# S14TrigNOTESnew

July 9, 2014 2:48 PM

TrigNOTESnew see below

 **11U** Date:____________ Name: ______________________

## Trigonometric Ratios Unit 5

Tentative TEST date_______________

## Big idea/Learning Goals

In this unit you will extend your knowledge of SOH CAH TOA to work with obtuse and reflex angles. This extension will involve the unit circle which will allow you to understand that inverse sine or inverse cosine or inverse tangent actually give more than one solution. You will learn how to find the exact ratios for special angles without using a calculator. You will then learn the differences between a trigonometric equation and a trigonometric identity. You will practice proving different trigonometric identities. Finally you will review Sine and Cosine laws that you've studied in grade 10 and see that the calculator doesn't always give you the solution that you need. You will learn how to interpret the results from the calculations and decide on the proper answer.

Corrections for the textbook answers:
Sec 5.6 #8 4138.9 m #9 97.6
Sect 5.8 #9 7°

## Success Criteria

☐ I understand the new topics for this unit if I can do the practice questions in the textbook/handouts

finish exp + Day 8

Day 9 short

Day 10

| Date | pages | Topics | # of quest. done? You may be asked to show them | Questions I had difficulty with ask teacher before test! |
|---|---|---|---|---|
| | 2-3 | Primary and Secondary Trig Ratios<br>Section 5.1 & Handout #5,8,11,12,15,16 | | |
| | 4-6 | Obtuse & Reflex Angles on the Unit Circle – 1.5 days<br>Section 5.3 & Handout all replace $\pi = 180°$ | not 3.14 radians | |
| | 7-9 | Using the Unit circle – 1.5 days<br>Section 5.4 & Handout #4,5,6,8,12 | | |
| | 10-12 | Exact Values using Special Angles<br>Section 5.2 & two Handouts Handout 1 ALL | Handout 2 ALL | skip Handout #3 |
| | | QUIZ no calculators | | |
| | 13-14 | Trigonometric Identities & Proofs<br>Section 5.5 & Handout all 1st side | | |
| | 15-16 | Sine Law Review & Ambiguous Case<br>Section 5.6 #4,5,7,8 | | |
| | 17-18 | Cosine Law Review & Solve 3D Problems<br>Section 5.7 & 5.8 5.7 #4,8,10 5.8 #3,4 | | |
| | | REVIEW – 2 days? | | |

do practice quiz

**Reflect** – previous TEST mark __________, Overall mark now__________.

1

sine, cosine, tangent

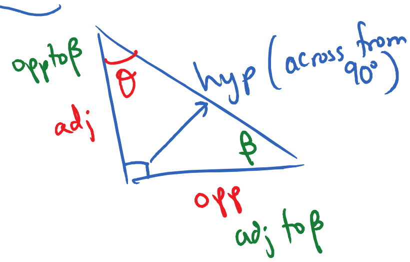
## Primary and Secondary Trig Ratios

1. Since you will be extending your trigonometry knowledge from grade 10 to be able to solve obtuse and reflex angles, it is a good idea to review the **primary trigonometric ratios,** or SOH CAH TOA, and Pythagorean Theorem. Summarize what you should know:

SOH CAH TOA

$\sin\theta = \frac{opp}{hyp}$ $\quad$ $\cos\theta = \frac{adj}{hyp}$ $\quad$ $\tan\theta = \frac{opp}{adj}$

$f(\theta)$

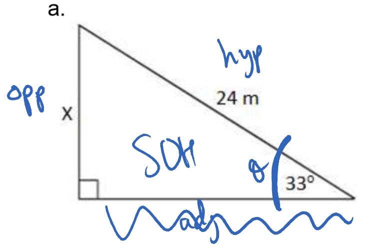



$a^2 + b^2 = c^2$

hyp.

2. Solve the following for X.

a.

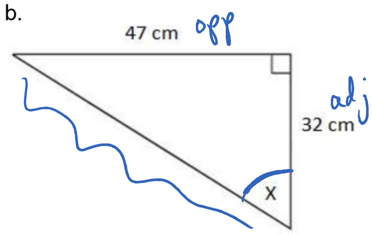



$\frac{\sin 33°}{1} = \frac{x}{24}$

$24(\sin 33) = x$

$13.1\text{m} = x$

b.

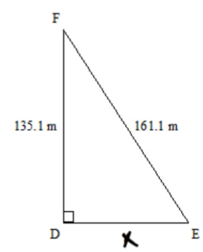



$\tan X = \frac{47}{32}$

$X = \tan^{-1}\left(\frac{47}{32}\right)$

$X = 56°$

c.




$a^2 + b^2 = c^2$

$135.1^2 + x^2 = 161.1^2$

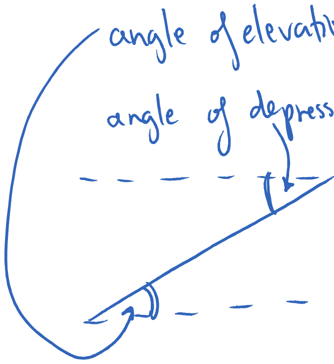
$\sqrt{x^2} = \sqrt{(161.1^2 - 135.1^2)}$

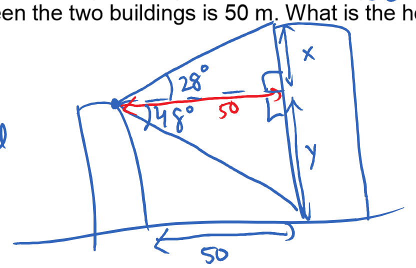
$x = 87.8\text{ m}$

3. From the top of a building, the angle of elevation of the top of a nearby building is 28° and the angle of depression of the bottom of the nearby building is 48°. The distance between the two buildings is 50 m. What is the height of the taller building?

angle of elevation - up from horizontal

angle of depression - down from horizontal






$\tan 28 = \frac{x}{50}$ $\quad$ $\tan 48 = \frac{y}{50}$

$50(\tan 28) = x$ $\quad$ $50(\tan 48) = y$

$26.6\text{m} = x$ $\quad$ $55.5 = y$

∴ the height is 82.1 m.

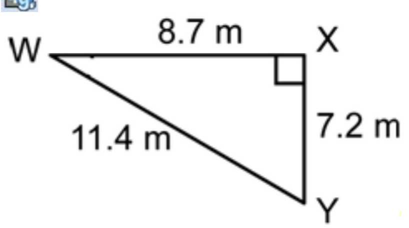
4. You've been using the primary trigonometric ratios, there are also secondary ones. Summarize what the **secondary trigonometric ratios** are:

cosecant
$\csc\theta = \frac{1}{\sin\theta} = \frac{hyp}{opp}$

secant
$\sec\theta = \frac{1}{\cos\theta} = \frac{hyp}{adj}$

cotangent
$\cot\theta = \frac{1}{\tan\theta} = \frac{adj}{opp}$

5. Find the following ratios (DON'T find angle unless asked)



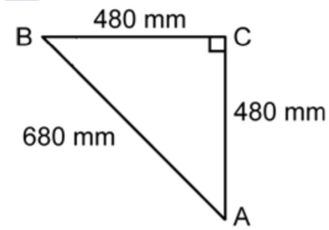

a. $\cos Y = \frac{adj}{hyp}$

$\cos Y = \frac{7.2}{11.4}$

b. $\cot W = \frac{adj}{opp}$

$\cot W = \frac{8.7}{7.2}$

6. Find the following ratios




a. $\csc A = \frac{680}{480}$

$= \frac{17}{12}$

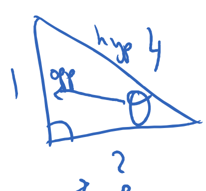
b. $\tan B = \frac{480}{480}$

$= 1$

7. If $\sin\theta = \frac{1}{4}$ $\frac{opp}{hyp}$ then show how to find the values of $\csc\theta$ and $\sec\theta$

reciprocals.

$\csc\theta = \frac{4}{1}$




$\sec\theta = \frac{hyp}{adj}$

$a^2+b^2=c^2$
$1^2+x^2=4^2$
$\sqrt{x^2}=\sqrt{4^2-1^2}$
$x=\sqrt{15}$
no decimals.

$\sec\theta = \frac{4}{\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}}$

$\sec\theta = \frac{4\sqrt{15}}{15}$

8. If $\cot\theta = 0.4732$ then what is $\theta$?

reciprocal

$\tan\theta = \frac{1}{0.4732}$

$\theta = \tan^{-1}\left(\frac{1}{0.4732}\right)$

$\theta \doteq 65^\circ$

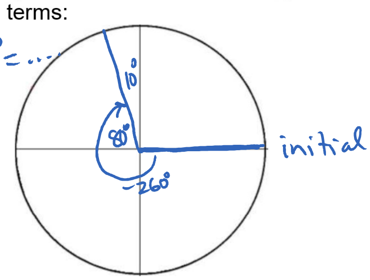
## Obtuse & Reflex Angles on the Unit Circle

r = 1

1. To understand the way obtuse and reflex angles relate to the unit circle, you need to learn some new definitions. Define(or show on a diagram) the following terms:

ex. $-260° = 100° = 460° = \dots$

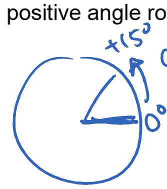
$\dots = -620$



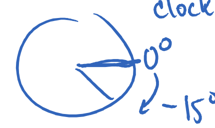

**terminal arm**
the arm that rotates around the circle. (attached to the origin.)

**initial position**
0° angle is at pos. x-axis.

**positive angle rotation**
counterclockwise




**negative angle rotation**
clockwise

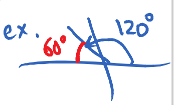



**co-terminal angles**
if you add/subtract 360° you land on the same terminal arm.

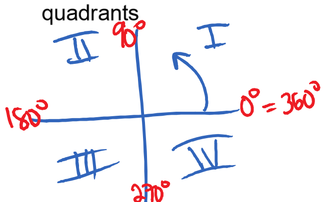
**standard position**
to begin rotation at pos. x-axis in counterclockise direction. (Not like Bearing.)

**principle angle**
$0° \le \theta \le 360°$
1st positive angle

**related acute angle (or reference angle)**
the acute angle made with x-axis (not y-axis) ex.




**quadrants**



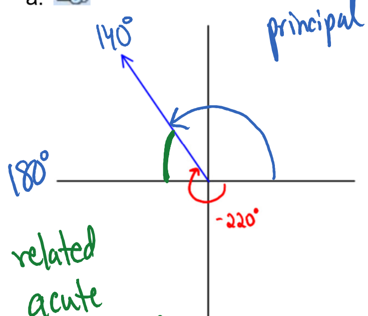

**acute angles**
$0° < \theta < 90°$

**obtuse angles**
$90° < \theta < 180°$

**reflex angles**
$\theta > 180°$

2. State the principal angle and the related acute angle, then state two more co-terminal angles.

a.

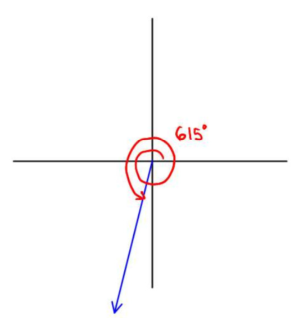



principal = 140°
$(-220° + 360°)$

related acute (beside x-axis) = 40°

coterminal = use angle provided (not related acute) +/- 360°
-580°, -940°, 500°, 860°, ...

b.




principal = 255°
related acute = 75°
* always positive

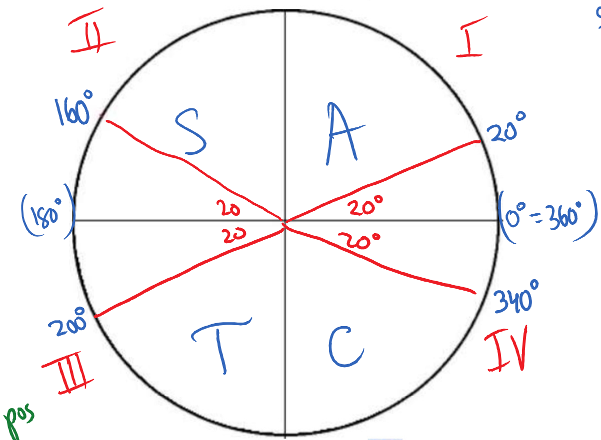
coterminal = 615°
= -105°
= -465°
= 975°
⋮

3. You must understand why sometimes ratios are negative and sometimes positive. Answer the following questions to see what the pattern is.

a. Draw the related acute angle $20^o$ in each quadrant, and state the principal angles each forms with the initial arm.

beside x-axis

b. In each quadrant, use the calculator to find all three primary trig ratios for the principal angle in that quadrant.




sin 160° = 0.3420
cos 160° = −0.9397
tan 160° = −0.3640
only sine is pos.

sin 20° = 0.3420
cos 20° = 0.9397
tan 20° = 0.3640
ALL pos.

sin 200° = −0.3420
cos 200° = −0.9397
tan 200° = 0.3640
only tan is pos

sin 340° = −0.3420
cos 340° = 0.9397
tan 340° = −0.3640
only cosine is pos.

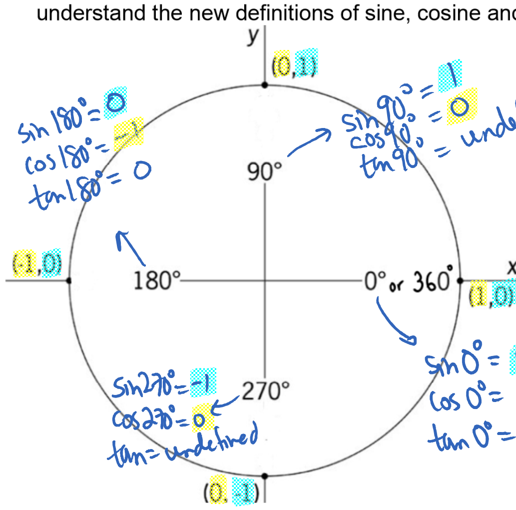
c. What do you notice?

For the same related acute angle all sine ratios are same magnitude but different sign, also all cosine... also all tangent...

d. What does the acronym CAST stand for?

Cosine is pos in IV
All are pos in I
S sine is pos in II
T tan is pos in III

4. The acronym CAST is not always useful. Angles can fall onto the x or y axes. Answer the following questions to really understand the new definitions of sine, cosine and tangent.




sin 180° = 0
cos 180° = −1
tan 180° = 0

sin 90° = 1
cos 90° = 0
tan 90° = undefined N/A.

sin 0° = 0
cos 0° = 1
tan 0° = 0

sin 270° = −1
cos 270° = 0
tan = undefined

a. Use the calculator to find all three primary trig ratios of all the angles that fall onto the axes

b. Compare your answers to the coordinate points that the answers correspond to, what do you notice?

Sine – is the y-coordinate on the unit circle (r=1)

Cosine – is the x-coordinate on the unit circle

tangent – is the slope of terminal arm

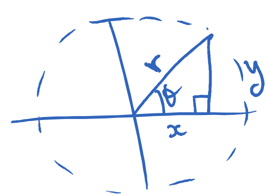
c. What are the new definitions of sine, cosine and tangent? (Keep in mind the cicle doesn't have to have a radius of 1.)
Also, what does the pythagorean theorem remind you of here?

Know memorize!

$\sin\theta = \frac{opp}{hyp} = \frac{y}{r}$

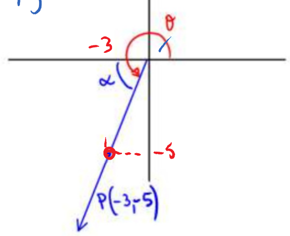
$\cos\theta = \frac{adj}{hyp} = \frac{x}{r}$

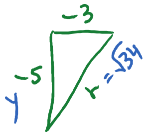
$\tan\theta = \frac{opp}{adj} = \frac{y}{x}$ ← rise / ← run



5. You CANNOT rely on the calculator to give you the answers that you need anymore, to see this, answer the following questions.

r is always positive
x, y can neg / pos.




$a^2+b^2=c^2$

$x^2+y^2=r^2$

$(-3)^2+(-5)^2=r^2$

$\sqrt{34}=r$

a. For the point P( –3, – 5) find the following ratios:

b. Now use the inverse buttons for EACH ratio to find the angle $\theta$

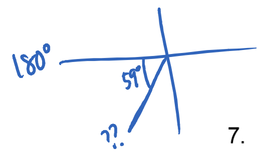
$\sin\theta = \frac{y}{r} = \frac{-5}{\sqrt{34}} \cdot \frac{\sqrt{34}}{\sqrt{34}} = \frac{-5\sqrt{34}}{34} \rightarrow \theta = \sin^{-1}\left(\frac{-5\sqrt{34}}{34}\right) = -59°$

$\cos\theta = \frac{x}{r} = \frac{-3}{\sqrt{34}} = \frac{-3\sqrt{34}}{34} \rightarrow \theta = \cos^{-1}\left(\frac{-3\sqrt{34}}{34}\right) = 121°$

$\tan\theta = \frac{y}{x} = \frac{-5}{-3} = \frac{5}{3} \rightarrow \theta = \tan^{-1}\left(\frac{5}{3}\right) = 59°$

c. What do you notice with answers the calculator gives you for $\theta$?

all answers are diffent but if you find the related acute = 59°




$\therefore \theta = 239°$

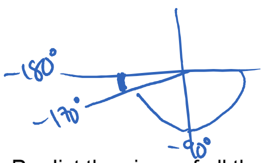
d. What must be done when you use inverse buttons when dealing with obtuse or reflex angles?

You need to draw a picture + use symmetry and the related acute angle (always pos.)

like Quiz

6. For the angle $-170^o$

a. Find the related acute or the reference angle




$\therefore$ related acute $= 10°$

pos angle w x-axis

b. Predict the signs of all the primary trig ratios. Explain your choice using ~~CAST~~ **and** using the new definitions of the trig ratios.

$\sin(-170°) = +/-$ since sine is y (below x-axis)

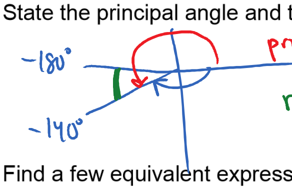
$\cos(-170°) = +/-$ since cosine is x (left of origin)

$\tan(-170°) = +/-$ since terminal arm has pos. slope.

7. For $\tan(-140^o)$

a. Sketch the given angle

b. State the principal angle and the related acute angle




principal = 220°

related acute = 40°

c. Find a few equivalent expressions to $\tan(-140^o)$ that give the same answer for the ratio.

$\tan(-140°) = \tan(40°)$ in I

$= -\tan(-40°)$ in IV

$= -\tan(320°)$ in IV

$= -\tan(140°)$ in II

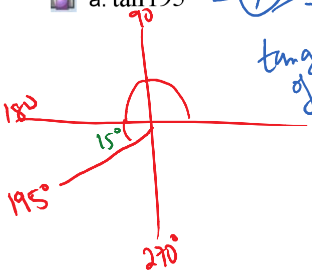
$= \tan(220°)$ in III

$= \tan(400°)$ in I

## Using the Unit Circle

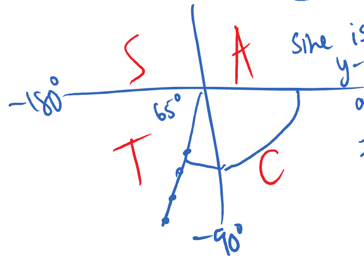
1. Predict whether each value will be positive or negative. Explain the MEANING of each ratio.

a. $\tan 195^o$ = + / −



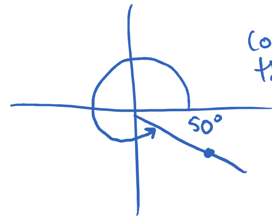

tangent is slope of terminal arm.

b. $\sin(-115^o)$ = + / −




Sine is the y-coordinate and it's below the origin.

c. $\cos 670^o$ = cos 310° = + / − (−360°)




Cosine is the x-coord. and it's on right side of origin.

2. For all of the above state an equivalent trigonometric expression ~~that uses the **smallest possible negative angle**~~, but gives the same value of the ratio.

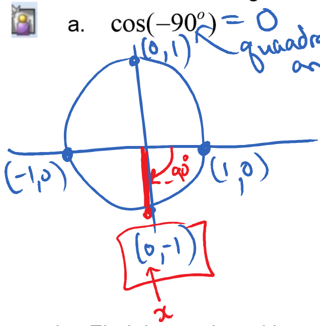
ⓐ tan 195° = tan 15° in I
= − tan 165° in II
= − tan 345° in IV

ⓑ sin (−115°) = − sin 65° in I
= − sin 115° in II
= sin 245° in III
= sin 295° in IV

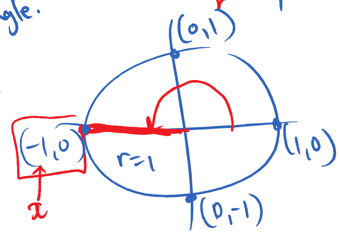
ⓒ cos 670° = cos 50° in I
= − cos 130° in II
= − cos 230° in III
= cos 310° in IV

3. Find the following ratios without using the calculator.

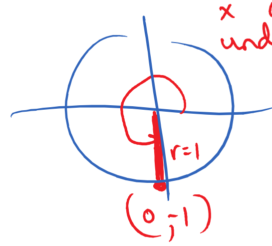
a. $\cos(-90^o)$ = 0 quadrantal angle.



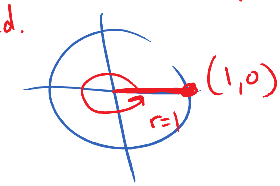

b. $\cos 180^o = \frac{x}{r} = \frac{-1}{1} = -1$




c. $\tan 270^o = \frac{y}{x} = \frac{-1}{0}$ undefined.

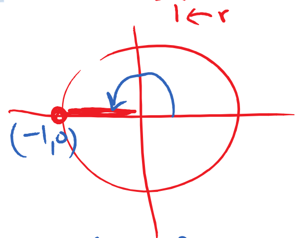



d. $\sin 360^o = \frac{y}{r} = \frac{0}{1} = 0$



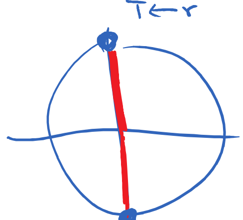

4. Find the angles without using the calculator. → $\boxed{0^\circ \le \theta \le 360^\circ}$

a. $\cos\theta = -1$ ← x, 1 ← r




θ = 180°

b. $\cos\theta = 0$ ← x, 1 ← r



θ = 90°, 270° …
or
θ = 270°, −90° …

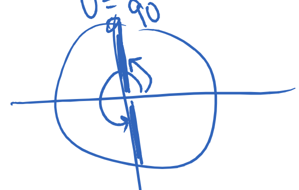
quote positives

c. $\sin\theta = 1$ ← y

θ = 90°


d. $\tan\theta = undefined$

θ = 90°



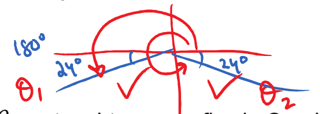
θ = 270°

5. For the ratio $\sin\theta = \frac{-2y}{5r}$, the angle $\theta$ is in standard position i.e. $0^o \le \theta \le 360^o$.

neg. y, below origin.

a. How many answers for $\theta$ are there?

2 answers for $\theta$ since it can be in III or IV




b. Is $\theta$ acute, obtuse or reflex in Quadrant III or reflex in Quadrant IV?

c. Find all possible measures of $\theta$ in the given domain.

rough work: to find related acute drop all negatives

$\theta_r = \sin^{-1}\left(\frac{2}{5}\right)$

$\theta_r = 24^o$

Now use picture

$\theta_1 = 204^o$

$\theta_2 = 336^o$

9. For the ratio $\sec\theta = \frac{4r}{-3x}$, the angle $\theta$ is in standard position $0^o \le \theta \le 360^o$.

neg x, left of origin

a. Find all 5 other trig ratios for $\theta$

$\sin\theta = \frac{y}{r} = \frac{\pm\sqrt{7}}{4}$

$\cos\theta = \frac{x}{r} = \frac{-3}{4}$

$\tan\theta = \frac{y}{x} = \frac{\pm\sqrt{7}}{-3}$

$\csc\theta = \frac{r}{y} = \frac{4}{\pm\sqrt{7}} = \pm\frac{4\sqrt{7}}{7}$

$\cot\theta = \frac{x}{y} = \frac{-3}{\pm\sqrt{7}} = \pm\frac{3\sqrt{7}}{7}$

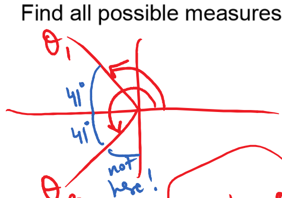
$x^2 + y^2 = r^2$

$(-3)^2 + y^2 = 4^2$

$y^2 = \sqrt{16-9}$

$y = \pm\sqrt{7}$

b. Find all possible measures of $\theta$ in the given domain




drop negatives.

rough: $\theta_r = \cos^{-1}\left(\frac{3}{4}\right)$

$\theta_r = 41^o$

related acute
- always pos
- near x-axis.

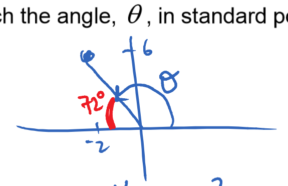
SOL: $\theta_1 = 139^o$

use pic.

$\theta_2 = 221^o$

7. For the point $P(-2,6)$

a. Sketch the angle, $\theta$, in standard position




b. Find $\cot\theta = \frac{x}{y} = \frac{-2}{6} = \frac{-1}{3}$

$\therefore \boxed{\cot\theta = -\frac{1}{3}}$

$\tan\theta = -3$

rough: $\theta_r = \tan^{-1}(3)$

$\theta_r = 72$

c. Find the angle $\theta$.

$\therefore \theta = 108^o$

from picture

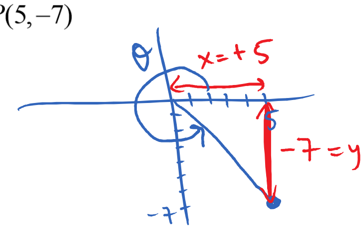
8. For the point $P(5,-7)$

a. Find $\csc\theta$

$= \frac{1}{\sin\theta}$

$= \frac{r}{y}$

$\therefore \boxed{\csc\theta = \frac{\sqrt{74}}{-7}}$




$a^2 + b^2 = c^2$

$(5^2) + (-7)^2 = r^2$

$\sqrt{74} = \sqrt{r^2}$

$\sqrt{74} = r$

b. Find the angle $\theta$.

$\sin\theta = \frac{-7}{\sqrt{74}}$

$\theta = \sin^{-1}\left(\frac{-7}{\sqrt{74}}\right)$

$\theta = -54^o$ in IV

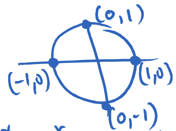
+360°

$\theta = 306^o$

How to find $\theta$

9. When trig ratios are 0, and sine/cosine are $\pm 1$ then:

1.) draw the circle




2.) Match the definition $\frac{x}{y}$ or $\frac{r}{x}$ ... with points to find the quadrantal angle(s) (on axes)

$\tan\theta = \pm 1$ not quadrantal but $\theta_r = 45°$

10. If trig ratios are something else:

1.) Isolate trig ratio
2.) Do rough work for $\theta_r$ (drop any negatives)
3.) Draw where angle(s) can be and place $\theta_r$ beside x-axis
4.) Use symmetry to figure out $\theta_1, \theta_2$ ...

11. Solve for angle if $0^o \le angle \le 360^o$

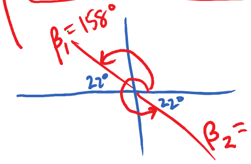
a. $2 = -5\tan(\beta)$ "beta"

$\frac{2}{-5} = \frac{-5\tan\beta}{-5}$

$\frac{2}{-5} = \tan\beta$ $\quad \frac{-2}{5}$ or $\frac{2}{-5}$

rough: $\beta_r = \tan^{-1}\left(\frac{2}{5}\right) = 22°$

look at original

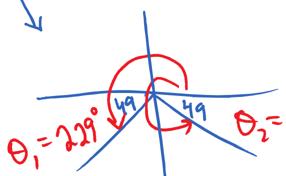



b. $-4\sin\theta - 3 = 0$

$-4\sin\theta = 3$

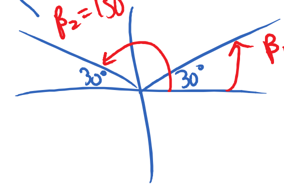
$\sin\theta = \frac{-3}{4} \frac{y}{r}$

rough: $\theta_r = \sin^{-1}\left(\frac{3}{4}\right) = 49°$




c. $\sin\beta = 0.5 \leftarrow y$
$\qquad 1 \leftarrow r$

rough: $\beta_r = \sin^{-1}(0.5) = 30°$

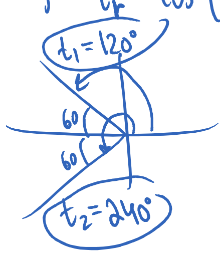



g. $2\cos t + 1 = 0$

$2\cos t = -1$

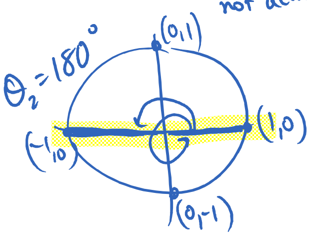
$\cos t = -\frac{1}{2}$

rough: $t_r = \cos^{-1}\left(\frac{1}{2}\right) = 60°$

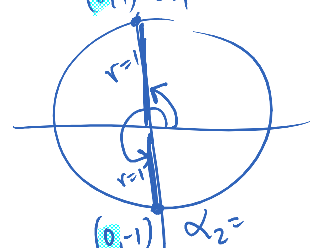



h. $\tan\theta = 0 \; \frac{y}{x}$ $\pm 1$

$\theta_r = \tan^{-1}(\cancel{0}) = 0°$ not acute.




i. $\cos\alpha = \frac{0}{1} \frac{x}{r}$

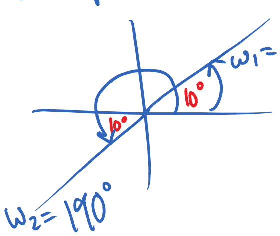



d. $\cot\omega = 5.64$

$\tan\omega = \frac{1}{5.64} = \frac{-1}{-5.64} = \frac{y}{x}$

rough: $\omega_r = \tan^{-1}\left(\frac{1}{5.64}\right) = 10°$

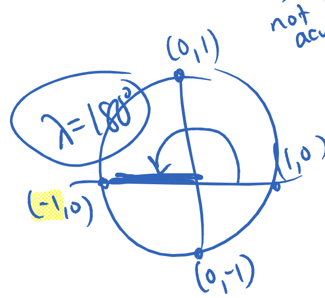
tan is pos = slope.




e. $\sec\lambda = \frac{-1}{1}$

reciprocal: $\cos\lambda = \frac{1}{-1} = \frac{x}{r} = \frac{-1}{1}$

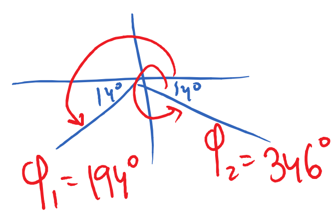
$\lambda_r = \cos^{-1}(1) = \cancel{0}$ not acute




f. $\csc\varphi = -4$

$\sin\varphi = \frac{-1}{4}$

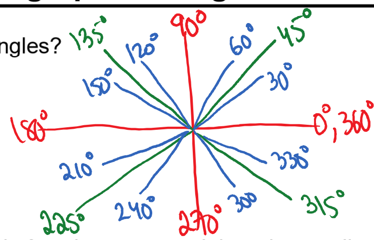
rough: $\varphi_r = \sin^{-1}\left(\frac{1}{4}\right) = 14°$



## Exact Values using Special Angles

use the circle for all "red" angles

1. What are special angles?




2. What answer is better to record of the two below and why?

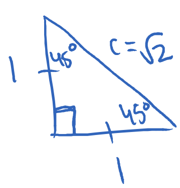
$\cos 30^o = 0.866025403...$ or $\cos 30^o = \frac{\sqrt{3}}{2}$

Better
no rounding error.

3. Almost everytime trig functions are used there is rounding error. However, it is possible to find exact values for some special angles. Draw two special triangles and explain where the side lengths come from.

Right Isosceles Δ

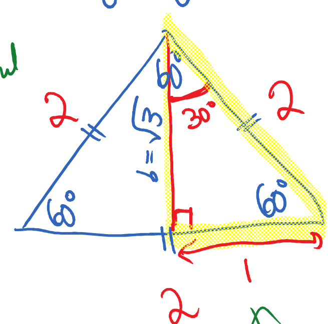
help with all "green" angles




$a^2+b^2=c^2$
$1^2+1^2=c^2$
$\sqrt{2}=c$

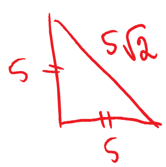
Half of Equilateral Δ.

helpful with all "Blue" angles




$a^2+b^2=c^2$
$1^2+b^2=2^2$
$b^2=2^2-1^2$
$b=\sqrt{3}$

4. Does it matter what is the size of the triangle used when dealing with ratios? Draw different sized triangles and label the dimensions. Show that ratios are still the same.



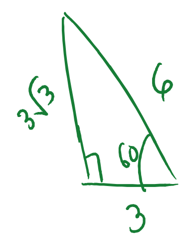

$\sin 45^o = \frac{opp}{hyp} = \frac{1}{\sqrt{2}}$

$= \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}$

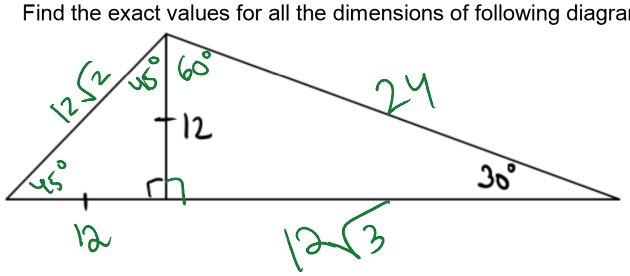
$\cos 60^o = \frac{adj}{hyp} = \frac{1}{2}$

$= \frac{3}{6}$

$= \frac{1}{2}$




5. Find the exact values for all the dimensions of following diagram

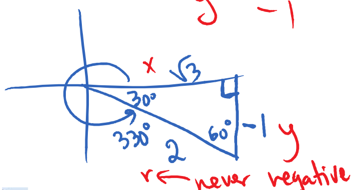



using "scale" (magnifying factor.)

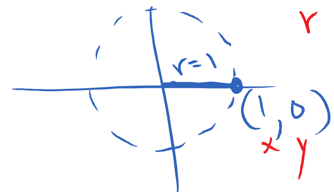
① Draw angle ② Draw Δ toward x-axis ③ Label 1, $\sqrt{3}$, 2 / 30°, 60°, 90° | 1, 1, $\sqrt{2}$ / 45°, 45°, 90° ④ insert any negatives (radius always pos) ⑤ use def.

6. Find the exact values of each of the following, without using the calculator.

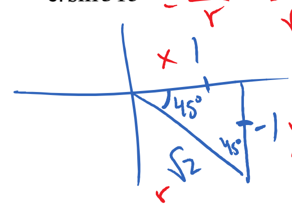
a. $\csc 330^o = \frac{r}{y} = \frac{2}{-1}$



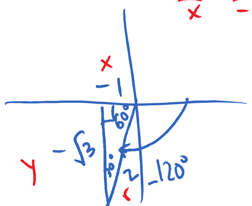

b. $\cos 720^o = \cos 0^o = \frac{x}{r} = \frac{1}{1}$



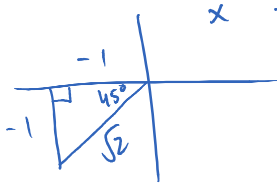

c. $\sin 315^o = \frac{y}{r} = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$



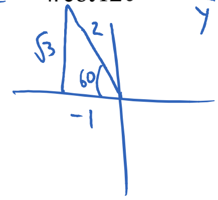

d. $\tan(-120^o) = \frac{y}{x} = \frac{-\sqrt{3}}{-1} = \sqrt{3}$




e. $\sec 225^0 = \frac{r}{x} = \frac{\sqrt{2}}{-1} = -\frac{\sqrt{2}}{1}$




f. $\cot 120^0 = \frac{x}{y} = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$




g. $\sin 270^o \cos 45^o - \cot 60^o \sec 150^o$

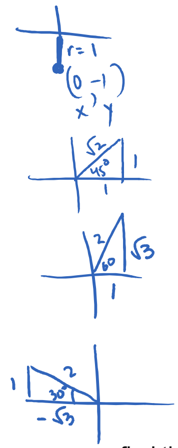
$= \left(\frac{y}{r}\right)\left(\frac{x}{r}\right) - \left(\frac{x}{y}\right)\left(\frac{r}{x}\right)$

$= \left(\frac{-1}{1}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{3}\right)\left(\frac{-2\sqrt{3}}{3}\right)$

$= -\frac{\sqrt{2}}{2} - \left(\frac{-2\sqrt{9}}{9}\right)$

$= -\frac{\sqrt{2}}{2} + \frac{6}{9}$

$= -\frac{\sqrt{2}\cdot 3}{2\cdot 3} + \frac{2\cdot 2}{3\cdot 2} = \frac{-3\sqrt{2}+4}{6}$




h. $2\csc 90^o - 3\tan 135^o \cos 210^o$

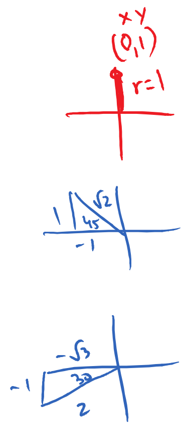
$= 2\left(\frac{r}{y}\right) - 3\left(\frac{y}{x}\right)\left(\frac{x}{r}\right)$

$= \frac{2}{1}\left(\frac{1}{1}\right) - \frac{3}{1}\left(\frac{1}{-1}\right)\left(\frac{-\sqrt{3}}{2}\right)$

$= \frac{2}{1} - \left(\frac{-3\sqrt{3}}{-2}\right)$

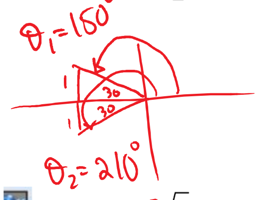
$= \frac{2\cdot 2}{1\cdot 2} - \frac{3\sqrt{3}}{2}$

$= \frac{4-3\sqrt{3}}{2}$




7. Without using the calculator explain how you can find the solutions for angles to the following

a. $\cos\theta = \frac{-\sqrt{3}}{2} = \frac{x}{r}$ $\quad y = \pm 1 \longleftrightarrow \theta_r = 30°$

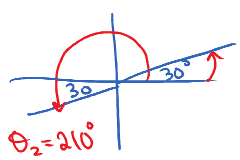



① Find the side y since across from it will be $\theta_r$

② Decide which quadrants the angle(s) are in

③ Find $\theta_1, \theta_2 \ldots$

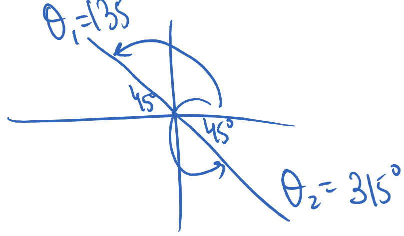
b. $\tan\theta = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$ ← y, ← x $\quad y = 1 \longleftrightarrow \theta_r = 30°$



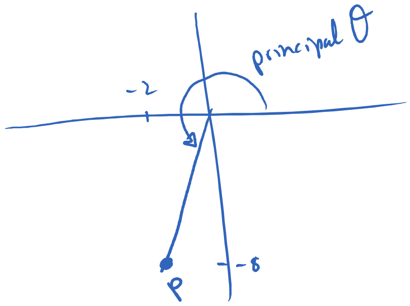

a. $\sin\theta = \frac{-\sqrt{3}}{2} = \frac{y}{r}$

$y = -\sqrt{3} \longleftrightarrow \theta_r = 60°$


b. $\tan\theta = \frac{-1}{1} = \frac{y}{x}$ $\quad y = 1 \longleftrightarrow$ ~~$\theta_r = 30°$~~ $\theta_r = 45°$



8. For the point $P(-2,-8)$ find the exact values of the three primary trig ratios for the principal angle that is made with the terminal arm with point P on it.



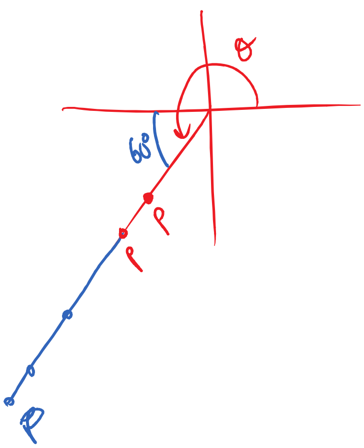

$\sin\theta = \frac{y}{r} = \frac{-8}{2\sqrt{17}} \cdot \frac{\sqrt{17}}{\sqrt{17}} = \frac{-4\sqrt{17}}{17}$

$\cos\theta = \frac{x}{r} = \frac{-\cancel{2}}{\cancel{2}\sqrt{17}} = \frac{-1}{\sqrt{17}} = \frac{-\sqrt{17}}{17}$

$\tan\theta = \frac{y}{x} = \frac{-8}{-2} = 4$

$x^2+y^2=r^2$
$(-2)^2+(-8)^2=r^2$
$4+64=r^2$
$\sqrt{68}=r$
$\sqrt{4}\sqrt{17}=r$
$2\sqrt{17}=r$

9. The angle $\theta$, is in Quadrant III, and $\sin\theta = -\frac{\sqrt{3}}{2}$ $\left(\frac{y}{r}\right)$. Point P lies on the terminal arm. Determine $\theta$, and state at least two possible coordinates for point P.




rough: $\theta_r = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ$

$y=\sqrt{3} \leftrightarrow \theta_r = 60^\circ$

$\therefore \theta = 240^\circ$

point $P(x,y)$

$P_1(-1, -\sqrt{3})$

$2\times$ any scale factor

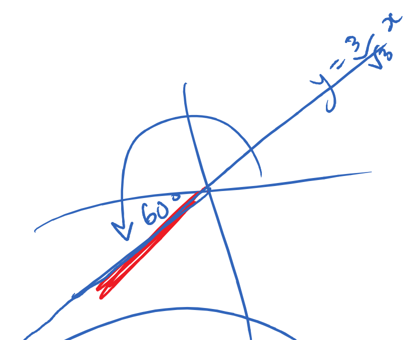
$P_2(-2, -2\sqrt{3})$

$P_3(-8, -8\sqrt{3})$

$\vdots$

$x^2+y^2=r^2$
$x^2+(-\sqrt{3})^2=2^2$
$\vdots$
$x=\pm 1$ on left.

10. The terminal arm of $\theta$ is in quadrant III and on the line $\sqrt{3}y - 3x = 0$. Determine the angle $\theta$ in standard position




$\therefore \theta = 240^\circ$

Rewrite in $y = mx + b$ (isolate $y$)

slope $= \tan\theta$

$\sqrt{3}y = 3x + 0$

$\boxed{y = \frac{3}{\sqrt{3}}x}$ equation of line

$\therefore m = \frac{3 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{\cancel{3}\sqrt{3}}{\cancel{3}} = \sqrt{3}$

OR

$\tan\theta = \frac{\sqrt{3}}{1} \begin{matrix} \leftarrow y \\ \leftarrow x \end{matrix}$

$\therefore y = \sqrt{3} \rightarrow \theta_r = 60^\circ$

## Trigonometric Identities & Proofs

1. Before you begin proving identities, it is important to understand the differences between the following words. Explain with the use of examples what each term means.

| EXPRESSION | EQUATION | IDENTITY |
|---|---|---|
| – no equals sign | – have equals sign | – have equals sign |
| ex. $2x+5$ | ex. $2x+5=7$ | ex. $2x+5-3x=-x+5$ |
| can't solve, can only simplify | – can solve, get unique solution(s) | – can't solve them since every value for $x$ works |
| | | – can prove them. |

2. Show that the following identities are true for any angle

a. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

| LS | RS |
|---|---|
| $\frac{y}{x}$ | $\left(\frac{y}{r}\right) \div \left(\frac{x}{r}\right)$ |
| | mult. by recip. |
| | $\frac{y}{r} \cdot \frac{r}{x}$ |
| | $\frac{y\cancel{r}}{\cancel{r}x} = \frac{y}{x}$ |

$\boxed{\therefore LS = RS}$

b. $\sin^2\theta + \cos^2\theta = 1$

| LS | RS |
|---|---|
| $\left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$ | |
| $\frac{y^2}{r^2} + \frac{x^2}{r^2}$ | |
| $\frac{y^2+x^2}{r^2}$ circle | |
| $\frac{r^2}{r^2} = 1$ | |

$\therefore LS = RS$

3. The proofs to identities are not unique, hence it is important to write out what you do in each step. List some strategies to try when doing proofs.

– change all trig ratios into sine / cosine.

$\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$ $\boxed{\csc\theta = \frac{1}{\sin\theta}}$ $\boxed{\sec\theta = \frac{1}{\cos\theta}}$ $\boxed{\cot\theta = \frac{\cos\theta}{\sin\theta}}$

Quotient identities. Reciprocal iD.

– consider starting from most complicated side

– try factoring or expanding

– try LCD or distribute denominator ⊛ if you see a potential cancellation

– use $\boxed{\sin^2\theta + \cos^2\theta = 1}$ → $1-\sin^2\theta = \cos^2\theta$, → $1-\cos^2\theta = \sin^2\theta$

Pythagorean iD.

• if you see binomials with $\sin\theta$ or $\cos\theta$ no squares then use conjugate(s)

$1-\cos\theta$

conjugate

$1+\cos\theta$

Things that you will lose marks for are:

- Not explaining steps or skipping steps
- For writting terms incorrectly eg. cos, sin, tan without $\theta$, or $\sin\theta^2$ when you mean $\sin^2\theta$ → means $(\sin\theta)^2$
- For incorrectly canceling or simplifying.
- Moving terms over the equals sign. This is not wrong to do, however the proofs in grade 11 are not particularly hard and if terms are moved proofs can become too simple.

4. Prove the following

LS | RS

a. $\sin\theta + \dfrac{\cos\theta}{\tan\theta} = \dfrac{1}{\cos\theta\tan\theta}$

LS:

$\sin\theta + \dfrac{\cos\theta}{\frac{\sin\theta}{\cos\theta}}$ quotient

mult. by recip.

$\sin\theta + \dfrac{\cos\theta}{1}\left(\dfrac{\cos\theta}{\sin\theta}\right)$

$\dfrac{\sin\theta \cdot \sin\theta}{\sin\theta \cdot 1} + \dfrac{\cos^2\theta}{\sin\theta}$

LCD

$\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta}$

pythag. iD.

$\dfrac{1}{\sin\theta}$

RS:

$\dfrac{1}{\cos\theta\left(\frac{\sin\theta}{\cos\theta}\right)}$ quot.

$\dfrac{1}{\sin\theta}$

$\therefore LS = RS$

LS | RS

b. $1-\sin^2\theta = \dfrac{\sin^2\theta}{\tan^2\theta}$

RS:

$\dfrac{\sin^2\theta}{\left(\frac{\sin^2\theta}{\cos^2\theta}\right)}$ quotient. iD.

$\sin^2\theta \div \left(\dfrac{\sin^2\theta}{\cos^2\theta}\right)$

$\sin^2\theta \times \left(\dfrac{\cos^2\theta}{\sin^2\theta}\right)$

mult. by recip.

$\cancel{\sin^2\theta}\left(\dfrac{\cos^2\theta}{\cancel{\sin^2\theta}}\right)$

$\cos^2\theta$

$1-\sin^2\theta$ pythag.

$\therefore LS = RS$

LS RS

c. $\sin^2\theta + 2\cos^2\theta - 1 = \cos^2\theta$

$\underbrace{\sin^2\theta + \cos^2\theta}_{\text{pythag.}} + \cos^2\theta - 1$

$\cancel{1} + \cos^2\theta - \cancel{1}$

$\cos^2\theta$

$\therefore LS = RS$

d. $\dfrac{\sin^2\theta}{1-\cos\theta} = 1+\cos\theta$

$\dfrac{\sin^2\theta}{(1-\cos\theta)} \cdot \dfrac{(1+\cos\theta)}{(1+\cos\theta)}$

mult. by conjugate

FOIL

$\dfrac{\sin^2\theta\,(1+\cos\theta)}{1-\cos\theta+\cos\theta-\cos^2\theta}$

pythag.

$\dfrac{\cancel{\sin^2\theta}\,(1+\cos\theta)}{\cancel{\sin^2\theta}}$

$\therefore LS = RS$

LS | RS

e. $\sec\theta\csc\theta - \cot\theta = \tan\theta$

reciprocal iD.

$\left(\dfrac{1}{\cos\theta}\right)\left(\dfrac{1}{\sin\theta}\right) - \left(\dfrac{\cos\theta}{\sin\theta}\right)$ quot.

$\dfrac{1}{\cos\theta\sin\theta} - \dfrac{\cos\theta}{\sin\theta}\cdot\dfrac{\cos\theta}{\cos\theta}$

LCD

$\dfrac{1-\cos^2\theta}{\cos\theta\sin\theta}$

pythag. iD.

$\dfrac{\sin^{\cancel{2}}\theta}{\cos\theta\cancel{\sin\theta}}$

quotient

$\dfrac{\sin\theta}{\cos\theta}$

$\tan\theta$

$\therefore LS = RS$

LS | RS

f. $(\sin x + \cos x)^2 = 1 + 2\sin x\cos x$

expand FOIL

$\sin^2 x + 2\sin x\cos x + \cos^2 x$

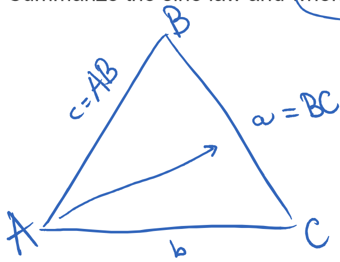
$\underbrace{\sin^2 x + \cos^2 x}_{\text{pythag.}} + 2\sin x\cos x$

$1 + 2\sin x\cos x$

$\therefore LS = RS$

## Sine Law Review & Ambiguous Case

1. Summarize the sine law and when you can use it.



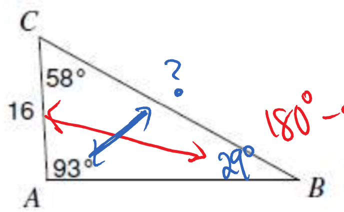

use it if you are given a "pair" of opposite angle + side.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \leftarrow \text{looking for angle}$$

OR

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \leftarrow \text{looking for side}$$

2. Find side BC




$180° - 58° - 93°$

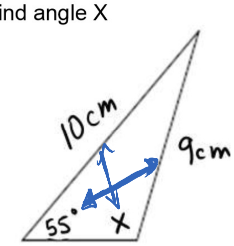
"half cross mult."

$$\frac{16}{\sin 29°} = \frac{BC}{\sin 93°}$$

$$\frac{16 \sin 93°}{\sin 29°} = BC$$

33 units. = BC

3. Find angle X




$$\frac{\sin 55°}{9} = \frac{\sin X}{10}$$

$$\frac{10 \sin 55}{9} = \sin X$$

$$\sin^{-1}\left(\frac{10 \sin 55}{9}\right) = X$$


66° = X not obtuse

want II

X = 180° − 66°

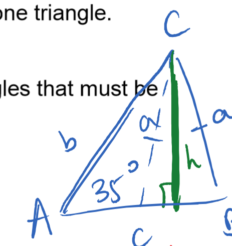
$\boxed{X = 114°}$

4. Sometimes solving for the angles using sine law, the calculator gives you the acute angle, when the problem actually requires an obtuse angle, like in one of the questions above. Why does this happen? And why don't you need to worry about this scenario with cosine law?

Because when you do $\sin^{-1}$(pos) you get acute I OR obtuse II } can create a △

However $\cos^{-1}$(pos) you get acute I OR reflex IV ← will not create a △ for real life scenarios.

Sometimes when the diagram is not given an **ambiguous case** is created. This can happen when you are given SSA. In this situation there can be two possible triangles that can be solved OR no triangles at all OR only one triangle.



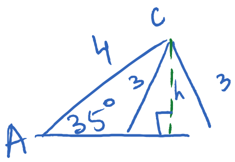

problem: don't know if "a" is long enough OR if it can be placed differently.

finding height would help.

5. Determine if there is no triangle to solve at all, or if there is one triangle or if there are two triangles that must be solved.

across from each other

a. In $\Delta ABC$ $\angle A = 35^o, a = 3, b = 4$



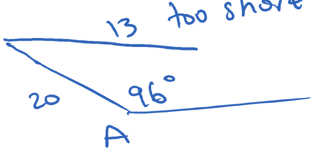

(1.) don't draw side a yet.

(2) find $h = 4\sin 35^\circ = 2.29$

(3.) Ask can "a" fit between 4 and h?

$\boxed{\therefore \text{ 2 triangles possible}}$

b. In $\Delta ABC$ $\angle A = 96^o, a = 13, b = 20$




13 too short

96° is Largest angle side across from it should be longest

$\boxed{\therefore \text{ no } \Delta}$

c. In $\Delta DEF$ $\angle E = 28^o, e = 24, f = 20$

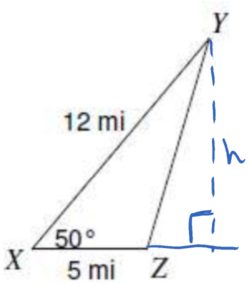

$h = 20\sin 28^\circ$

$h = 9.4$

24 won't fit between 20 and 9.4

$\boxed{\therefore \text{ one } \Delta}$

6. Find the area of the triangle




SOH CAH TOA

$\sin 50^\circ = \frac{h}{12}$

$12\sin 50^\circ = h$

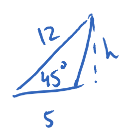
$9.2 = h$

$A = \frac{bh}{2}$

$A = \frac{(5)(12\sin 50)}{2}$

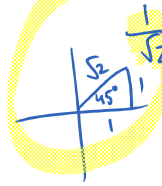
$A \doteq 23 \text{ mi}^2$

7. Same question as 6, just change angle to 45° and do exact value for the area.




$\sin 45^\circ = \frac{h}{12}$

$\frac{1}{\sqrt{2}} = \frac{h}{12}$




$\frac{\sqrt{2}}{2} = \frac{h}{12}$

$\frac{12\sqrt{2}}{2} = h$

$6\sqrt{2} = h$

$A = \frac{bh}{2}$

$A = \frac{(5)(6\sqrt{2})}{2}$

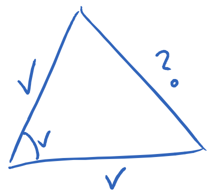
$A = \frac{30\sqrt{2}}{2}$

$A = 15\sqrt{2} \text{ mi}^2$

## Cosine Law Review & 3D Problems

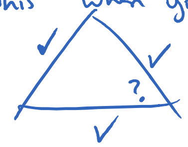
1. Summarize the cosine law and when you can use it.

$c^2 = a^2 + b^2 - 2ab(\cos C)$ ⟷ OR $\cos C = \dfrac{a^2+b^2-c^2}{2ab}$



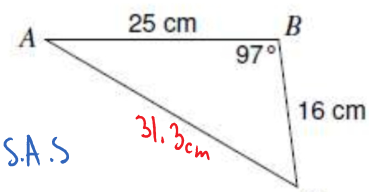

use this version when you're given S.A.S.

use this when you are given S.S.S.




2. Solve each triangle. This means find all sides, all angles.

a.




have S.A.S

find b:

$b^2 = a^2 + c^2 - 2ac\,(\cos B)$

$b^2 = 25^2 + 16^2 - 2(25)(16)\underbrace{\cos 97^\circ}_{\text{neg.}}$

$\sqrt{b^2} = \sqrt{978.49\ldots}$

$b = 31.3\text{ cm}$

have S.S.S

find C

$\cos C = \dfrac{a^2+b^2-c^2}{2ab}$

$\cos C = \dfrac{16^2+31.3^2-25^2}{2(16)(31.3)}$

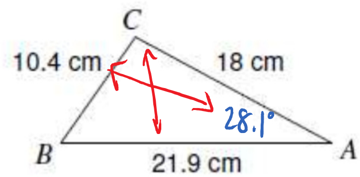
$C = \cos^{-1}\left(\dfrac{16^2+31.3^2-25^2}{2(16)(31.3)}\right)$

$C = 53^\circ$

find A: $180^\circ - 53^\circ - 97^\circ$

$A = 30^\circ$

b.




$\cos A = \dfrac{21.9^2 + 18^2 - (10.4)^2}{2(21.9)(18)}$

$\cos A = \dfrac{695.45}{788.4}$

$A = \cos^{-1}(\ \ )$

$A = 28.1^\circ$

$\dfrac{\sin C}{21.9} = \dfrac{\sin 28.1}{10.4}$

$\sin C = \dfrac{21.9 \sin 28.1^\circ}{10.4}$

$C = 82.7^\circ$ not obtuse like pic!

$\therefore C = 180 - 82.7^\circ$

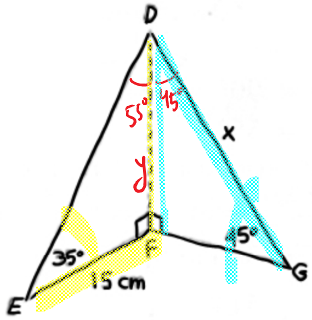
$\angle A = 28.1$

$\angle B = 54.6$

$\angle C = 97.3$

3. Real life situations are almost never flat 2D problems. Solve the following for x.

a.




find y:
SOH CAH TOA (or sine Law)

$\tan 35^\circ = \frac{y}{15}$

$15\tan 35^\circ = y$

$10.5 \doteq y$

find x:
SOH CAH TOA

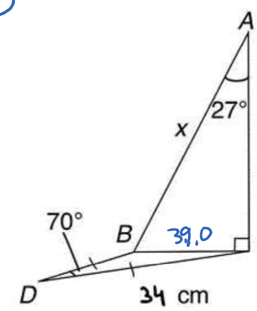
$\sin 45^\circ = \frac{10.5}{x}$

$\sin 45\, x = 10.5$

$x = \frac{10.5}{\sin 45^\circ}$

$x \doteq 14.9\text{ cm}$

b.




find y SAS.

$y^2 = (34)^2 + (34)^2 - 2(34)(34)\cos 70^\circ$

$y^2 = \sqrt{\ldots}$

$y = 39.0032\ldots$

find x
sine law or SOH CAH TOA

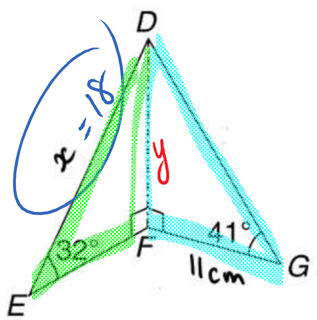
$\frac{\sin 27^\circ}{1} = \frac{39.0032}{x}$

$x \sin 27^\circ = 39.0032$

$x = \frac{39.0032\ldots}{\sin 27^\circ}$

$x = 85.9\text{ cm}$

c.




find y SOH CAH TOA

$\tan 41 = \frac{y}{11}$

$11\tan 41 = y$

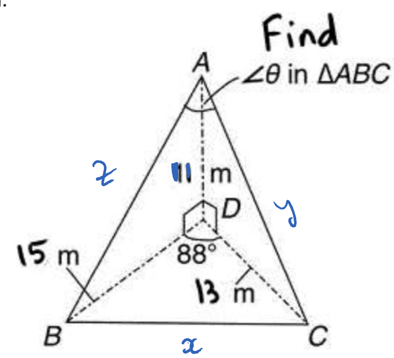
$9.56 \doteq y$

find x SOH CAH TOA

$\sin 32 = \frac{9.56}{x}$

$x = \frac{9.56}{\sin 32} \doteq 18.0$

d.




find x using SAS.

$x^2 = 15^2 + 13^2 - 2(15)(13)\cos 88$

$x = 19.5036\ldots$

find y using pythag.

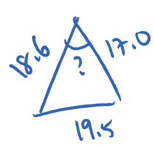
$y^2 = 11^2 + 13^2$

$y = 17.0294\ldots$

find z using pythag

$z^2 = 11^2 + 15^2$

$z = 18.6011\ldots$




find θ using SSS.

$\cos\theta = \frac{18.6^2 + 17.0^2 - 19.5^2}{2(18.6)(17.0)}$

$\theta \approx 66^\circ$