# Sinusoidals Unit 6

Tentative TEST date________________

## Big idea/Learning Goals

In this unit you will learn how trigonometry can be used to model wavelike relationships. These wavelike functions are called sinusoidal. You will study key properties that these functions have and use these properties to sketch these functions, to model real life situations and to solve trigonometric equations. In grade 12 you will continue studying these functions but instead of degree mode you will learn how to use the radian mode.

Corrections for the textbook answers:
Sec 6.1 #2b) period = 3 #4e) period = 5
Sec 6.3 #6 all should start at MIN #8b) 20cm c) 362cm
Sec 6.6 #13 y=-30cos(1.9x)+30 58.9 without rounding, if you round you can get very different answers!! #14 y=7cos(22.7x)+8
Sec 6.7 #5a) -30cos(18(t-2)) #5b) -9cos(18(t-2)) #9 a) k=1.4

## Success Criteria

- ☐ I am ready for this unit if I am confident in the following review topics
  - ☐ Simplifying expressions
  - ☐ Solving equations
  - ☐ Factoring
  - ☐ SOH CAH TOA
  - ☐ Sine and Cosine laws
  - ☐ Transformations
  - ☐ Function notation
  - ☐ Inverses
  - ☐ Domain & range
  - ☐ Radicals
  - ☐ Unit circle
  - ☐ Exact trig ratios for special angles
  - ☐ Trig identities
- ☐ I understand the new topics for this unit if I can do the practice questions in the textbook/handouts

| Date | pages | Topics | # of quest. done? You may be asked to show them |
|---|---|---|---|
| | 2-4 | Periodic Function Properties<br>Section 6.1 | |
| | 5-7 | Sinusoidal Function Properties<br>Section 6.2 | |
| | 8-10 | Interpreting Sinusoidals<br>Section 6.3 | |
| | 11-14 | Transformations & Sketching Sinusoidals<br>Section 6.4 & 6.5 | |
| | 15-18 | Modelling with Sinusoidals<br>Section 6.6 & Handout | |
| | 19-21 | Solve Problems with Sinusoidals<br>Section 6.7 & Handout | |
| | | REVIEW | |

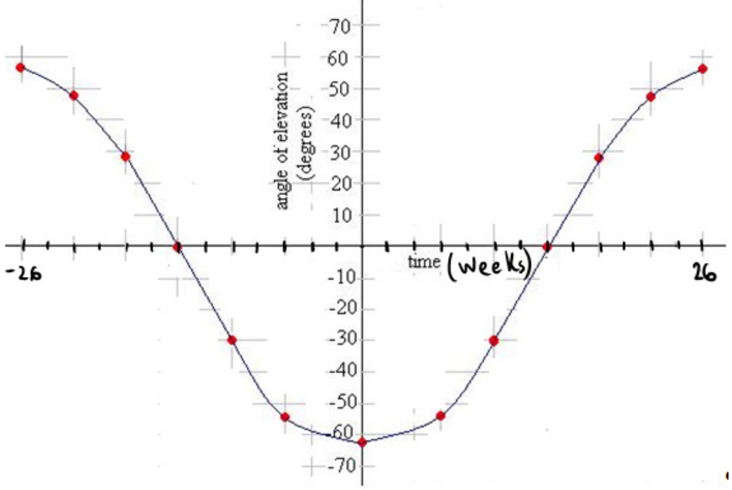
**Reflect** – previous TEST mark ___________, Overall mark now___________.
Looking back, what can you improve upon?

# Periodic Function Properties

There are many situations in real life that repeat in cycles.For example, tides, daylight hours, temperature for the year, heartbeat, volume of air in lungs, rides on ferris wheels, the list can go on. This trend that repeats in cycles is called **periodic** phenomenon. The length of the cycle is called the **period**. The average value of peaks and troughs is the **axis** of the function, and the distance from the axis to the maximum, or from the axis to the minimum is called the **amplitude**.

1. Summarize the equations that you can use to find the period from the graph, the axis, the amplitude and the range.

2. After the sun rises, its angle of elevation increases rapidly at first, then more slowly, reaching a maximum in 26 weeks. Then the angle decreases until sunset.




a. Would you consider this trend periodic? Explain.

b. When does sunrise occur at this time of the year, for this particular spot on Earth?

c. What is the period? What does it represent? What is longer the night or the day for this situation?

d. What is the axis? What does it represent?

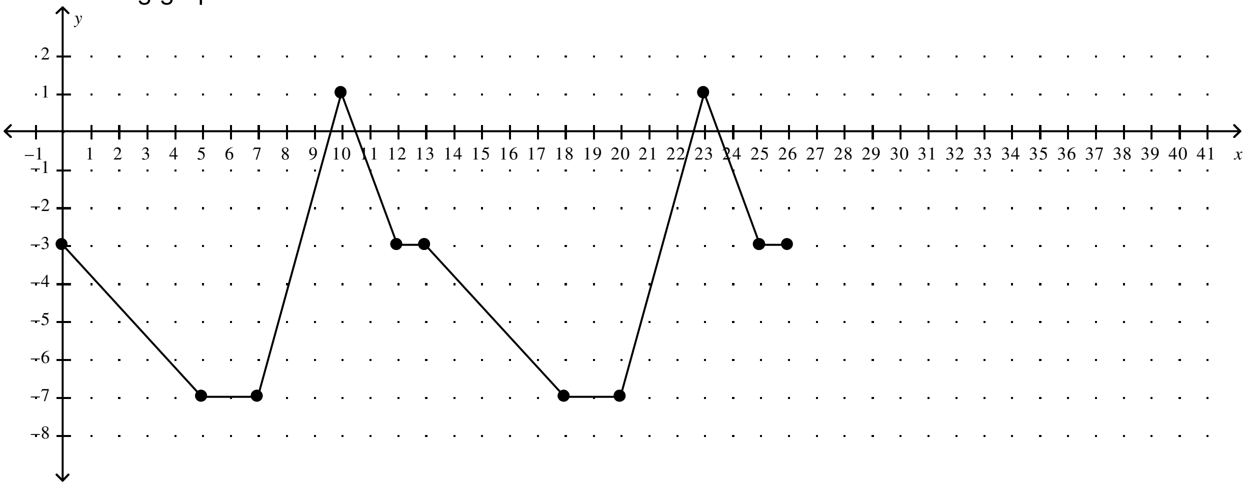
e. What is the amplitude? What does it represent?

f. What is the range? What does it represent?

g. Extrapolate the angle of elevation in 32 weeks and interpolate the angle of elevation in 12 weeks.

h. During what weeks is the angle of elevation of the sun above 30 degrees?

3. The movement of a factory machine that cuts grooves in metal to create a required template pattern is shown on the following graph.



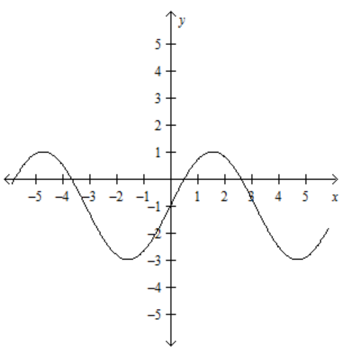

a. Describe what the machine could be doing at each part of the graph. Would you consider this trend periodic?

b. What is the axis, amplitude, range and period?

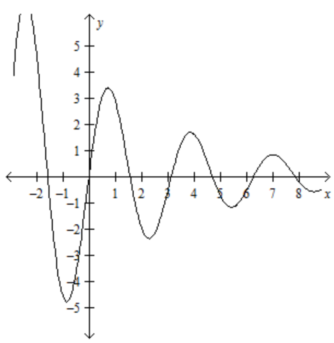
c. How fast is the machine moving on its way up?

4. Decide if each graph or description or table of values is periodic or not.

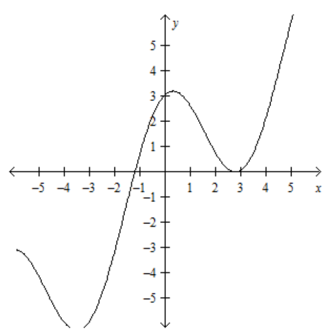
a.



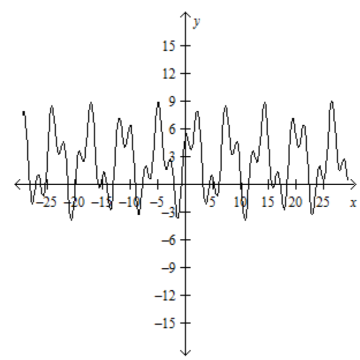
b.



c.



d.



e. Dependent = the horizontal distance travelled by the grandfather clock's pendulum
Independent = time

f. Dependent = interest on the money invested at 5%
Independent = principal deposited

g. Dependent = the height of the pedal on a moving bicycle
Independent = minutes

h.

| x | y |
|---|---|
| -3 | 7 |
| 1 | 4 |
| 5 | 1 |
| 9 | 4 |
| 13 | 7 |
| 17 | 4 |
| 21 | 1 |

i.

| x | y |
|---|---|
| 0 | -2 |
| 2 | 3 |
| 4 | 0 |
| 6 | -2 |
| 8 | -4 |
| 10 | 0 |
| 12 | 3 |

5. For all the graphs and tables of periodic situations state the values of the period, axis, amplitude, and range.

6. Sketch a periodic graph with the following characteristics:
   a. period of 8 units, amplitude of 3, and axis at –2
   b. period or ¾ units and range between –4 and –1

7. A buoy, bobbing up and down in the water of a depth of 16 feet. As waves move past it, the buoy moves from its highest point to its lowest point and back to its highest point. The water on average has 5 waves every 40 seconds. The distance between its highest and lowest points is 3 feet.
   a. Sketch for 3 cycles and state the domain.

   b. State the increasing and decreasing intervals.

# Sinusoidal Function Properties

Some of the questions in the textbook require you to graph with technology. There are lots of applets you can use online, or you can download a free program to use on your computer offline.

Graphing technology often does not have degrees as independent variable. It has radians – which you will learn in gr.12
For now, if the equation has a degree symbol in it set $k = \frac{\pi}{180^o}$ as a multiple of the given k value so that the graph will appear correctly.

**Online Graphing Calculator**
**http://my.hrw.com/math06_07/nsmedia/tools/Graph_Calculator/graphCalc.html**

**Download GeoGebra (offline and online)**
**http://www.geogebra.org/cms/en/download**, select **webstart**, for offline
select **appletstart**, for online

1. Use technology to graph the following and decide if they are periodic or not. Sketch small pictures below.

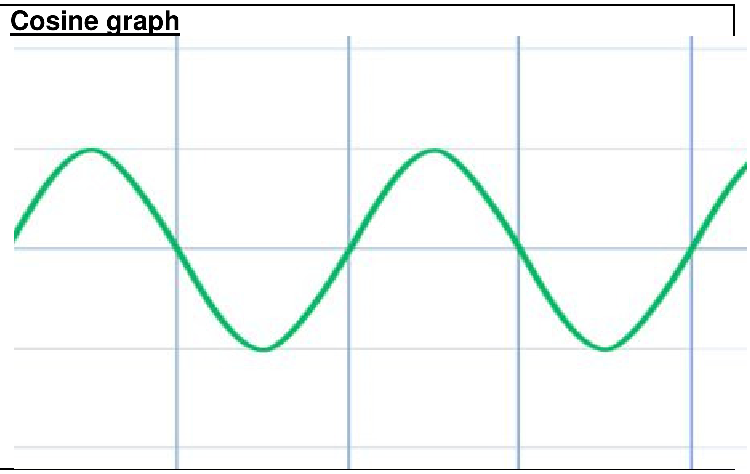
a. $y = (1.1)^x \sin 2x$
b. $y = 10\sin x - 15$
c. $y = \tan x$
d. $y = 4\sin^2 2x - \cos x$
e. $y = -3\cos(2x - 4)$

2. **Sinusoidal** graphs resemble a regular symmetrical looking wave. Only two of the above periodic functions are considered sinusoidal. Can you guess which ones?

3. It is time to develop the parent graphs of the functions that give a sinusoidal wave. Place and label the x and y axes in the following graphs in the proper places so that it would represent a sine function and a cosine function.

**Sine graph**

**Cosine graph**



4. For each function, outline one complete cycle and highlight the 5 key points on the cycle. State reminders of how to begin sketching each type of graph.

5. Recall the meaning of sine and cosine ratios. Connect these meanings to why sine starts at the axis and cosine starts at max.

6. In general any point on the circle can be defined as
   a. For unit circles, of radius 1:

   b. For circles of radius r:

   c. What is the new location of the point (6, 0) that was rotated about 55°?

7. For $f(x)=\sin x$ and $g(x)=\cos x$ sketch both graphs on the same grid

Find all the x values in the domain $-180^o \le x \le 180^o$ such that
   a. $f(x)=0$

   b. $g(x)=-1$

   c. $f(x)=g(x)$

8. All towers and skyscrapers are designed to sway with the wind. When standing on the glass floor of the CN tower the equation of the horizontal sway is $h(x) = 40\sin(30.023x)°$, where h is the horizontal sway in centimetres and x is the time in seconds.

   a. Use table of values to help you graph this:

| x | h |
|---|---|
| 0 | |
| 3 | |
| 6 | |
| 9 | |
| 12 | |
| 15 | |

   b. What is the period? What does it represent?

   c. State the maximum and minimum values of sway and the times at which they occur.

   d. State the mean value of sway and the time at which it occurs.

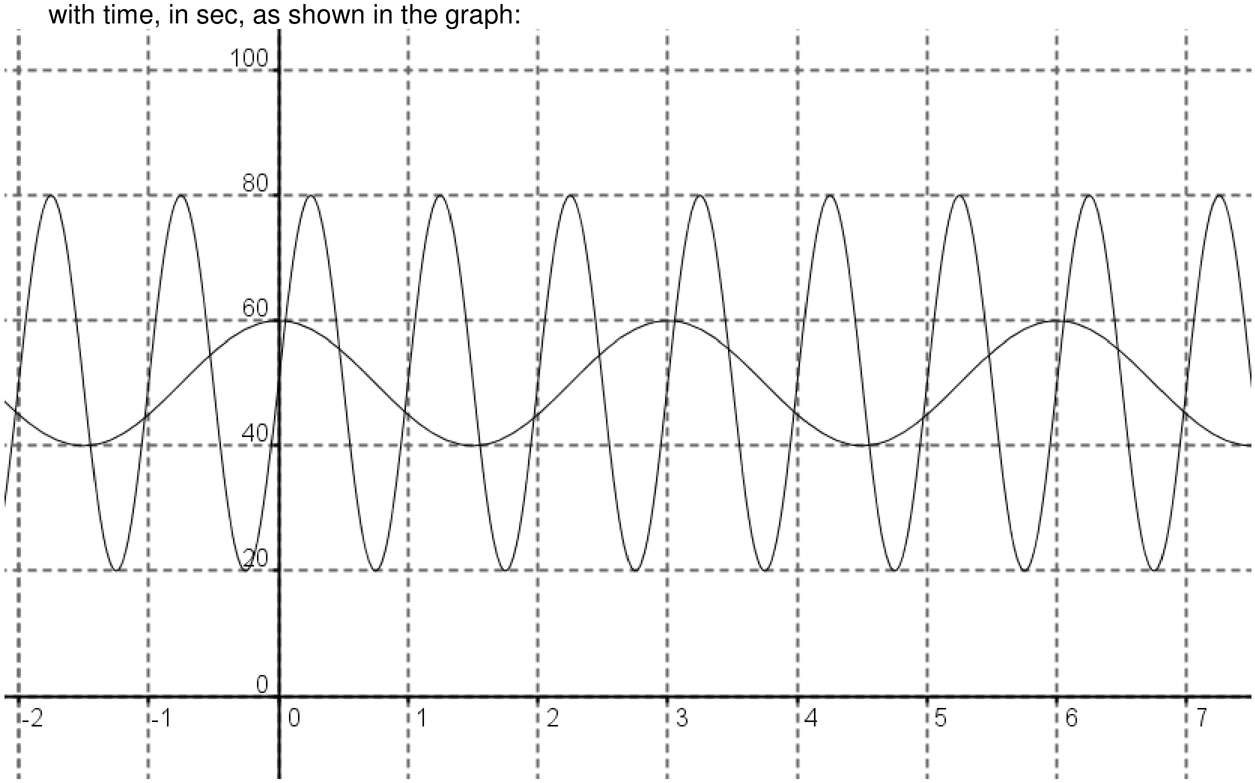
   e. If a guest arrives on the glass floor at time = 0, how many seconds will have elapsed before the guest has swayed 20 cm from the horizontal?

USING GRAPH: USING EQUATION

   f. Find $h(2.034)$, what does it represent?

# Interpreting Sinusoidals

1. Two weights attached to the end of two springs are bouncing up and down. As they bounce their height, in cm, varies with time, in sec, as shown in the graph:

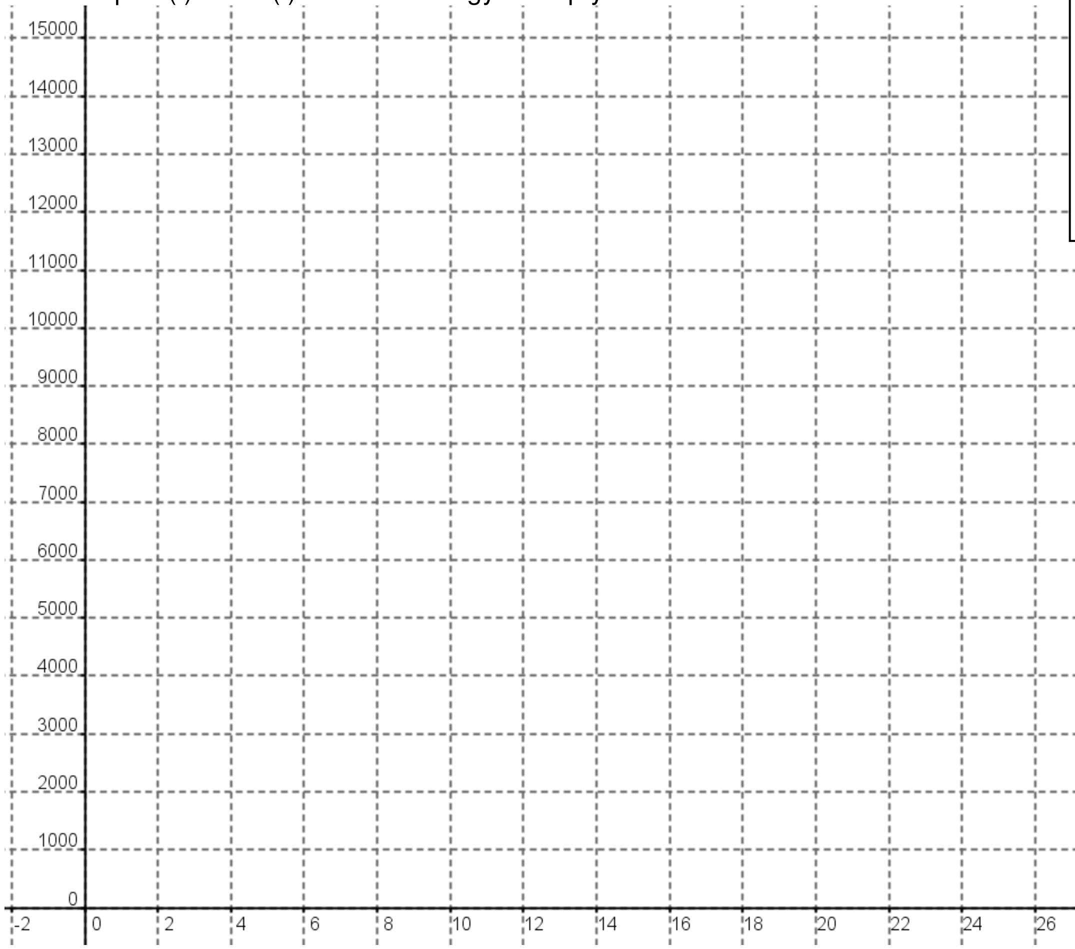



Compare and contrast the bounces of these weights.

2. The population, F, of foxes in the region is modelled by the function $F(t) = 500\sin(15t)° + 1000$, where t is the time in months. The population, R, of rabbits in the same region is modelled by the function, $R(t) = 5000\sin(15t + 90)° + 10000$

> Graphing technology often does not have degrees as independent variable. It has radians – which you will learn in gr.12
>
> For now, if your equation has a degree symbol in it set $k = \dfrac{\pi}{180^o}$ as a multiple of the given k value value so that the graph will appear correctly.

a. Graph F(t) and R(t). Use technology to help you.

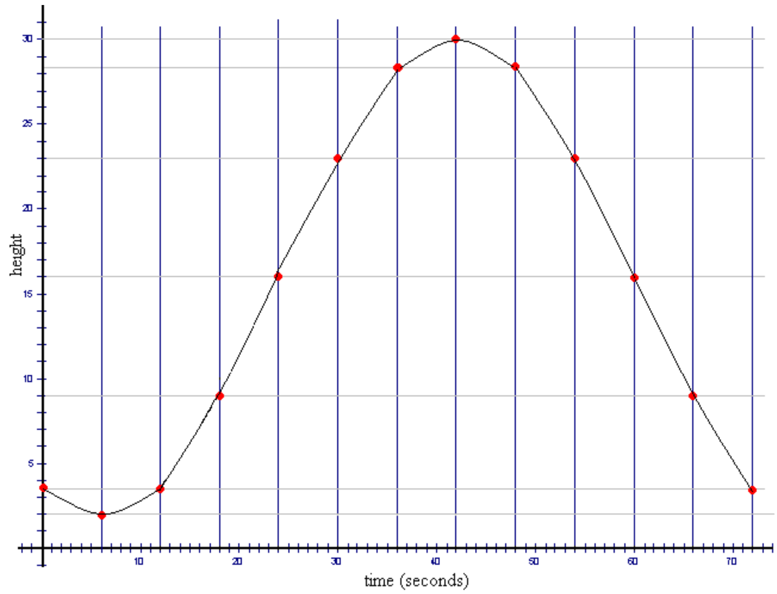



b. State the maximum and minimum values and the month in which they occur for both species in the chart below

| | Month for Max | Max Value | Month for Min | Min Value | Month for Mean | Mean Value |
|---|---|---|---|---|---|---|
| Fox | | | | | | |
| Rabbit | | | | | | |

c. Describe the relationships between the maximum, minimum and mean points of the two curves in terms of the lifestyles of the rabbits and foxes and list possible causes for the relationships.

3. As you ride a Ferris wheel, your distance from the ground varies sinusoidally with time according to the equation $h(t) = 14\sin(5(t-24))^{o} + 16$ where h is height in meters and t is time in seconds. The graph of this model is below




a. What is the radius of the wheel? What part of the equation gives you this?

b. Where is the centre of the wheel located? What part of the equation gives you this?

c. How long does it take for this Ferris wheel to complete one full revolution? What part of the equation gives you this?

d. If the Ferris wheel was sped up, what part of the equation will change?

e. If the Ferris wheel rotated in the other direction, sketch the resulting graph on the same grid. What part of the equation will change?

f. How far off the ground did you board the Ferris wheel?

g. You took a video of the whole ride. The video is 7 minutes 12 seconds long, how many revolutions did the wheel make during this ride?

h. At 3 minutes and 14 sec of the video you were level with a nearby building, how tall is that building?

i. When was the last time you were at maximum height before you had to get off?

# Transformations & Sketching Sinusoidals

1. In real life it is very rare to have degrees as the independent variable. Usually it is time or horizontal distance. What type of transformation can cancel the degrees out?

2. You can also tell from the equation whether degrees is the unit to be used on the x-axis. Look at the placement of the degree symbol in the equation and decide whether x represents degrees or if the degrees are cancelled out.

a. $y = \sin 3^o x$  b. $y = \sin(x - 7^o)$  c. $y = \sin x + 5$  d. $y = \sin(2x - 4)^o + 8$

Note: In grade 12, you will learn how to use radian mode and you will no longer have to write the degree symbol, since radian measure actually has no units. Therefore, equations with no units, like $y = \sin x + 5$, are assumed to be in radians unless within the question it says the domain is $0^o \le x \le 360^o$.

3. Review what kind of transformations do the constants control in the equations $y = a\cos(k(x-d)) + c$ or $y = a\sin(k(x-d)) + c$.

4. State the transformations of the following.

a. $y = -2\cos(0.75x)^o + 3$  b. $y = 2.5\sin(2x - 60^o)$

5. State the domain and range for the functions above if you are told you want to have 3 cycles in the domain.

You can sketch sinusoidals in 3 ways:

- by applying step-by-step transformations to the parent function using different colour for each step OR
- by using a table of values OR
- by using key characteristics

6. PRACTICE sketching using different methods

a. $y = -2\cos(0.75x)^{o} + 3$ - step by step method using different colours for each transformation, applied in correct order

b. $y = 2.5\sin(2x - 60^{o})$ - table of values method

7. Summarize how to sketch by finding all the key characteristics from a given equation.

8. State all the key characteristics of the following then sketch

a. $y = -2\sin(3\theta + 180) - 4$

b. $y = 3\cos(2x - 120)^o + 5$

c. $y=-50\cos(2\theta-270)-10$

d. $y=235\sin(8x+288)^{o}+300$

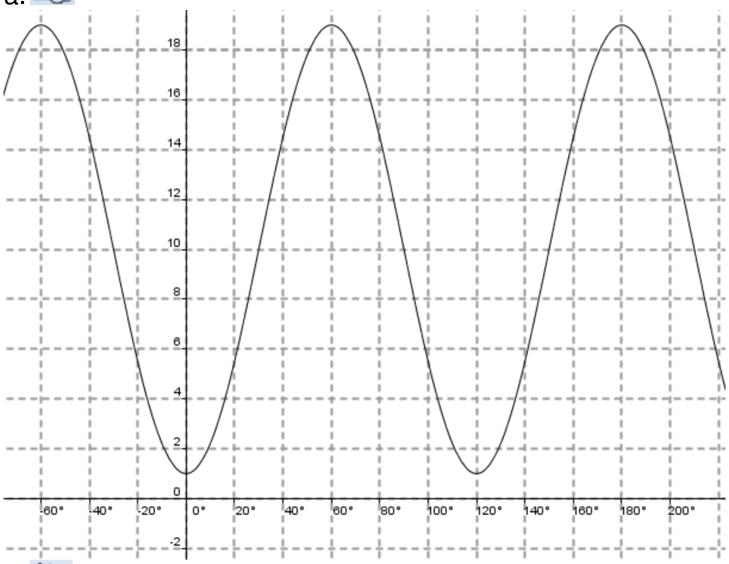
9. Find the equation for cosine graph if you are told that the range is $-6\le y\le 2$ and period is $240^{o}$. Assume there are no reflections or horizontal shifts.

# Modelling with Sinusoidals

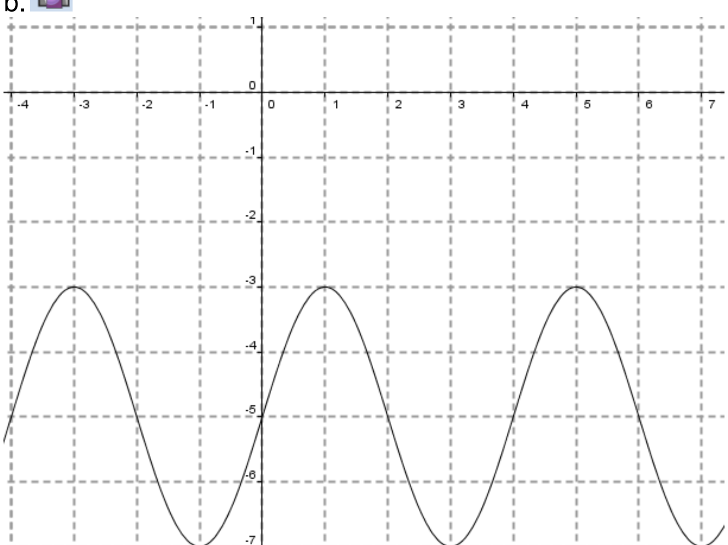
1. Summarize how you can find the equation from a given graph.

2. Find two possible equations for sine and two possible equations for cosine for each of the following

a.



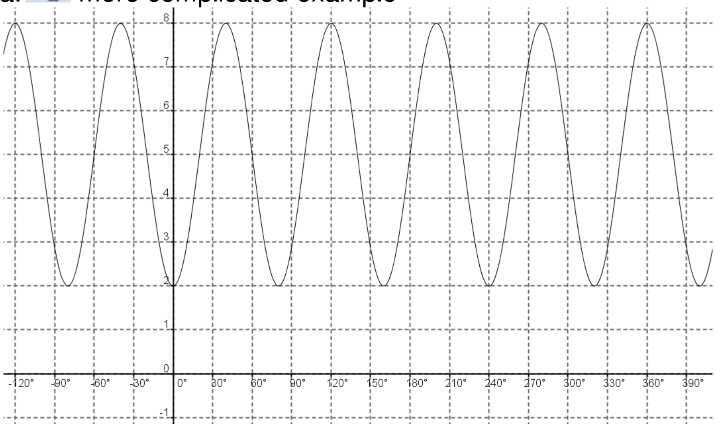

b.



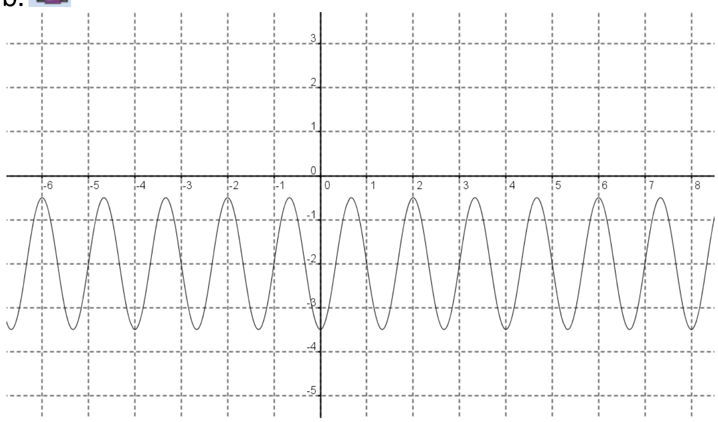

3. In the graphs above, notice that x-axis is labelled differently, how does that affect the equation that you write down?

4. Find one possible equation for sine and one for cosine for each of the following

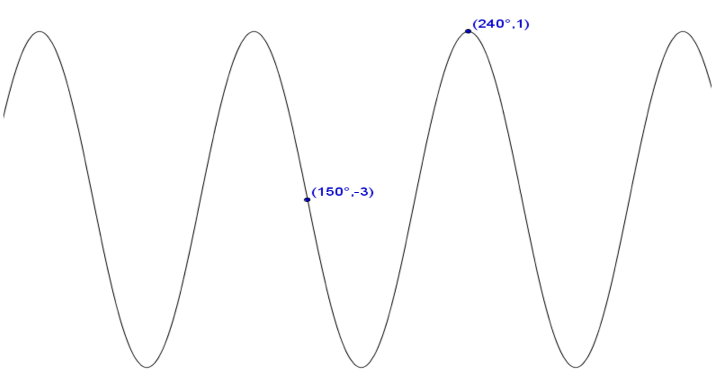
a.  more complicated example




b. 



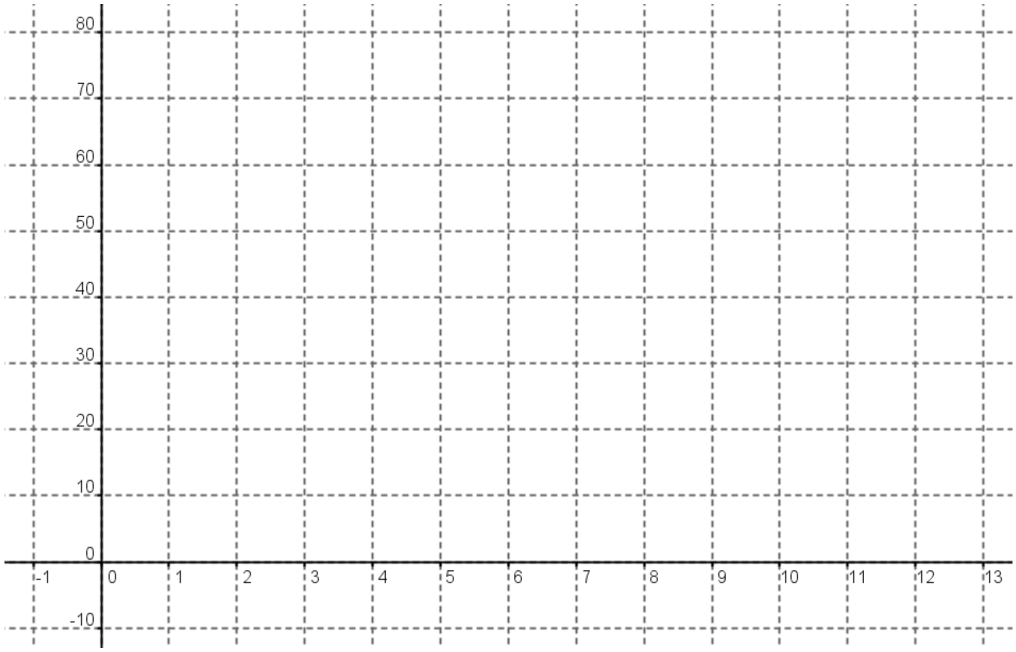

 5. Find an equation for the following graph



6. The average monthly temperature of a city in degrees Fahrenheit are given below

| Jan | Feb | Mar | Apr | May | June | July | Aug | Sept | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 32 | 35 | 44 | 53 | 63 | 73 | 77 | 76 | 69 | 57 | 47 | 37 |

a. Sketch the data




b. Find an approximate equation that would model this data. Assume that x=1 is the start of January.

c. Use the equation to find the approximate monthly temperature for the middle of August.

d. Most of the households turn on the heat if the temperature falls below 64°F. For what domain do most households use heating in their homes?

e. What part(s) of the equation will change if the data was taken from a warmer climate?

7. Summarize how to find the period and the k value if you are given the speed in revolutions per second. Use the example, 10 revolutions in 35 seconds to help you explain.

8. A conveyor belt is powered by two circular pulleys, one of radius 5m the other of 3m. Both pulleys have serial numbers on them, but when the conveyor belt starts, the serial numbers are in different positions. The bigger pulley, A, has the serial number starting at maximum height, the smaller pulley, B, has the serial number starting at minimum height which is 2m away from the floor. The pulleys are also spinning at different speeds but have the same minimum height. Pulley A completes 5 revolutions in 2 minutes. Pulley B is faster, completing 5 revolutions in a minute.
   a. Find the period for each pulley.

   b. Sketch both functions.

   c. Find the equations that model the heights.

   d. Analyze the meaning of all constants and variables for the equation that describes pulley A, in the context of the problem.

## Solve Problems with Sinusoidals

1. A weight attached to the end of a long spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. You start a stopwatch. When the stopwatch reads 0.3 seconds, the weight first reaches a high point 60 cm above the floor. The next low point, 40 cm above the floor, occurs at 1.8 seconds. (Don't assume that at time zero the weight is at the minimum!)
   a. Sketch a graph of this sinusoidal function.

   b. Write an equation expressing distance from the floor in terms of the number of seconds the stopwatch reads.

   c. What was the distance from the floor when you started the watch?

   d. Predict the time at which the weight was 59 cm above the floor for the first time and for the second time.

2. You are on an 8-seat Ferris wheel at an unknown height when the ride starts. It takes you 24 seconds to reach the top of the wheel 26m above the ground. The loading platform is 4m high. The wheel makes a complete revolution in 60 seconds.
   a. Create a diagram/sketch to show your location on the ride and the key features of the wheel.

   b. Find the sinusoidal equation that models your ride on the Ferris wheel.

   c. How high above the ground will your seat be after 90 seconds?

   d. Find the two times within one cycle when the height is at 24m.

HINT for TIPS p372 #8 text uses distance travelled. Use circumference formula $C = 2\pi r$ for horizontal distance travelled by the wheel.

3. You are at Risser's Beach, N.S., to search for interesting shells. At 2:00 p.m. on June 19, the tide is in (i.e., the water is at its deepest). At that time you find that the depth of the water at the end of the breakwater is 15 meters. At 8:00 p.m. the same day when the tide is out, you find that the depth of the water is 11 meters. Assume that the depth of the water varies sinusoidally with time.

   a. Derive an equation expressing depth in terms of the number of hours that have elapsed since 12:00 noon on June 19.

   b. Use your mathematical model to predict the depth of the water at 7:00 a.m. on June 20.

   c. At what time will the first low tide occur on June 20?

   d. What is the earliest time on June 20 that the water will be at 12.7 meters deep?