 x - 1 = \cos^2 x$

d. $\dfrac{\sin^2 x}{1-\cos x} = 1 + \cos x$

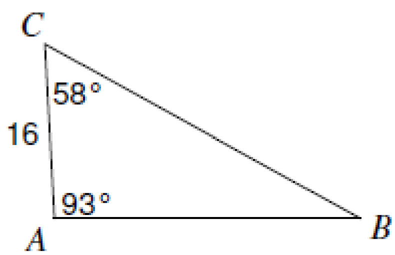
e. $\sec\theta\csc\theta - \cot\theta = \tan\theta$

f. $(\sin x + \cos x)^2 = 1 + 2\sin x\cos x$

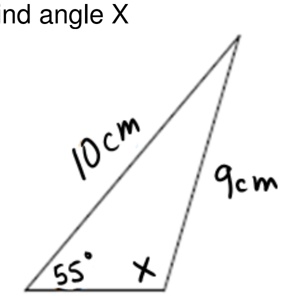
Sine Law Review & Ambiguous Case

1. Summarize the sine law and when you can use it.

2. Find side BC



3. Find angle X



4. Sometimes solving for the angles using sine law, the calculator gives you the acute angle, when the problem actually requires an obtuse angle, like in one of the questions above. Why does this happen? And why don't you need to worry about this scenario with cosine law?

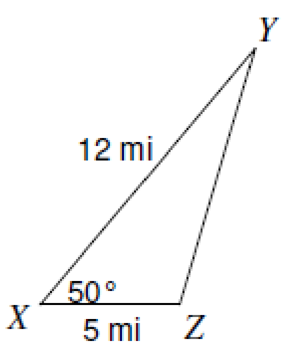
Sometimes when the diagram is not given an **ambiguous case** is created. This can happen when you are given SSA. In this situation there can be two possible triangles that can be solved OR no triangles at all OR only one triangle.

5. Determine if there is no triangle to solve at all, or if there is one triangle or if there are two triangles that must be solved.
   a. In ΔABC $\angle A=35^o, a=3, b=4$

   b. In ΔABC $\angle A=96^o, a=13, b=20$

   c. In ΔDEF $\angle E=28^o, e=24, f=20$

6. Find the area of the triangle



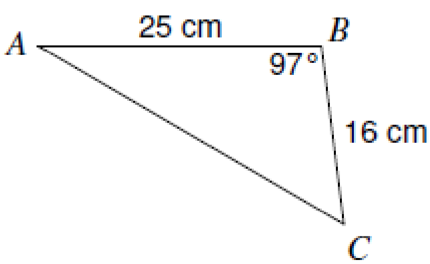
7. Same question as 6, just change angle to 45° and do exact value for the area.

Cosine Law Review & 3D Problems

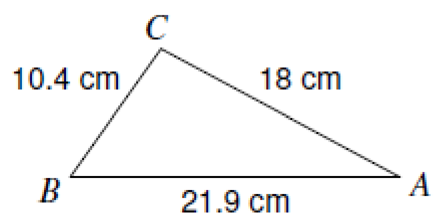
1. Summarize the cosine law and when you can use it.

2. Solve each triangle. This means find all sides, all angles.

a.

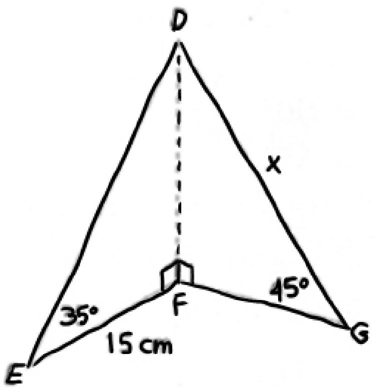


b.



3. Real life situations are almost never flat 2D problems. Solve the following for x.

a.



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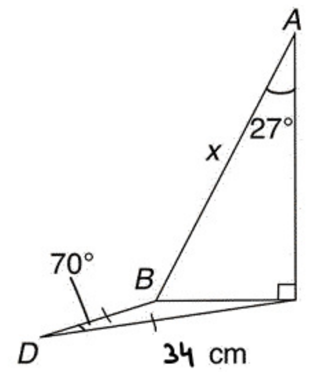
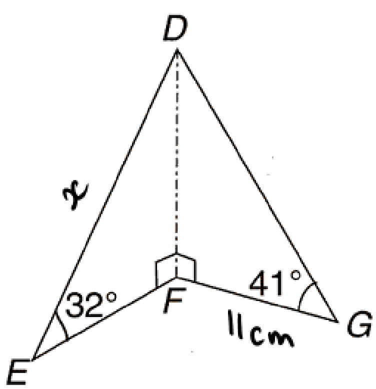


c.



d.