Trigonometry Extra Practice

1. The coordinates of a point P on the terminal arm of an $\angle\theta$ in standard position are given, where $0^o \le \theta \le 360^o$. Determine the exact values of sin θ, cos θ, and tan θ.

a. P(4, 5) b. P(-2, 7) c. (-3, -6) d. P(7, -4)

2. Find the exact value of each trigonometric ratio.

a. $\cos 30^o$ b. $\tan 225^o$ c. $\sin 210^o$ d. $\cos 150^o$

e. $\sin 30^o$ f. $\cos 315^o$ g. $\tan 300^o$ h. $\sin 330^o$

3. Angle θ is in standard position with its terminal arm in the stated quadrant, and $0^o \le \theta \le 360^o$. Find the exact values of the other two trigonometric ratios.

a. $\sin\theta = \frac{2}{5}$, quadrant II b. $\sin\theta = -\frac{2}{5}$, quadrant III c. $\tan\theta = -\frac{5}{6}$, quadrant IV

4. If $0^o \le \theta \le 360^o$, find the possible measure of $\angle$A.

a. $\sin A = \frac{1}{2}$ b. $\cos A = \frac{1}{\sqrt{2}}$ c. $\tan A = -\sqrt{3}$ d. $\tan A = \sqrt{3}$

5. Prove each identity.

a. $\frac{1-\sin^2 x}{\cos x} = \cos x$

b. $\frac{\tan x}{\sin x} = \frac{1}{\cos x}$

c. $\frac{\sin x \cos x}{\tan x} = 1 - \sin^2 x$

d. $\cos^2 x + \frac{\sin x \cos x}{\tan x} = 2\cos^2 x$

e. $1 + \tan^2 x = \frac{1}{\cos^2 x}$

f. $\cos^2 x - \sin^2 x = 2\cos^2 x - 1$

g. $\frac{1}{\sin x} - \sin x = \frac{\cos x}{\tan x}$

h. $\frac{1-\tan^2 x}{1+\tan^2 x} = \cos^2 x - \sin^2 x$

i. $(\sin x - \cos x)(\sin x + \cos x) = 2\sin^2 x - 1$

j. $(\sin x - \cos x)^2 = 1 - 2\sin x \cos x$

k. $1 + \tan^2 x = \frac{1}{\cos^2 x}$

l. $\cos^2 x - \cos^4 x = \cos^2 x \sin^2 x$

m. $\left(1-\cos^2 x\right)\left(1+\tan^2 x\right)=\tan^2 x$

n. $\dfrac{\sin x}{1-\cos x}-\dfrac{1+\cos x}{\sin x}=0$

6. Solve each equation for $0^o \le \theta \le 360^o$.

a. $\cos x = 0$

b. $2\sin x - 1 = 0$

c. $\tan x = -1$

d. $\sqrt{2}\sin x = 1$

e. $2\cos x - 3 = 0$

f. $2\sin x + \sqrt{3} = 0$

g. $\sqrt{2}\cos x + 1 = 0$

h. $\cos x - 1 = 0$

i. $\tan x = \sqrt{3}$

7. Solve each equation for $0^o \le \theta \le 360^o$. (TIPS)

a. $\cos^2 x - 1 = \sin^2 x$

b. $2\cos^2 x + 3\cos x = -1$

c. $\sin^2 x + \sin x - 2 = 0$

d. $\sin x \cos x - \sin x = 0$

e. $2\sin^2 x + 1 = 3\sin x$

f. $\cos x + 1 = 2\sin^2 x$

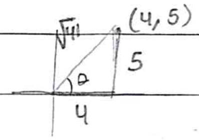
g. $2\cos^2 x - 5\cos x + 3 = 0$

h. $2\sin^2 x - 7\sin x = 4$

i. $\cos^2 x + 4 = 4\cos x$

Aaron

1a)



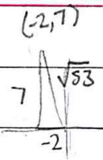
$c^2 = a^2 + b^2$
$c^2 = 5^2 + 4^2$
$c^2 = 25 + 16$
$c = \sqrt{41}$

$\sin\theta = \frac{y}{r} = \frac{5}{\sqrt{41}} = \frac{5\sqrt{41}}{41}$

$\cos\theta = \frac{x}{r} = \frac{4}{\sqrt{41}} = \frac{4\sqrt{41}}{41}$

$\tan\theta = \frac{y}{x} = \frac{5}{4}$

b)



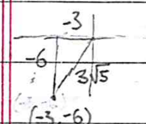
$c^2 = a^2 + b^2$
$c^2 = 7^2 + (-2)^2$
$c^2 = 49 + 4$
$c = \sqrt{53}$

$\sin\theta = \frac{y}{r} = \frac{7}{\sqrt{53}} = \frac{7\sqrt{53}}{53}$

$\cos\theta = \frac{x}{r} = \frac{-2}{\sqrt{53}} = \frac{-2\sqrt{53}}{53}$

$\tan\theta = \frac{y}{x} = \frac{7}{-2} = -\frac{7}{2}$

c)



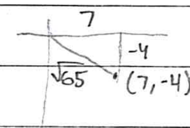
$c^2 = a^2 + b^2$
$c^2 = (-3)^2 + (-6)^2$
$c^2 = 9 + 36$
$c = \sqrt{45} = 3\sqrt{5}$

$\sin\theta = \frac{y}{r} = \frac{-6}{3\sqrt{5}} = \frac{-2}{\sqrt{5}} = \frac{-2\sqrt{5}}{5}$

$\cos\theta = \frac{x}{r} = \frac{-3}{3\sqrt{5}} = \frac{-1}{\sqrt{5}} = \frac{-\sqrt{5}}{5}$

$\tan\theta = \frac{y}{x} = \frac{-6}{-3} = 2$

d)



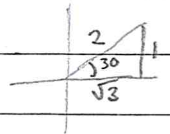
$c^2 = a^2 + b^2$
$c^2 = 7^2 + (-4)^2$
$c^2 = 49 + 16$
$c = \sqrt{65}$

$\sin\theta = \frac{y}{r} = \frac{-4}{\sqrt{65}} = \frac{-4\sqrt{65}}{65}$

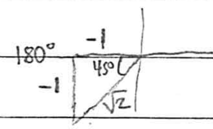
$\cos\theta = \frac{x}{r} = \frac{7}{\sqrt{65}} = \frac{7\sqrt{65}}{65}$

$\tan\theta = \frac{y}{x} = \frac{-4}{7}$

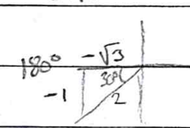
2a) $\cos 30° = \frac{x}{r} = \frac{\sqrt{3}}{2}$



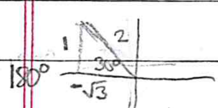
b) $\tan 225° = \frac{y}{x} = \frac{-1}{-1} = 1$



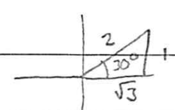
c) $\sin 210° = \frac{y}{r} = \frac{-1}{2}$



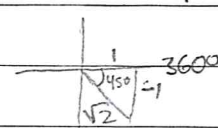
d) $\cos 150° = \frac{x}{r} = \frac{-\sqrt{3}}{2}$



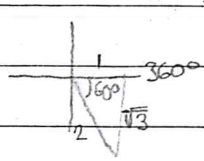
e) $\sin 30° = \frac{y}{r} = \frac{1}{2}$



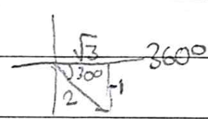
f) $\cos 315° = \frac{x}{r} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$



g) $\tan 300° = \frac{y}{x} = \frac{-\sqrt{3}}{1} = -\sqrt{3}$



h) $\sin 330° = \frac{y}{r} = \frac{-1}{2}$



3a) $\sin\theta = \frac{2}{5}$, quad II

pythagorean ID

$\sin^2\theta = \frac{4}{25}$

$1-\cos^2\theta = \frac{4}{25}$

$-\cos^2\theta = \frac{-21}{25}$

$\cos^2\theta = 21/25$ in quad II

$\cos\theta = -\sqrt{21}/5$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{2/5}{\sqrt{21}/5}$

$= \frac{2}{-\sqrt{21}}$

$\tan\theta = \frac{-2\sqrt{21}}{21}$

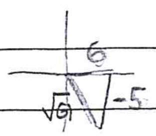
b) $\sin\theta = \frac{-2}{5} \rightsquigarrow \sin^2\theta = 4/25$

see above work

$\cos\theta = -\sqrt{21}/5$ in quad III

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-2/5}{-\sqrt{21}/5} = \frac{2}{\sqrt{21}} = \frac{2\sqrt{21}}{21}$

c) $\tan\theta = -\frac{5}{6}$, quad IV



$c^2 = a^2 + b^2$

$c^2 = 5^2 + (-6)^2$

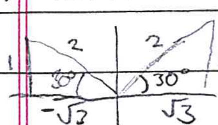
$c^2 = 25 + 36$

$c = \sqrt{61}$

$\sin\theta = y/r = -5/\sqrt{61} = -5\sqrt{61}/61$

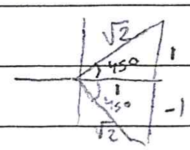
$\cos\theta = x/r = 6/\sqrt{61} = 6\sqrt{61}/61$

4a) $\sin A = 1/2$



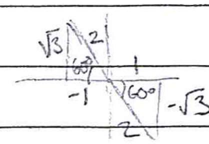
$A = 30°, 150°$

b) $\cos A = 1/\sqrt{2}$



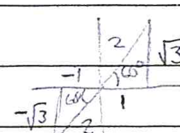
$A = 45°, 315°$

c) $\tan A = -\sqrt{3}$



$A = 120°, 300°$

d) $\tan A = \sqrt{3}$



$A = 60°, 240°$

5a) $\frac{1-\sin^2 x}{\cos x} = \cos x$

Pythagorean ID

$\frac{\cos^2 x}{\cos x} = \cos x$

Simplify

$\cos x = \cos x$

b) $\frac{\tan x}{\sin x} = \frac{1}{\cos x}$

Quotient ID

$\frac{\sin x}{\sin x \cos x} = \frac{1}{\cos x}$

Simplify

$\frac{1}{\cos x} = \frac{1}{\cos x}$

5c) 5c

$$\frac{\sin x\cos x}{\tan x} = 1-\sin^2 x$$ } Pythagorean ID

Quotient ID ↳ $$\frac{\sin x\cos^2 x}{\sin x} = \cos^2 x$$

Simplify ↳ $$\cos^2 x = \cos^2 x$$

d) Quotient ID ↳ $$\cos^2 x + \frac{\sin x\cos x}{\tan x} = 2\cos^2 x$$

Simplify ↳ $$\cos^2 x + \frac{\sin x\cos^2 x}{\sin x} = 2\cos^2 x$$

Simplify ↳ $$\cos^2 x + \cos^2 x = 2\cos^2 x$$

$$2\cos^2 x = 2\cos^2 x$$

e) Pythagorean ID ↳ $$1+\tan^2 x = \frac{1}{\cos^2 x}$$ } Reciprocal ID

$$\sec^2 x = \sec^2 x$$

f) $$\cos^2 x - \sin^2 x = 2\cos^2 x - 1$$ } Pythagorean ID

$$\cos^2 x - (1-\cos^2 x) = 2\cos^2 x - 1$$ } Simplify

$$2\cos^2 x - 1 = 2\cos^2 x - 1$$

g) $$\frac{1}{\sin x} - \sin x = \frac{\cos x}{\tan x}$$ } LCD

$$\frac{1-\sin^2 x}{\sin x} = \frac{\cos x}{\tan x}$$ } Quotient ID

Pythagorean ID ↳ $$\frac{\cos^2 x}{\sin x} = \frac{\cos^2 x}{\sin x}$$

h) $$\frac{1-\tan^2 x}{1+\tan^2 x} = \cos^2 x - \sin^2 x$$

Pythag ID ↳ $$\frac{1-\tan^2 x}{\sec^2 x} = \cos^2 x - \sin^2 x$$

Recip. ID ↳ $$\cos^2 x(1-\tan^2 x) = \cos^2 x - \sin^2 x$$

Distribute + Quotient ID ↳ $$\cos^2 x - \frac{\cos^2 x\sin^2 x}{\cos^2 x} = \cos^2 x - \sin^2 x$$

Simplify ↳ $$\cos^2 x - \sin^2 x = \cos^2 x - \sin^2 x$$

i) $$(\sin x - \cos x)(\sin x + \cos x) = 2\sin^2 x - 1$$

Expand ↳ $$\sin^2 x - \cos^2 x = 2\sin^2 x - 1$$

Pythag ID ↳ $$\sin^2 x - (1-\sin^2 x) = 2\sin^2 x - 1$$

Simplify ↳ $$2\sin^2 x - 1 = 2\sin^2 x - 1$$

j) $$(\sin x - \cos x)^2 = 1 - 2\sin x\cos x$$

Expand ↳ $$\sin^2 x + \cos^2 x - 2\sin x\cos x = 1 - 2\sin x\cos x$$

Pythag ID ↳ $$1 - 2\sin x\cos x = 1 - 2\sin x\cos x$$

same as e) ← k) $$1+\tan^2 x = \frac{1}{\cos^2 x}$$ } Quotient ID

Pythag ID ↳ $$\sec^2 x = \sec^2 x$$

l) $$\cos^2 x - \cos^4 x = \cos^2 x\sin^2 x$$

Factor ↳ $$\cos^2 x(1-\cos^2 x) = \cos^2 x\sin^2 x$$

Pythag. ID ↳ $$\cos^2 x\sin^2 x = \cos^2 x\sin^2 x$$

m) Pythag ID ↳ $$(1-\cos^2 x)(1+\tan^2 x) = \tan^2 x$$ } Quotient ID

$$\sin^2 x\sec^2 x = \frac{\sin^2 x}{\cos^2 x}$$

Recip. ID ↳ $$\frac{\sin^2 x}{\cos^2 x} = \frac{\sin^2 x}{\cos^2 x}$$

n) $$\frac{\sin x}{1-\cos x} - \frac{1+\cos x}{\sin x} = 0$$ } LCD

$$\frac{\sin^2 x - (1+\cos x)(1-\cos x)}{(\sin x)(1-\cos x)} = 0$$ } expand

$$\frac{\sin^2 x - (1-\cos^2 x)}{\text{si}}$$

5n.) $\dfrac{\sin x}{1-\cos x} - \dfrac{1+\cos x}{\sin x} = 0$ LCD

$\dfrac{\sin^2 x - (1+\cos x)(1-\cos x)}{\sin x(1-\cos x)} = 0$ Expand

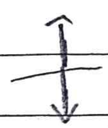
$\dfrac{\sin^2 x - (1-\cos^2 x)}{\sin x(1-\cos x)} = 0$ Pythag ID

$\dfrac{\sin^2 x - \sin^2 x}{\sin x(1-\cos x)} = 0$ Simplify

$\dfrac{0}{\sin x(1-\cos x)} = 0$

$0 = 0$

6a) $\cos x = 0$,

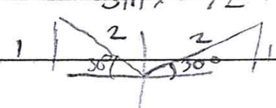


$x = 90°, 270°$

b) $2\sin x - 1 = 0$

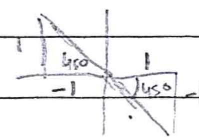
$2\sin x = 1$

$\sin x = 1/2$



$x = 30°, 150°$

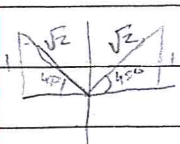
c) $\tan x = -1$



$x = 135°, 315°$

d) $\sqrt{2}\sin x = 1$

$\sin x = 1/\sqrt{2}$



$x = 45°, 135°$

e) $2\cos x - 3 = 0$

$\cos x = 3/2$

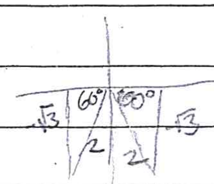
no solution

($\cos x$ ranges from $[-1, 1]$ and $3/2$ is not in this range)

(for $x \in \mathbb{R}$)

f) $2\sin x + \sqrt{3} = 0$

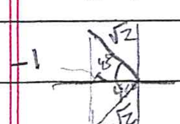
$\sin x = -\sqrt{3}/2$



$x = 240°, 300°$

g) $\sqrt{2}\cos x + 1 = 0$

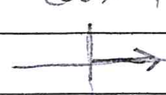
$\cos x = -1/\sqrt{2}$



$x = 135°, 225°$

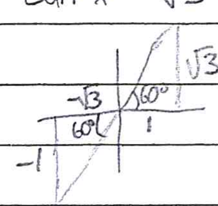
h) $\cos x - 1 = 0$

$\cos x = 1$



$x = 0°, 360°$

i) $\tan x = \sqrt{3}$



$x = 60°, 240°$

7a) $\cos^2 x - 1 = \sin^2 x$

Pythag's $-\sin^2 x = \sin^2 x$

$\sin x = 0$

$x = 0^\circ, 180^\circ, 360^\circ$

b) $2\cos^2 x + 3\cos x = -1$

$2\cos^2 x + 3\cos x + 1 = 0$

2 1

$(2\cos x + 1)(\cos x + 1) = 0$

$2\cos x + 1 = 0$

$\cos x = -1/2$

60°

$x = 120^\circ, 240^\circ$

$\cos x + 1 = 0$

$\cos x = -1$

$x = 180^\circ$

$\therefore x = 120^\circ, 180^\circ, 240^\circ$

c) $\sin^2 x + \sin x - 2 = 0$

1 −1 +2

$(\sin x - 1)(\sin x + 2) = 0$

$\sin x - 1 = 0$

$\sin x = 1$

$x = 90^\circ$

$\sin x + 2 = 0$

no sol.

$x = 90^\circ$

d) $\sin x \cos x - \sin x = 0$

$\sin x(\cos x - 1) = 0$

$\sin x = 0$

$x = 0^\circ, 180^\circ, 360^\circ$

$\cos x - 1 = 0$

$\cos x = 1$

$x = 0^\circ, 360^\circ$

$\therefore x = 0^\circ, 180^\circ, 360^\circ$

e) $2\sin^2 x + 1 = 3\sin x$

$2\sin^2 x - 3\sin x + 1 = 0$

2 −1 −1

$(\sin x - 1)(2\sin x - 1) = 0$

$\sin x - 1 = 0$

$\sin x = 1$

$x = 90^\circ$

$2\sin x - 1 = 0$

$\sin x = 1/2$

30°

$x = 30^\circ, 150^\circ$

$\therefore x = 30^\circ, 90^\circ, 150^\circ$

f) $\cos x + 1 = 2\sin^2 x$

$\cos x + 1 = 2(1 - \cos^2 x)$

$\cos x + 1 = 2 - 2\cos^2 x$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$\cos x = 1/2$

60° 60°

$x = 60^\circ, 300^\circ$

$\cos x = -1$

$x = 180^\circ$

$\therefore x = 60^\circ, 180^\circ, 300^\circ$

g) $2\cos^2 x - 5\cos x + 3 = 0$

2 −3 −1

$(\cos x - 1)(2\cos x - 3) = 0$

$\cos x - 1 = 0$

$\cos x = 1$

$x = 0^\circ, 360^\circ$

$2\cos x - 3 = 0$

$\cos x = 3/2$

no sol.

$\therefore x = 0^\circ, 360^\circ$

h. $2\sin^2 x - 7\sin x = 4$

$2\sin^2 x - 7\sin x - 4 = 0$

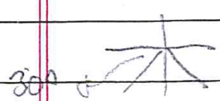
$\frac{1}{2}$ $\quad$ $\frac{1}{-4}$

$(2\sin x + 1)(\sin x - 4) = 0$

$2\sin x + 1 = 0$ $\quad$ $\sin x - 4 = 0$

$\sin x = -1/2$ $\quad$ $\sin x = 4$

no sol.



$x = 210°, 330°$

i) $\cos^2 x + 4 = 4\cos x$

$\cos^2 x - 4\cos x + 4 = 0$

1 $\quad$ -2, -2

$(\cos x - 2)^2 = 0$

$\cos x - 2 = 0$

$\cos x = 2$

no sol.