S14SinusoidalsNOTESnew

July 9, 2014 2:53 PM

SinusoidalsNOTESnew

see below

Sinusoidals Unit 6

Tentative TEST date______________

Big idea/Learning Goals

In this unit you will learn how trigonometry can be used to model wavelike relationships. These wavelike functions are called sinusoidal. You will study key properties that these functions have and use these properties to sketch these functions, to model real life situations and to solve trigonometric equations. In grade 12 you will continue studying these functions but instead of degree mode you will learn how to use the radian mode.

Corrections for the textbook answers:
Sec 6.1 #2b) period = 3 #4e) period = 5
Sec 6.3 #6 all should start at MIN
Sec 6.6 #13 y=-30cos(1.9x)+30 58.9 without rounding, if you round you can get very different answers!! #14 y=7cos(22.7x)+8
Sec 6.7 #5a) -30cos(18(t-2)) #5b) -9cos(18(t-2)) #9 a) k=1.4

Success Criteria

☐ I understand the new topics for this unit if I can do the practice questions in the textbook/handouts

take up practice Quiz + Day 11

Day 12 + start seq.

Date	pages	Topics	# of quest. done? You may be asked to show them	Questions I had difficulty with ask teacher before test!
	2-4	Periodic Function Properties Section 6.1 #2,3,4,5,6,8,9,11		
	5-8	Sinusoidal Functions – 2 days Section 6.2 & 6.3 6.2 # 3,4,9,12 6.3 # 1,5,6,8		
	9-12	Transformations & Sketching Sinusoidals Section 6.4 & 6.5 #6,7,10		
	13-19	Modelling & Problem Solving with Sinusoidals – 3 days Section 6.6 & 6.7 Handouts 6.6 Handout, all	6.7 Handout	ALL
		REVIEW		

Reflect – previous TEST mark __________, Overall mark now__________.

Periodic Function Properties

There are many situations in real life that repeat in cycles.For example, tides, daylight hours, temperature for the year, heartbeat, volume of air in lungs, rides on ferris wheels, the list can go on. This trend that repeats in cycles is called **periodic** phenomenon. The length of the cycle is called the **period**. The average value of peaks and troughs is the **axis** of the function, and the distance from the axis to the maximum, or from the axis to the minimum is called the **amplitude**.

pos.

1. Summarize the equations that you can use to find the period from the graph, the axis, the amplitude and the range.

period = pick 2 points that are in the same place of adjacent cycles

pos. $\boxed{\text{period} = \text{right pt. } x - \text{left pt. } x}$

$\boxed{\text{axis} = \frac{MAX + MIN}{2}}$

pos. amplitude = MAX − axis = axis − MIN = $\frac{MAX - MIN}{2}$

Range = {y∈R, MIN ≤ y ≤ MAX}

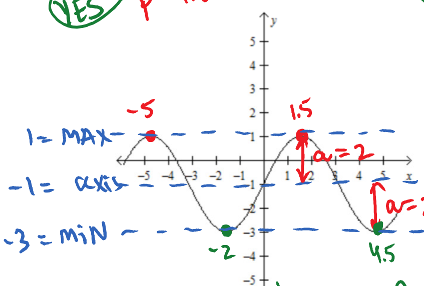
2. Decide if each graph or description or table of values is periodic or not.

a. YES p = 1.5 − −5 = 6.5

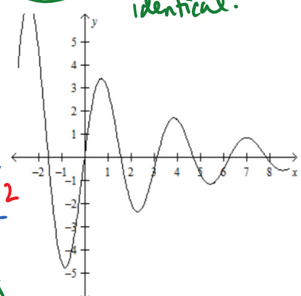
b. NO − amplitude not identical.

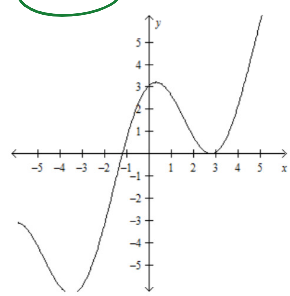
c. NO

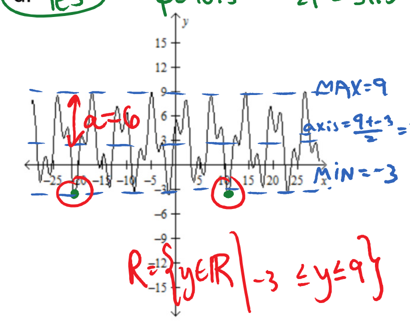
d. YES p = 10.5 − −21 = 31.5



R={y∈R | −3 ≤ y ≤ 1}







R={y∈R | −3 ≤ y ≤ 9}

e. Dependent = the horizontal distance travelled by the grandfather clock's pendulum y
Independent = time x YES.

f. Dependent = interest on the money invested at 5%
Independent = principal deposited NO

g. Dependent = the height of the pedal on a moving bicycle
Independent = minutes YES assume constant speed.

h. YES.

x	y
-3	7
1	4
5	1
9	4
13	7
17	4
21	1

i. NO for what we see.

x	y
0	-2
2	3
4	0
6	-2
8	-4
10	0
12	3

3. For all the graphs and tables of **periodic** situations state the values of the period, axis, amplitude, and range.

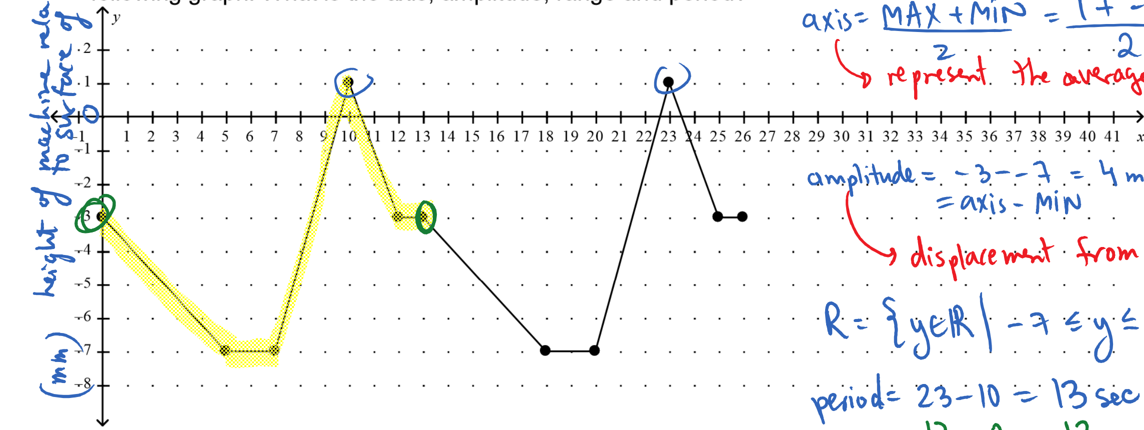
(h) period = 13 − −3 = 16 21 − 5 = 16

axis = $\frac{7+1}{2} = \frac{8}{2} = 4$

amp = MAX − axis = 7 − 4 = 3

Range = {y∈R | 1 ≤ y ≤ 7}

4. The movement of a factory machine that cuts grooves in metal to create a required template pattern is shown on the following graph. What is the axis, amplitude, range and period?



axis $= \frac{MAX + MIN}{2} = \frac{1 + -7}{2} = -3$ mm

represent the average height.

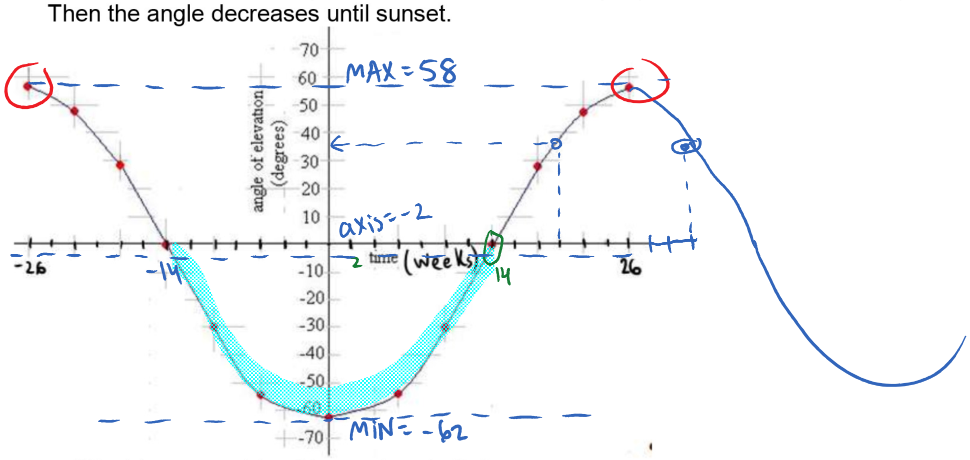
amplitude $= -3 - -7 = 4$ mm
$= $ axis $-$ MIN

displacement from mean (average)

$R = \{y \in \mathbb{R} \mid -7 \le y \le 1\}$

period $= 23 - 10 = 13$ sec
$13 - 0 = 13$

5. After the sun rises, its angle of elevation increases rapidly at first, then more slowly, reaching a maximum in 26 weeks. Then the angle decreases until sunset.



a. Would you consider this trend periodic? YES.

b. When does sunrise occur at this time of the year, for this particular spot on Earth?

14 weeks from now

c. What is the period? What is longer the night or the day for this situation?

$p = 26 - -26 = 52$ weeks $= 1$ yr.

Nighttime $= 14 + 14 = 28$ weeks
Daytime $= 24$ weeks.

d. What is the axis? What does it represent?

axis $= \frac{58 + -62}{2} = -2°$ – average angle of the sun

e. What is the amplitude?

amplitude $=$ MAX $-$ axis $= 58 - -2 = 60$

f. What is the range?

range $= \{y \in \mathbb{R} \mid -62 \le y \le 58\}$

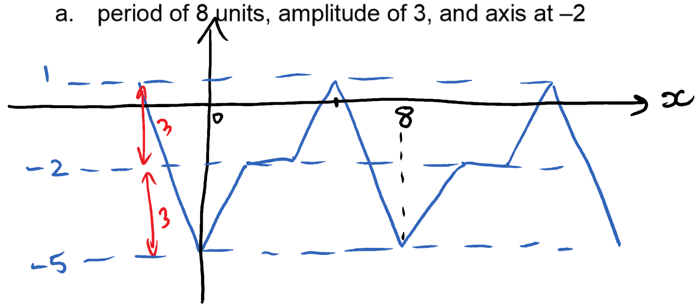
g. Extrapolate the angle of elevation in 32 weeks

beyond the data

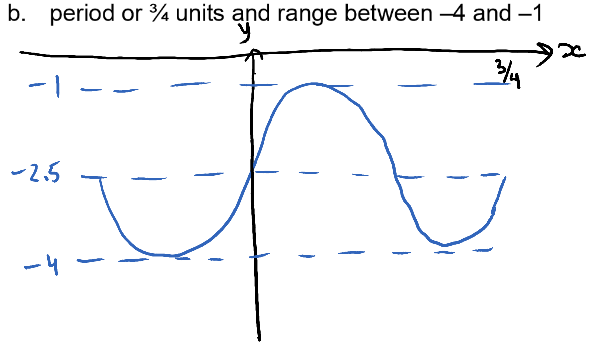
About 37 degrees (the angle of elevation)

6. Sketch a periodic graph with the following characteristics:

   a. period of 8 units, amplitude of 3, and axis at –2



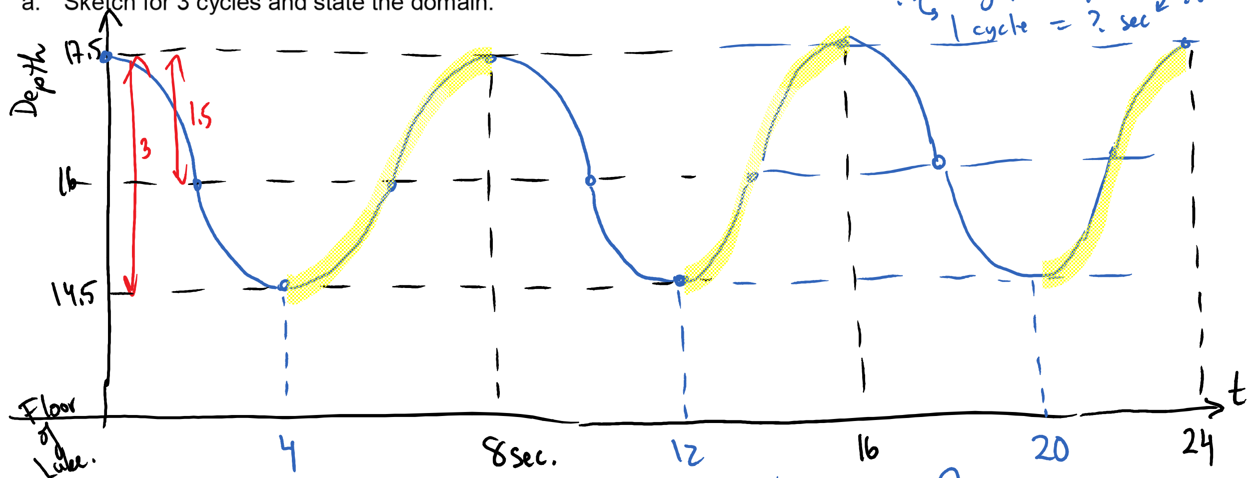
   b. period or ¾ units and range between –4 and –1



7. A buoy, bobbing up and down in the water of a depth of 16 feet. As waves move past it, the buoy moves from its highest point to its lowest point and back to its highest point. The water on average has 5 waves every 40 seconds. The distance between its highest and lowest points is 3 feet.

   a. Sketch for 3 cycles and state the domain.

÷5 5 cycles = 40 sec ÷5
1 cycle = ? sec



$D = \{t \in \mathbb{R} \mid 0 \le t \le 24\}$

   b. State the increasing and decreasing intervals.

increasing on: $4 \le t \le 8$, $12 \le t \le 16$, $20 \le t \le 24$

decreasing: $0 \le t \le 4$, $8 \le t \le 12$, $16 \le t \le 20$

Sinusoidal Function Properties

Some of the questions in the textbook require you to graph with technology. There are lots of applets you can use online desmos.com

1. Use technology to graph the following and decide if they are periodic or not. Sketch small pictures below.

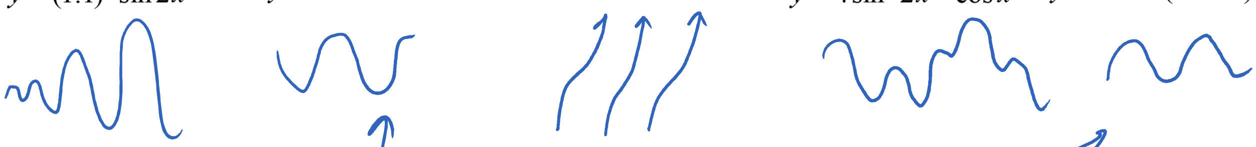
a. not periodic $y=(1.1)^x \sin 2x$

b. periodic $y=10\sin x-15$

c. periodic $y=\tan x$

d. periodic $y=4\sin^2 2x-\cos x$

e. periodic $y=-3\cos(2x-4)$

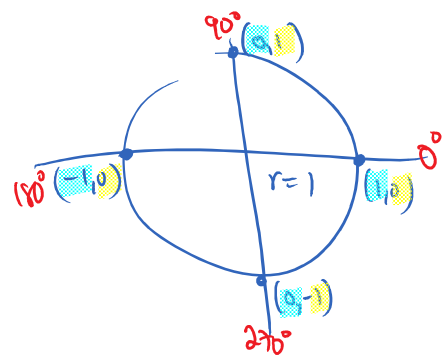


2. **Sinusoidal** graphs resemble a simplest symmetrical looking waves. Only two of the above periodic functions are considered sinusoidal. Can you guess which we will study in this course?

b , e are both periodic AND sinusoidal.

3. Recall the meaning of sine and cosine ratios and fill in the table

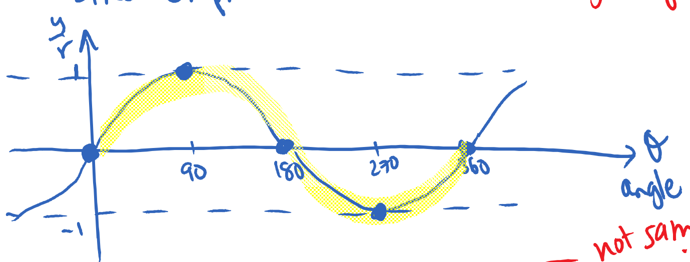
θ → x	0°	90°	180°	270°	360°
sinx	$\sin 0° = \frac{y}{r} = \frac{0}{1}$	$\sin 90° = 1$	$\sin 180° = 0$	$\sin 270° = -1$	$\sin 360° = 0$
cosx	$\cos 0° = \frac{x}{r} = \frac{1}{1}$	0	−1	0	1

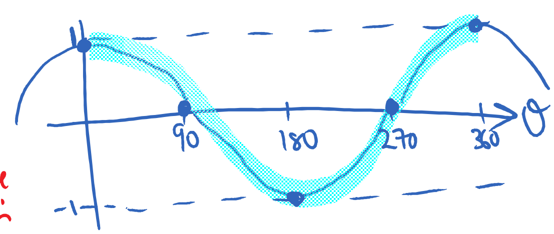


4. PARENT GRAPHS of the functions that give a sinusoidal wave. For each function, outline one complete cycle and highlight the 5 key points on the cycle. State reminders of how to begin sketching each type of graph.

Sine Graph :→ starts at axis then goes up

Cosine Graph → starts at MAX then goes down





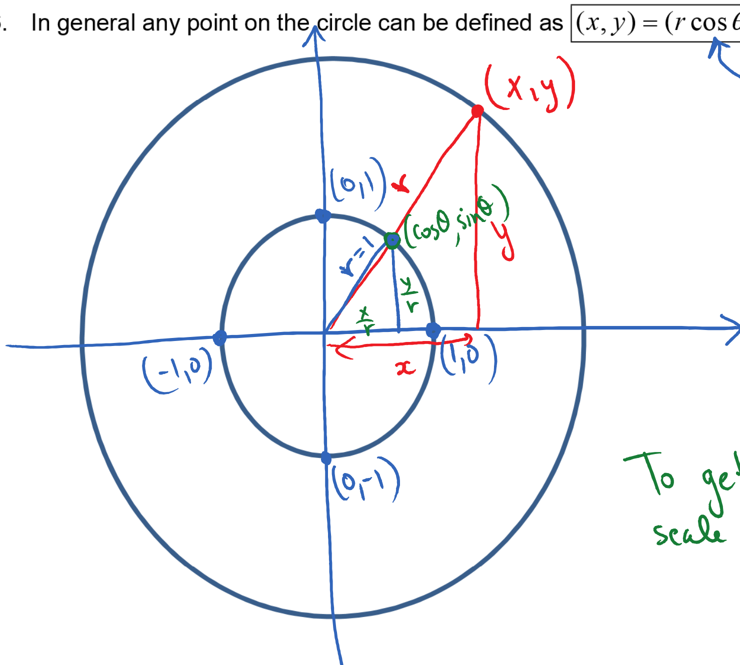
not same :(

5. Note about Input and output variables

y = output of the ratio (for sinθ it's y/r)

x = input of the ratio (the angle θ)

6. In general any point on the circle can be defined as $\boxed{(x,y)=(r\cos\theta, r\sin\theta)}$. Explain why.

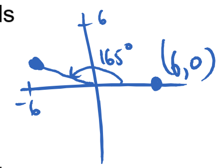


$\sin\theta = \frac{y}{r}$ $\quad\cos\theta = \frac{x}{r}$

$r\sin\theta = y$ $\quad r\cos\theta = x$

To get the small Δ labels scale down the big Δ by "r" (÷r)

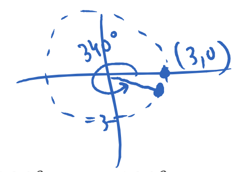
a. What is the new location of the point (6, 0) that was rotated about 165°? approximate to 2 decimals



$r = 6$

new pt $= (r\cos\theta, r\sin\theta) = (6\cos 165°, 6\sin 165°) = (-5.80, 1.55)$

b. What is the new location of the point (3, 0) that was rotated about 340°? approximate to 2 decimals



$r = 3$

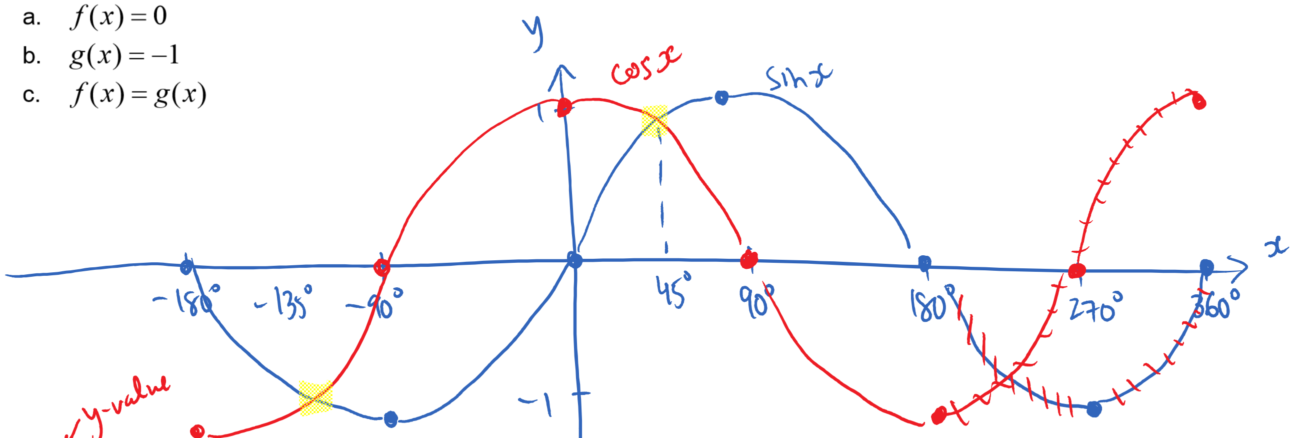
new pt. $= (r\cos\theta, r\sin\theta) = (3\cos 340°, 3\sin 340°) = (2.82, -1.03)$

7. For $f(x)=\sin x$ and $g(x)=\cos x$ Sketch on the same grid to find all the x values in the domain $-180^o \le x \le 180^o$ such that

a. $f(x)=0$

b. $g(x)=-1$

c. $f(x)=g(x)$



ⓐ $f(\theta)=0$ ← y-value

$\sin\theta = 0$

$\theta = 0°, 180°, -180°$

ⓑ $g(x)=-1$

$\cos x = -1$ ← y-value

angles

$x = 180°, -180°$

ⓒ $f(x)=g(x)$

$\sin x = \cos x$

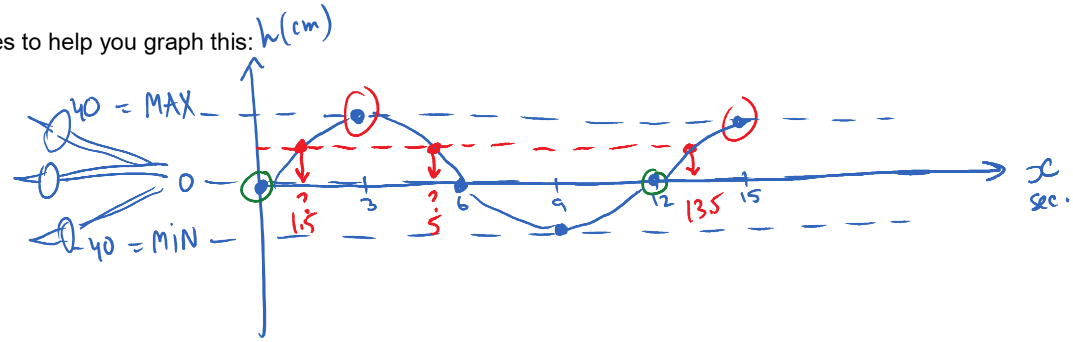
graphs meet at

$x = 45°, -135°$

8. All towers and skyscrapers are designed to sway with the wind. When standing on the glass floor of the CN tower the equation of the horizontal sway is $h(x)=40\sin(30.023x)^\circ$, where h is the horizontal sway in centimetres and x is the time in seconds.

a. Use table of values to help you graph this:

x	h
0	0
3	40
6	0
9	-40
12	0
15	40



b. What is the period? What does it represent?

$p = 15 - 3$ It represents the time of one full "sway"

$p = 12$ sec

c. State the maximum and minimum values of sway and the times at which they occur.

+ period.

MAX = 40 at $x = 3, 15, 27, 39, \ldots \rightarrow x_n = 12n - 9, n \in \mathbb{Z}$

MIN = -40 at $x = 9, 21, 33, \ldots$

d. State the mean value of sway and the time at which it occurs.

average. = axis = 0 cm at $x = 0, 6, 12, 18, 24, \ldots$ $x_n = 6n - 6, n \in \mathbb{Z}$

+ p/2

e. If a guest arrives on the glass floor at time = 0, how many seconds will have elapsed before the guest has swayed +20 cm from the horizontal?

USING GRAPH: → assume to the right.

not a very accurate guess

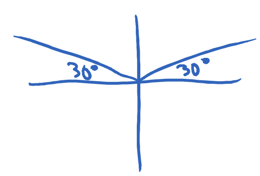
$x = \cancel{1.5}, \cancel{5}, 13.5, \ldots$ etc.

USING EQUATION sub h = 20 cm

$20 = 40\sin(\underbrace{30.023^\circ x}_{\text{let } \theta = 30.023^\circ x})$

$\frac{20}{40} = \frac{40}{40}\sin\theta$

$\frac{1}{2} = \sin\theta$ $y = 1 \Leftrightarrow \theta_r = 30^\circ$



$\theta_1 = \boxed{\frac{30^\circ}{30.023} = \frac{30.023x_1}{30.023}}$

$1 \doteq x_1$

$\theta_2 = \frac{150^\circ}{30.023} = \frac{30.023x_2}{30.023}$

$5 \doteq x_2$

add period = 12

to find $x_3 = x_1 + 12 = 13$

$x_4 = x_2 + 12 = 17$

$x_5 = x_3 + 12 = 25$

⋮

f. Find $h(2.034)$, what does it represent?

$h(2.034) = 40\sin(30.023(2.034))$

$h(2.034) \doteq 35$

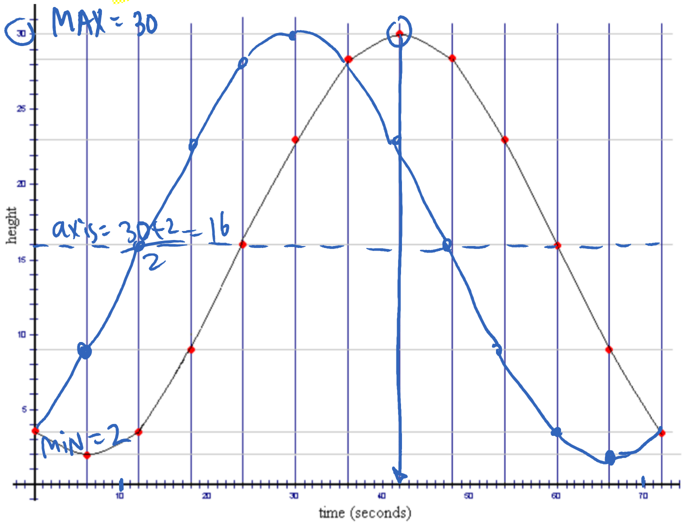
input x sec

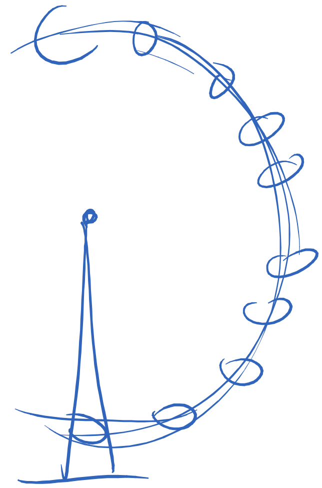
output y cm of horiz. sway.

it represents that at time 2.034 sec tower is 35 cm right of vertical.

7

1. As you ride a Ferris wheel, your distance from the ground varies sinusoidally with time according to the equation $h(t) = 14\sin(5(t-24))^{o} + 16$ where h is height in meters and t is time in seconds. The graph of this model is below





a. What is the radius of the wheel? What part of the equation gives you this?

radius = amplitude = max - axis = 30 − 16 = 14

The amplitude = |a| never negative.

b. Where is the centre of the wheel located? What part of the equation gives you this?

centre of wheel = axis = $\frac{MAX + MIN}{2}$ = 16

The axis = c

c. How long does it take for this Ferris wheel to complete one full revolution? What part of the equation gives you this?

period = 72

$\boxed{\text{period} = \frac{360^{\circ}}{|k|}} = \frac{360^{\circ}}{5} = 72$

d. If the Ferris wheel was sped up, what part of the equation will change?

The k will be bigger

e. If the Ferris wheel rotated in the other direction, sketch the resulting graph on the same grid. What part of the equation will change?

The d will change (shift left/right)

f. How far off the ground did you board the Ferris wheel?

t=0 h = 3.5 m

g. You took a video of the whole ride. The video is 7 minutes 12 seconds long, how many revolutions did the wheel make during this ride?

×60

420+12 = 432 sec ÷ 72 sec ∴ 6 revolutions

h. At 3 minutes and 14 sec of the video you were level with a nearby building, how tall is that building?

×60

180 + 14 = 194 sec

$h(194) = 14\sin(5(194-24)) + 16 \doteq 26.7$ m

i. When was the last time you were at maximum height before you had to get off?

x = ?

MAX at 42 sec → but this at beginning.

+72

+72

⋮ until you are before → the end at 432 sec

at 402 sec (6 min 42 sec)

8

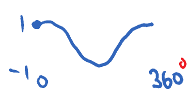
a → vertical stretch / compression
$|a|>1$ $|a|<1$
↳ reflection in x-axis $a<0$

k → horizontal stretch / compression
$|k|<1$ $|k|>1$
↳ reflection in y-axis $k<0$

d → shifts left / right * hidden if k and x have no bracket between them
* switch sign

c → shifts up/down

parent: $y = \cos(x)$



$|a|$ amplitude $a=-2$ → vertical stretch
↳ reflection in x-axis

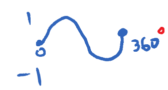
$k=0.75$ → horiz. stretch

$d=0$ →

axis → $c=3$ → shift up

$2(x-30)$

parent $y = \sin(x)$



$a=2.5$ vertical stretch

$k=2$ horiz. compress

$d=30$ shift 30 right

$c=0$

Range $= \{y \in \mathbb{R} \mid MIN \le y \le MAX\}$

$\boxed{MIN = c - |a|}$
$\boxed{MAX = c + |a|}$

5 - - - -
3 - - - -
1 - - - -

$\boxed{k = \frac{360°}{p}}$ same

$\boxed{p = \frac{360°}{|k|}}$

$D = \{x \in \mathbb{R}\}$ however we want only 3 cycles.

$D = \{x \in \mathbb{R} \mid 0 \le x \le 3p\}$

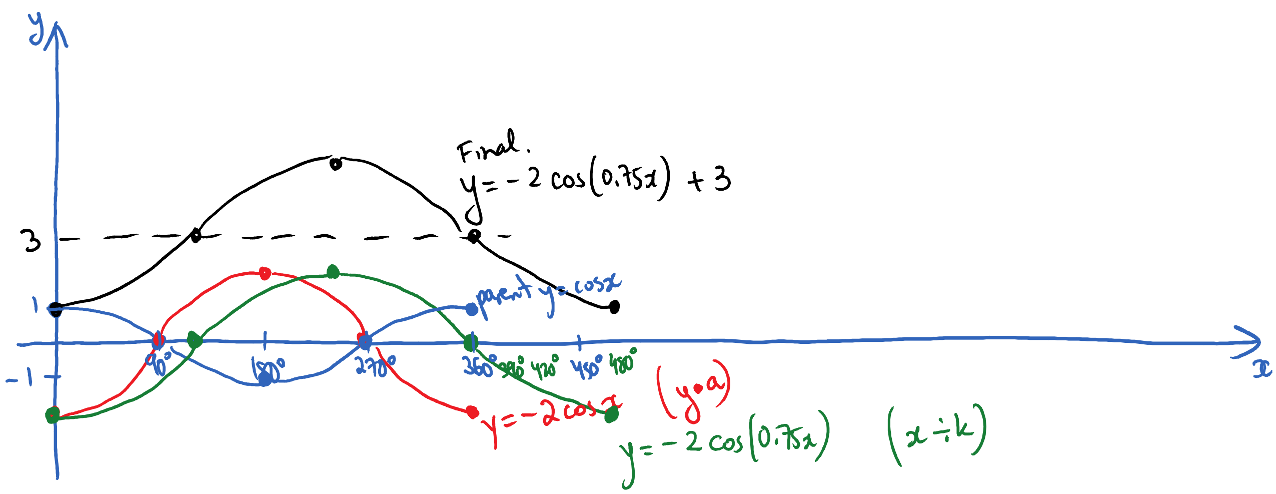
ⓐ $D = \{x \in \mathbb{R} \mid 0 \le x \le 1440°\}$ ← $p = \frac{360°}{0.75}$
$3p = \underbrace{\downarrow \times 3}$

$R = \{y \in \mathbb{R} \mid 1 \le y \le 5\}$

ⓑ $D = \{x \in \mathbb{R} \mid 0 \le x \le 540°\}$

$R = \{y \in \mathbb{R} \mid -2.5 \le y \le 2.5\}$

- by using $(x,y) \to \left(\frac{x}{k} \pm d,\ y \cdot a \pm c\right)$



period $= \frac{360°}{|k|} = \frac{360°}{2} = 180°$

- start the table at "d" $= 30°$
- find each section length $= \frac{\text{period}}{4}$ → use this as the skip count on x's.

$\frac{180°}{4} = 45°$

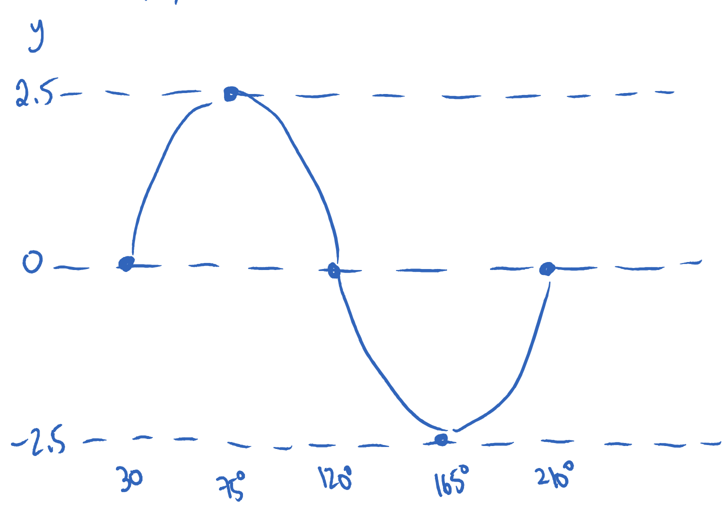
x	30°	75°	120°	165°	210°
y	0	2.5	0	-2.5	0

+45°

MAX/MIN

"c"

"5 points in a cycle"

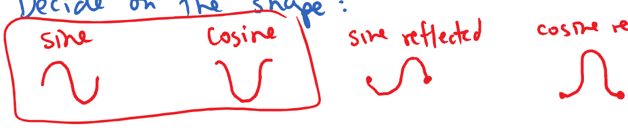


5. Summarize how to sketch by finding all the key characteristics from a given equation.

(1.) Draw 3 dotted horizontal lines
- - - - - MAX = $c + |a|$
- - - - - axis = c
- - - - - MIN = $c - |a|$

(2.) Find period $= \frac{360^\circ}{|k|}$

(3.) Decide on the shape :

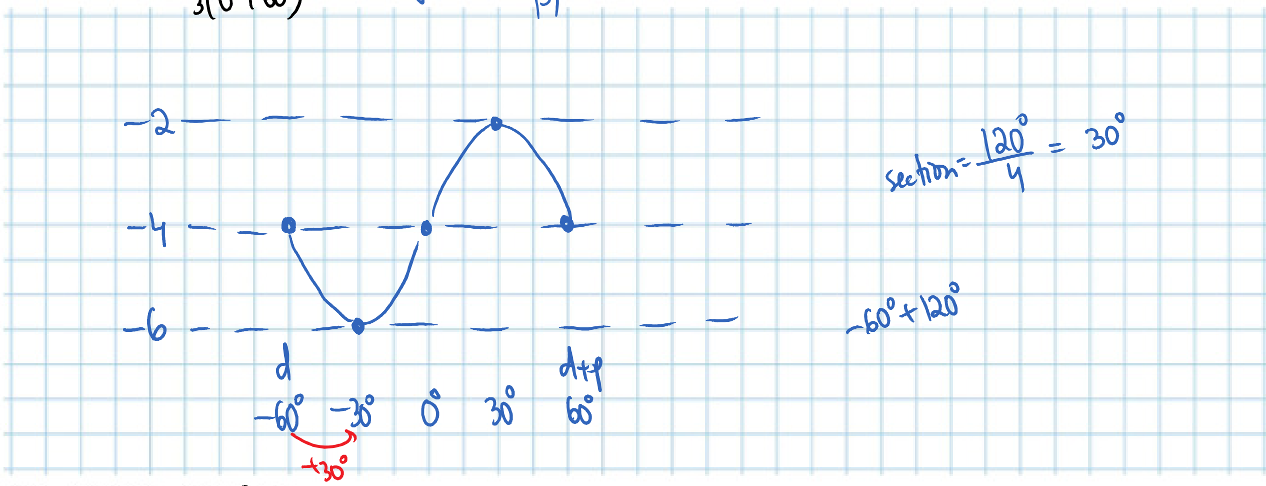


(4.) Label the 5 horizontal points
1st pt = d = "phase shift"
last pt = d + p
other points use skip count by section length $= \frac{\text{period}}{4}$

6. State all the key characteristics of the following then sketch

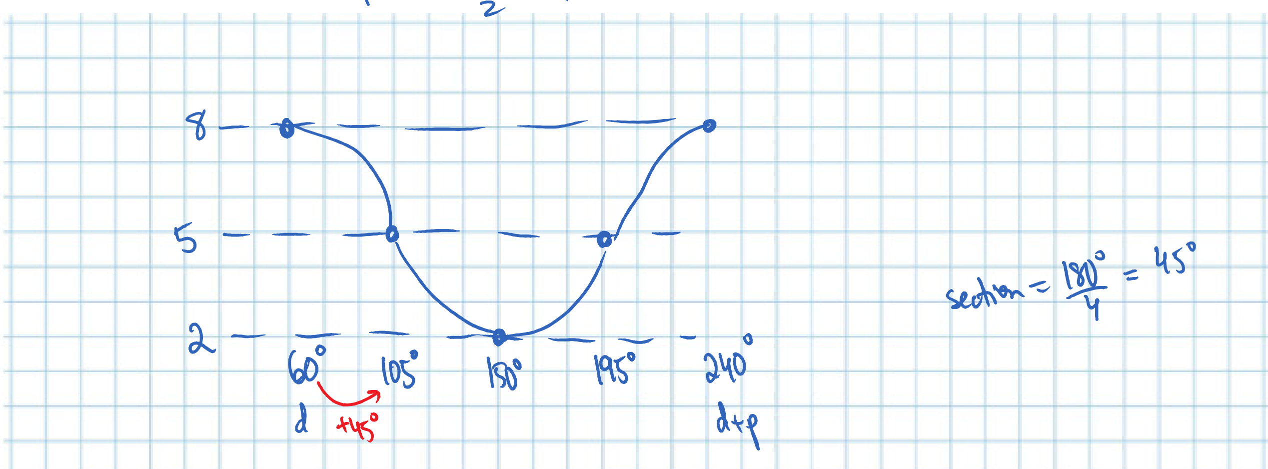
a. $y = -2\sin(3\theta + 180) - 4$

amp $= |-2| = 2$
axis $= -4$
$3(\theta + 60)$
phase shift $= -60^\circ = d$
period $= \frac{360^\circ}{|3|} = 120^\circ$



b. $y = 3\cos(2x - 120)^\circ + 5$

amp = 3
axis = 5
phase shift = d = 60°
period $= \frac{360^\circ}{2} = 180^\circ$



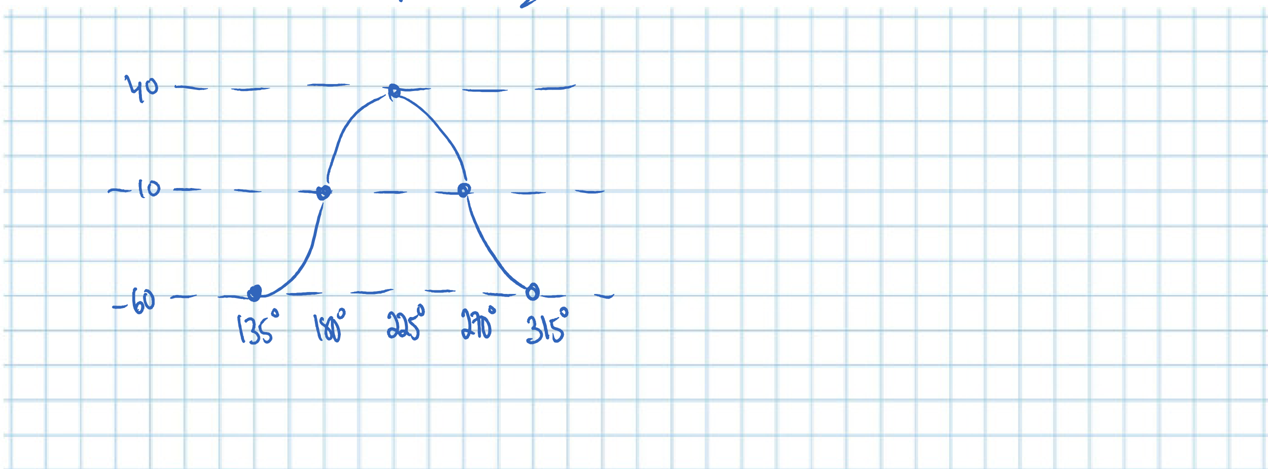
c. $y = -50\cos(2\theta - 270) - 10$

amp = 50

axis = -10

phase shift = 135°

period = $\frac{360°}{2}$ = 180°



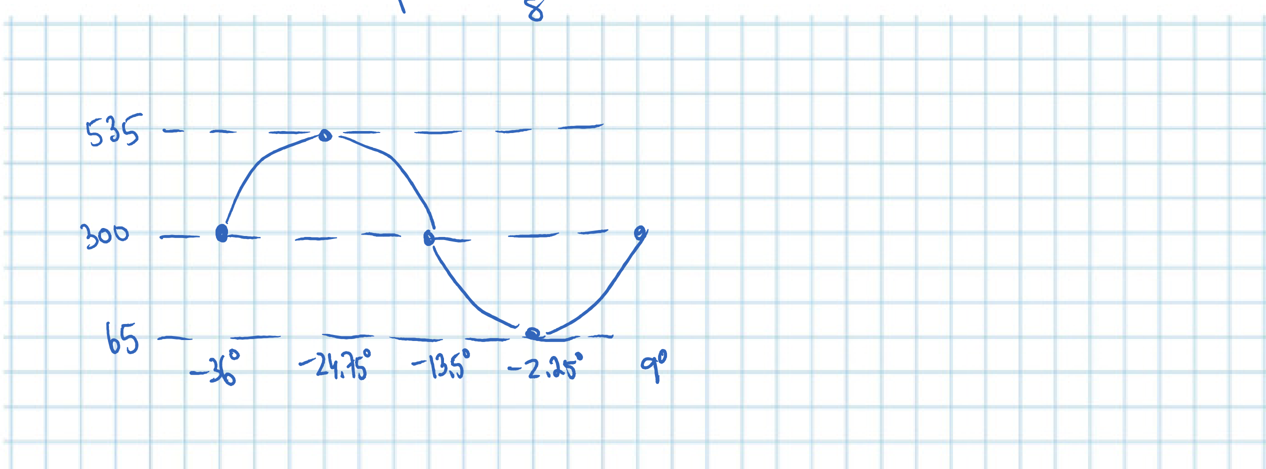
d. $y = 235\sin(8x + 288)^{o} + 300$

amp = 235

axis = 300

phase shift = -36°

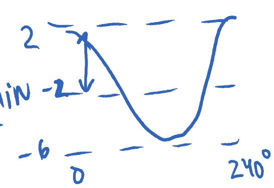
period = $\frac{360°}{8}$ = 45°



7. Find the equation for cosine graph if you are told that the range is $-6 \le y \le 2$ and period is 240^{o}. Assume there are no reflections or horizontal shifts.

$y = a\cos(k(x-d)) + c$

$y = 4\cos(1.5(x-0)) - 2$



$c = \frac{MAX + MIN}{2}$

$k = \frac{360°}{p} = \frac{360°}{240°} = 1.5$

12

Modelling with Sinusoidals

1. Summarize how you can find the equation from a given graph.

$\boxed{c = \text{axis} = \frac{MAX + MIN}{2}}$

$\boxed{\text{period} = \underset{(\text{right})}{\text{last pt}} - \underset{(\text{left})}{\text{1st pt.}}} \rightarrow \boxed{k = \frac{360^\circ}{p}}$

$\boxed{a = \underset{(\text{radius})}{\text{amplitude}} = \frac{MAX - MIN}{2} = MAX - \text{axis} = \text{axis} - MIN}$

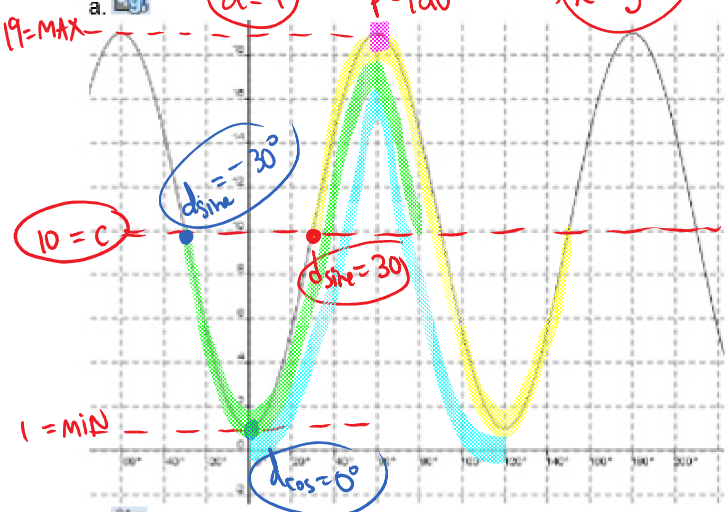
phase shift: d_{sine} = 1st pt (horizontally) of a cycle — starts at axis

d_{cose} = 1st pt. (horiz.) of a cycle — starts at MAX

2. Find two possible equations for sine and two possible equations for cosine for each of the following

a.

a=9 p=120° → k=3



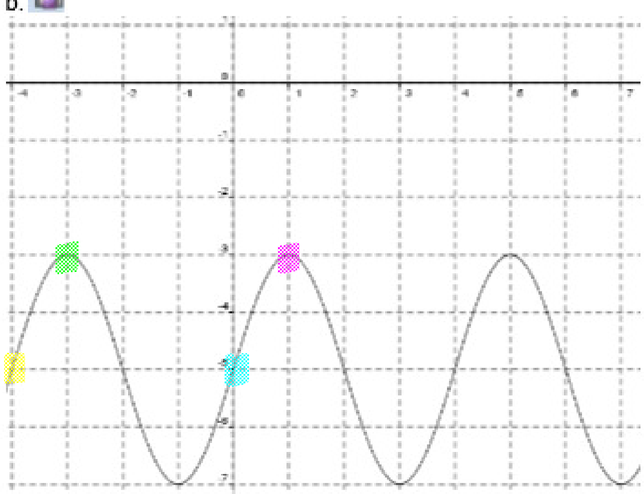
$y = 9 \sin[3(x - 30^\circ)] + 10$

$y = -9 \sin[3(x + 30^\circ)] + 10$
↑ reflected.

$y = -9 \cos 3x + 10$

$y = +9 \cos[3(x - 60^\circ)] + 10$

b.



$y = 2 \sin[90^\circ(x+4)] - 5$

$y = 2 \sin[90^\circ(x - 0)] - 5$
-2 $(x+2)$
-2 $(x-2)$

$y = 2 \cos[90^\circ(x+3)] - 5$

$y = 2 \cos[90^\circ(x-1)] - 5$
-2 $(x+1)$
-2 $(x-3)$

3. In the graphs above, notice that x-axis is labelled differently, how does that affect the equation that you write down?

ⓐ period = 120° $\therefore k = \frac{360^\circ}{120^\circ} = 3$ no degrees on k

ⓑ period = 4 $\therefore k = \frac{360^\circ}{4} = 90^\circ$ with degree on k.

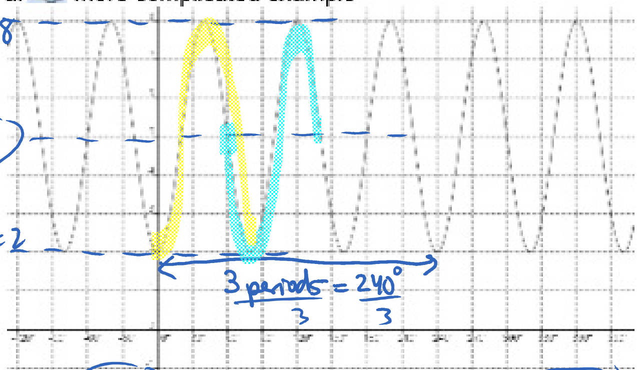
4. Find one possible equation for sine and one for cosine for each of the following

a. more complicated example

MAX=8

c=5

MIN=2



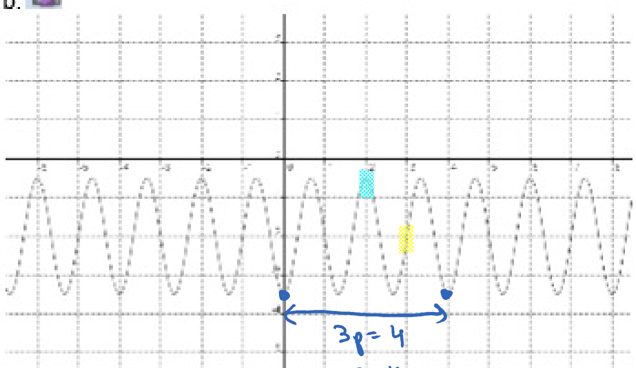
$\frac{3 \text{ periods}}{3} = \frac{240°}{3}$

$a=3$ $\quad p=80°$ $\quad \therefore k=\frac{360°}{80°} = 4.5=k$

$y = -3\sin[4.5(x-60°)]+5$

$y = -3\cos[4.5(x+0)]+5$

b.



$3p=4$

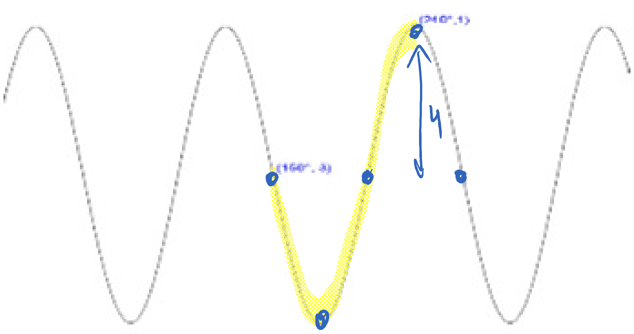
$p=\frac{4}{3}$

$\therefore k = \frac{360°}{\left(\frac{4}{3}\right)} = 360° \div \frac{4}{3} = 360° \times \frac{3}{4} = 270°$

$y = 1.5\sin[270°(x-3)]-2$

$y = 1.5\cos[270°(x-2)]-2$

5. Find an equation for the following graph



4

$y=-4\sin[3(x-150)]-3$

$+4\cos \quad -240°$

$\frac{3}{4}p=90°$

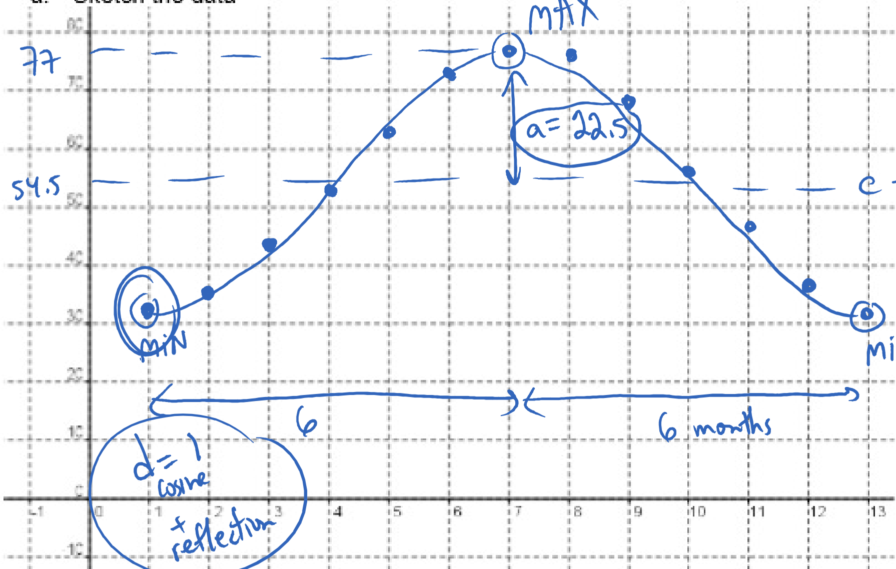
$p=120°$ $\quad \therefore k=\frac{360°}{120°}=3$

6. The average monthly temperature of a city in degrees Fahrenheit are given below

assume Jan = 1

Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
32	35	44	53	63	73	77	76	69	57	47	37

a. Sketch the data



$c = \frac{MAX + MIN}{2} = \frac{77 + 32}{2}$

$c = 54.5$

$p = 12$ months

$\therefore k = \frac{360°}{12} = 30° = k$

b. Find an approximate equation that would model this data. Assume that x=1 is the start of January.

temp. → $y = -22.5 \cos[30°(x-1)] + 54.5$ ← months

c. Use the equation to find the approximate monthly temperature for the middle of August

$x = 8.5$

$y = -22.5 \cos[30°(8.5-1)] + 54.5$

$= 70.4$ degrees Fahrenheit.

d. Most of the households turn on the heat if the temperature falls below 64ºF. For what domain do most households use heating in their homes?

$64 = -22.5 \cos[\underbrace{30°(x-1)}]+54.5$

let $\theta = 30°(x-1)$

$64 = -22.5 \cos\theta + 54.5$

-54.5

$\frac{9.5}{-22.5} = \frac{-22.5 \cos\theta}{-22.5}$

$-0.4222 = \cos\theta$

rough: $\theta_r = \cos^{-1}(0.4222)$

$\theta_r = 65°$

$\theta_1 = 115° = 30°(x_1 - 1)$
$\frac{115°}{30°} + 1$, $\frac{30°}{30°}$

$x_1 = 4.8$ end of Apr.

$\theta_2 = 245° = 30°(x_2 - 1)$

$x_2 = 9.2$ beginning of Sept.

add period 12

$x_3 = 16.8$ end of Apr of next yr.

∴ Heat is turned on for $9.2 \le x \le 16.8$

e. What part(s) of the equation will change if the data was taken from a warmer climate?

The "c" can be higher (warmer average temperature)

The "a" can be smaller (the range of temperature can vary less)

The "d" can be shifted (MAX can be in another month)

"k" will not since it's related to period = 1yr!! always.

7. Summarize how to find the period and the k value if you are given the speed in revolutions per second. Use the example, 10 revolutions in 35 seconds to help you explain.

cycle or period

$10p = 35$ sec $\quad \therefore p = 3.5$ sec

$\therefore k = \frac{360°}{3.5} = \frac{360°}{(7/2)} = \frac{720°}{7}$ leave exact.

8. A conveyor belt is powered by a pulley of radius 5m. The pulley has a serial number on the edge, starting at maximum height, minimum height is 2m away from the floor. The pulley completes 5 revolutions in 2 minutes.

a. Sketch height versus time + label period in minutes

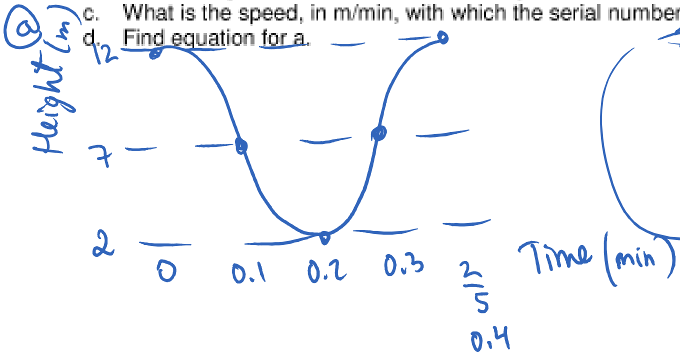
b. Sketch height versus distance traveled by the serial number + label period in meters

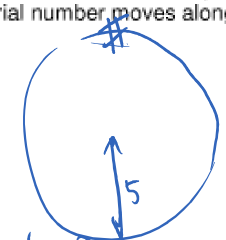
c. What is the speed, in m/min, with which the serial number moves along the circle?

d. Find equation for a.

$5p = 2$ min $\quad \therefore p = \frac{2}{5}$ min

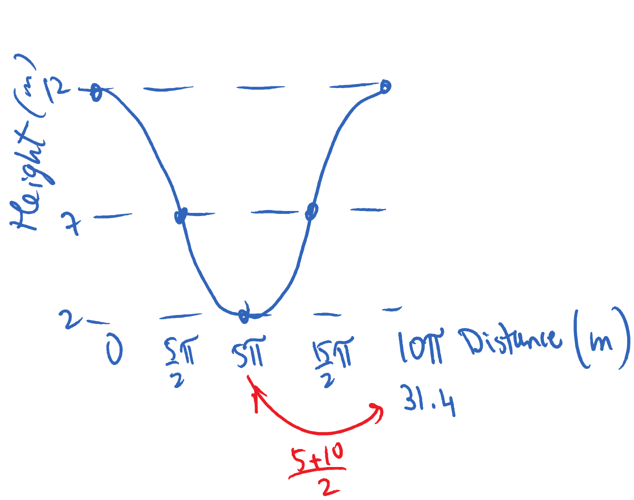
(a)



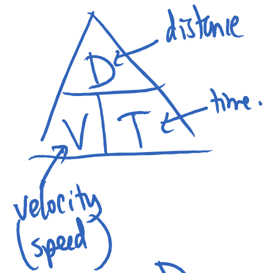


Distance travelled by # is circumference $\boxed{C = 2\pi r}$ $\boxed{C = \pi d}$

(b)



(c)



$V = \frac{D}{T}$

$V = \frac{10\pi}{2/5} = 25\pi = 78.5$ m/min

e. Analyze the meaning of all constants and variables for the equation that describes question a, in the context of the problem. Do not use words like amplitude or axis, describe in relation to real life.

(d) $y = 5\cos[900°(x - 0)] + 7$

(e) y = final height for any given x

x = given time

5 = radius of wheel

7 = axle of wheel

$900° = 5\pi$ radians (k – no meaning in degrees however in radians k = angular speed)

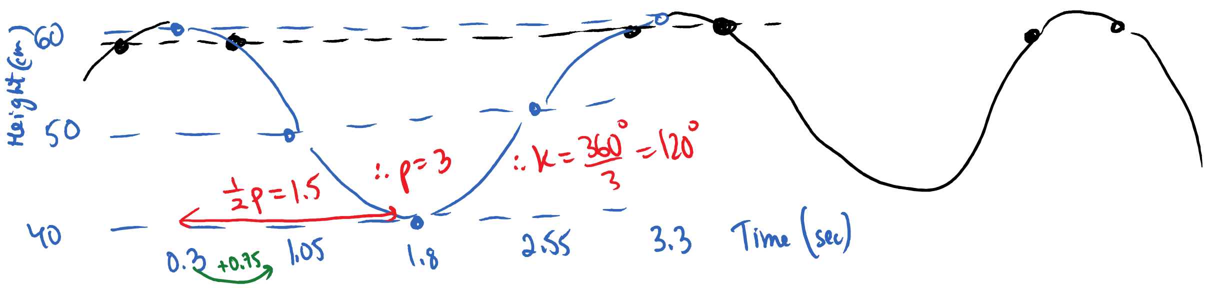
How to convert:

$\frac{78.5 \text{ m}}{1 \text{ min}} \times \frac{1 \text{ km}}{1000 \text{ m}} \times \frac{60 \text{ min}}{1 \text{ hr}}$

$\frac{4712.4 \text{ km}}{1000 \text{ hr}} = 4.7$ km/h

Solve Problems with Sinusoidals

1. A weight attached to the end of a long spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. You start a stopwatch. When the stopwatch reads 0.3 seconds, the weight first reaches a high point 60 cm above the floor. The next low point, 40 cm above the floor, occurs at 1.8 seconds. (Don't assume that at time zero the weight is at the minimum!)
   a. Sketch a graph of this sinusoidal function.



   b. Write an equation expressing distance from the floor in terms of the number of seconds the stopwatch reads.

$$y = 10\cos[120^\circ(x - 0.3)] + 50$$

distance from floor or height

seconds

   c. What was the distance from the floor when you started the watch?

sub $x = 0$

$$y = 10\cos(120^\circ(0 - 0.3)) + 50 = 58.1 \text{ cm}$$

   d. Predict the time at which the object was 59 cm above the floor for the first time and for the second time.

$$59 = 10\cos[120^\circ(x - 0.3)] + 50$$

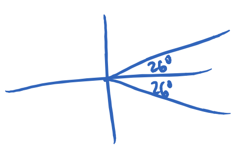
let $\theta = 120^\circ(x - 0.3)$

$$9 = 10\cos\theta$$

$$\frac{9}{10} = \cos\theta$$

rough: $\theta_r = \cos^{-1}\left(\frac{9}{10}\right)$

$\theta_r = 26^\circ$



$\theta_1 = 26^\circ = 120^\circ(x_1 - 0.3)$ → $x_1 = 0.5$ sec

$\theta_2 = 334^\circ = 120^\circ(x_2 - 0.3)$ → $x_2 = 3.1$ sec

add period 3

$x_3 = 3.5$

$x_4 = 6.1$

⋮

2. You are on an 8-seat Ferris wheel at an unknown height when the ride starts. It takes you 24 seconds to reach the top of the wheel 26m above the ground. The loading platform is 4m high. Your seat revolves at a speed of 4km/h.

a. Convert speed to m/sec
b. Find the distance travelled in one revolution
c. Find time it takes for one revolution
d. Sketch height versus time + label period in seconds
e. Find the sinusoidal equations that models height versus time

(a) $V = 4\,\frac{km}{hr} \times \frac{1000\,m}{1\,km} \times \frac{1\,hr}{3600\,sec}$

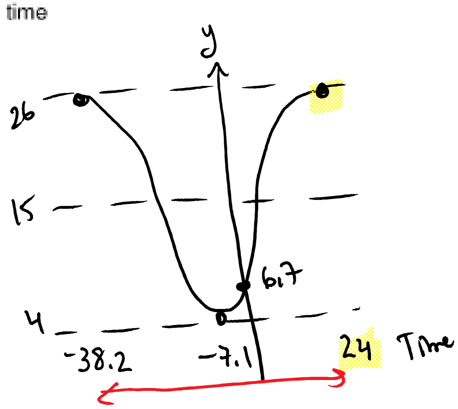
$V = \frac{10}{9}\,\frac{m}{sec}$

(b) $D = 2\pi r$ ← one cycle

$D = 22\pi$

(c) $\therefore T = \frac{D}{V} = \frac{22\pi}{\left(\frac{10}{9}\right)} = \frac{99\pi}{5}$ sec → period = 62.2 sec

(d)



$\therefore k = \frac{360°}{\left(\frac{99\pi}{5}\right)} = \frac{200}{11\pi}$

(e) $y = 11\cos\left[\frac{200}{11\pi}(x-24)\right] + 15$

height

K is "nice" in Radians ∴

f. How high above the ground will your seat be after 90 seconds?

$y = 11\cos\left(\frac{200}{11\pi}(90-24)\right) + 15 = 25.2\ m$

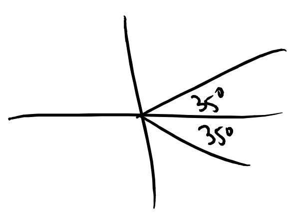
g. Find the two times within one cycle when the height is at 24m.

$24 = 11\cos\left(\frac{200}{11\pi}(x-24)\right) + 15$

let $\theta = \frac{200}{11\pi}(x-24)$

$\frac{9}{11} = \cos\theta$

rough: $\theta_r = \cos^{-1}\left(\frac{9}{11}\right) = 35°$



$\theta_1 = 35° = \frac{200}{11\pi}(x_1 - 24)$ → $x_1 = 30.1$ sec

$\theta_2 = 325° = \frac{200}{11\pi}(x_2 - 24)$ → $x_2 = 80.1$ sec

3. You are at Risser's Beach, N.S., to search for interesting shells. At 2:00 p.m. on June 19, the tide is in (i.e., the water is at its deepest). At that time you find that the depth of the water at the end of the breakwater is 15 meters. At 8:00 p.m. the same day when the tide is out, you find that the depth of the water is 11 meters. Assume that the depth of the water varies sinusoidally with time.

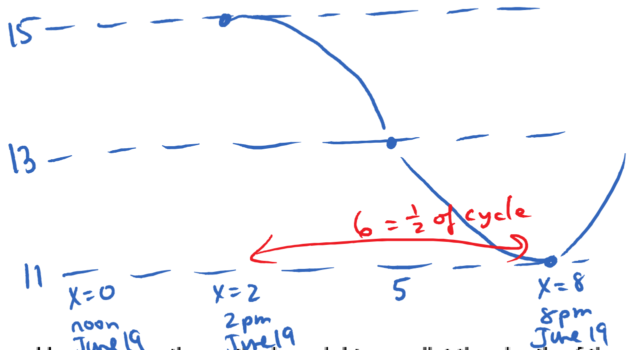
MAX

MIN

a. Derive an equation expressing depth in terms of the number of hours that have elapsed since 12:00 noon on June 19.

period = 12

x=0 at noon of June 19th



$y = 2\cos[30°(x-2)] + 13$

b. Use your mathematical model to predict the depth of the water at 7:00 a.m. on June 20.

equation

$x = 19$

$y = 2\cos[30°(19-2)] + 13 \doteq 11.3$ m

c. At what time will the first low tide occur on June 20?

sub $y = 11$ + solve for x

OR use symmetry

MIN is at $x=8$ 8pm June 19 + period 12 $x=20$

∴ 8am on June 20th

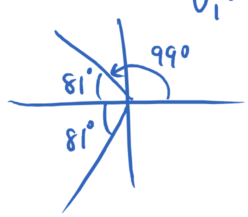
d. What is the earliest time on June 20 that the water will be at 12.7 meters deep?

$12.7 = 2\cos\theta + 13$ let $\theta = 30°(x-2)$

$\frac{-0.3}{2} = \cos\theta$

$\theta_1 = \cos^{-1}\left(\frac{-0.3}{2}\right) = 99° = 30°(x-2)$

$5.3 = x_1$ → ~5pm on June 19



$\theta_2 = 261° = 30°(x-2)$

+12

$10.7 = x_2$ ~ almost 11pm on June 19

$x_3 = 17.3$ → $\boxed{\text{5 am on June 20}}$