Sequences and Series Unit 7

NAME		Learn with	Teach to
HW	/60		
presentation			
peer	/5		
teacher	/10		
TOTAL	/75		

Big idea/Learning Goals

This unit involves discrete functions. Discrete is the opposite of continuous. When you have a domain of real numbers the function you get is **continuous**, where as if the domain is integers/whole/natural numbers, which are separated with spaces, the function is **discrete** or unconnected. Sequences of numbers are different from a series of numbers. **Sequences** are lists of numbers, while **series** are sums of numbers. You will study how to model a pattern sequence with an equation and how to use the equation to solve problems. Equations can be defined explicitly as well as recursively. A **recursion** formula defines terms by using previous terms of the sequence. Finally you will discover how sequences and series are used in real life applications.

Corrections for the textbook answers:
Sec 7.1 #11   62 months

Sec 7.3 #1 incorrect solution. $t_n = t_{n-1} - t_{n-2}$ $t_1 = 1, t_2 = 5$ and $t_{10} = -1$

Sec 7.3 #4a) $t_n = 1 - n$, if n odd, $t_n = n$, if n even, -12344   #8   4950
Sec 7.5 #9 $6945
Sec 7.7 #4e) fifth term 270x^2

Success Criteria

Date	pages	Topics	# of quest. done? You may be asked to show them	Questions I had **difficulty with** ask teacher before test!
	2-5	Introduction to Sequences – 2days Section 7.3 #2, 5,7abcdef,10 + Journal # 1 EXTRA two Handouts	/9	
		Learn your topic to present		
	6-7	Arithmetic Sequences Section 7.1 #5def, 6cd, 9cd, 10ab, 11, 13ef, 15 + Journal # 2 EXTRA Handout online	/13	
	8-9	Geometric Sequences Section 7.2 #5cdef, 8cd, 9cd, 11, 12, 13, 14ab + Journal # 3 EXTRA Handout online	/13	
	10-11	Arithmetic Series Section 7.5 #5bcd, 6abd, 7ef, 8ab, 9, 11 + Journal # 4 EXTRA Handout online	/12	
	12-13	Geometric Series Section 7.6 #3cd, 5abf, 6ef, 7, 11, 12abc + Journal # 5 EXTRA Handout online	/12	
	14-17	Pascal's Triangle & Binomial Expansion & Fibonacci Section 7.7 #5abcdef OR Handout #15-20 AND 7.4 #3abc + Journal # 6	/9	
	18	EXTRA Decimals to Fractions REVIEW		

Reflect – previous TEST mark _____, Overall mark now_____.

Introduction to Sequences

1. The sequences you will study will always have a pattern to enable you to find an equation explicitly or recursively for it.
   The pattern may be arithmetic or geometric or neither.
   Arithmetic pattern comes from consecutive terms having a constant _____ between them.
   Geometric pattern comes from consecutive terms having a constant _____ between them.

2. To define sequences you must use specific notation. Summarize what these mean
   n t_1 or a d r

   t_6 S_7 S_n t_n t_{n-1}

a) Which fornulas are explicit,and which recursive ? b) Find 1st four terms, decide if arithmetic/geometric/neither.

3.

$$t_n = \frac{3^n}{2^{n+1}}$$

4.

$$t_n = t_{n-1} + 2n - 1, \quad t_1 = 1$$

5.

$$t_n = \frac{(-1)^n + 1}{n^3}$$

6.

$$t_n = t_{n-1} + 2 t_{n-2}, \quad t_1 = 1, \quad t_2 = 3$$

For each of the following identify if it is:

a. Arithmetic/geometric/neither
b. Sequences/series
c. Finite/infinite

and then find

d. Next term
e. General explicit formula for t_n.
f. Recursive formula, if possible

7. $-4, -7, -10, -13, \_\_\_, ..., -52.$

8. $32 + 16 + 8 + 4 + \_\_\_ + ...$

Develop general formulas:

9. $a, a+d, a+2d, a+3d, _____, ...$

10. $a, ar, ar^2, ar^3, _____, ...$

For each of the following state
      a. Arithmetic/geometric/neither
      b. Next term
      c. General explicit formula for t_n.
      d. Recursive formula, if possible

11. $2 + 6 + 10 + 14 + \underline{\quad} + \ldots + 66.$

12. $5x, 7x, 9x, \underline{\quad}, \ldots, 99x.$

13. $x^4, x^6, x^8, \underline{\quad}, \ldots, x^{24}.$

14. $500 - 200 + 80 - 32 + \underline{\quad} \ldots$

What if neither?

For each of the following
    a. Show that the seq is neither arith or geo

b. state General explicit formula for t_n.
c. state Recursive formula,

15. $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \underline{\quad}, \dots$

16.

$$\frac{1}{2}, \frac{1}{2}, \frac{3}{8}, \frac{1}{4}, \frac{5}{32}, \dots$$

17. $\dfrac{1}{8}, \dfrac{4}{12}, \dfrac{9}{16}, \dfrac{16}{20}, \underline{\quad}, \dots$

18. $\dfrac{1}{2}, \dfrac{8}{5}, \dfrac{27}{10}, \dfrac{64}{17}, \underline{\quad}, \dots$

Arithmetic Sequences

1. Summarize the general term formula for any arithmetic sequence. Explain what each letter represents.

2. Use function notation to define the arithmetic sequence above. What function type best models it?

3. Why must the domain be different than usual real numbers? Use graphs to explain..

4. Determine a) the explicit formula for a general term, b) the recursive formula, and c) the 30th term of $-9, -6, -3, 0, ...$

5. Determine the number of terms in the following sequence $3, 15, 27, ..., 495$

6. Determine the general term, t_n, for the arithmetic sequence where $t_{10} = 300, t_{15} = 325$.

7. Prove whether the following are arithmetic sequences or not.

a. $t_n = (n+3)(n-5)$

b. $f(n) = \dfrac{n^2 - 4}{2 + n}$

c. $t_1 = 3, \ t_n = 3 + 2t_{n-1}$

8. Marisa deposits $12000 in a savings account that pays simple interest at 2%. She makes no other deposits.
    (**Simple interest** means that the interest grows by a constant amount each year.)
    If you have seen finance formulas in a previous course, don't use them here, use sequence formulas only
    a. Write the sequence of the year-end balances over four years
    b. Determine the general term formula in function notation
    c. At the end of some year the balance in the account is $13200. For how long has the original deposit earned interest?

9. If $a = 6, d = -\dfrac{1}{4}$ and $t_n = -10$ find n

10. If $t_4 = 7x - 3$, and $d = 5 + x$ find a

Geometric Sequences

1. Summarize the general term formula for any geometric sequence. Explain what each letter represents.

2. If the outputs are from the geometric sequence, what function type best models it. Explain.

3. Determine a) the explicit formula for the general term, b) the recursive formula, and c) the 30th term of $4, 12, 36, 108, ...$

4. Determine the number of terms in the following sequence $6, -12, 24, -48, ..., -768$.

5. Determine the general term, t_n, for the geometric sequence where $t_5 = 768, t_9 = 196608$.

6. Marisa deposits $12000 in a savings account that pays compound annual interest at 2%. She makes no other deposits. (**Compound interest** means that the interest grows by a an increasing amount each year, because the interest is calculated on the amount deposited as well as on the interest already earned so far.)

    If you have seen finance formulas in a previous course, don't use them here, use sequence formulas only
    a. Write the sequence of the year-end balances over four years
    b. Determine the general term formula in function notation.
    c. At the end of some year the balance in the account is $13248.97. For how long has the original deposit earned interest?

Notice the answers to simple interest (done in the last lesson) and compound interest calculations are different. This is what you will study in the next unit.

7. If $a = 2, r = 3$ and $t_n = 1458$ find n

8. If $t_5 = 350 y^x$, and $r = 5 y^{3-2x}$ find a

Arithmetic Series

1. Summarize the TWO sum formulas below. Explain what each letter represents.

2. What type of function is the sum formula?

3.
   The altitude of a plane is 200 km at 12 noon when the pilot begins her descent.
   If the plane's altitude is 196 km one minute later, 192 km one minute after that
   and so on. Determine when the plane will reach an altitude of 68 km.

4. Colin has begun building a pyramid with dominoes. In total, he has 6519 dominoes to work with. On the bottom row he has used 175 dominoes, and each row of his pyramid has a fixed number of dominoes less than the row below it. Unfortunately, Colin runs out of dominoes before he can finish his pyramid, but he is able to finish the last row he was working on before running out of dominoes. The last row contained 71 dominoes. How many rows of the pyramid was Colin able to complete?

5. There are 20 terms in an arithmetic series, the 5th term is -8 and the 9th terms is 4. Find the sum of the series

6. The 3rd term of an arithmetic series is 9, and the sum of the first 10 terms is -60. Find the partial sum of first 20 terms.

Explanation for why formula works: adding all the terms twice helps to see how formula was developed, mathematician Karl Gauss did this when he was 10 years old in year 1787.

Geometric Series

1. Summarize the TWO sum formulas below. Explain what each letter represents.

2. What type of function is the sum formula if the input is n? If the input is r?

3. Find how many terms in this sequence using t_n
   $$5+20+80+...+20480$$

4. Find the sum of the geometric series using S_n
   $$5+20+80+...+20480$$

5. A doctor prescibes 200mg of medication on the first day of treatment. The dosage is halved on each successive day for one week. To the nearest milligram, what is the total amount of medication administered?

6. Amy drops a ball from a height of 16m. Each time the ball touches the ground, it bounces up to 5/8 of the maximum height of the previous bounce. Determine the total vertical distance the ball has travelled when it touches the ground on the seventh bounce.

Explanation for why formula works:

For the series $S_9 = 3 + 15 + 75 + ... + 1\ 171\ 875$ find the sum without the formula

Hint subtract $rS_9 - S_9$ and notice what happens. Note that $t_1 = 3, t_9 = 1171875, t_{10} = 5859375$, how does that relate to the formula given on the previous page?

Pascal's Triangle & Binomial Expansion

1. Expand and simplify each binomial. Look for patterns.

a. $(a+b)^0$

b. $(a+b)^1$

c. $(a+b)^2$

d. $(a+b)^3$

e. $(a+b)^4$

f. $(a+b)^5$

2. Complete the pattern in the Pascal's triangle
   - The value of the first cell and the last cell in every row is ONE
   - The value of any other cell in the triangle is equal to the SUM of the two cells immediately above it.
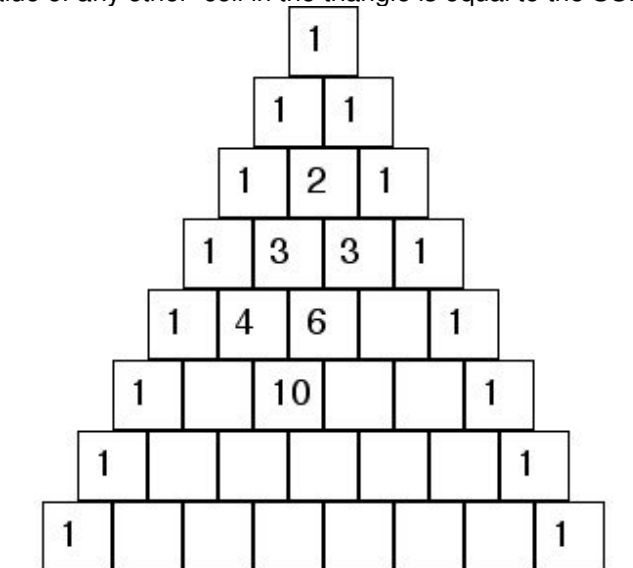
3. Use the found pattern of the Pascal's triangle to expand the following

a. $(2x+1)^5$

b. $(\sqrt{m}-3)^6$

c. $(3x^3-\dfrac{2}{x})^4$

Fibonacci Sequence

The information below is taken from:
www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html

The Fibonacci sequence actually first appeared in Indian mathematics in 200 BC, then in the west, Leonardo of Pisa, known as Fibonacci, posed an idealized (biologically unrealistic) rabbit puzzle in 1202 AD, that also relates to Fibonacci numbers. Here it is:

"A newly born pair of rabbits, one male, one female, are put in a field; rabbits are able to mate at the age of one month so that at the end of its second month a female can produce another pair of rabbits; rabbits never die and a mating pair always produces one new pair (one male, one female) every month from the second month on. How many pairs will there be in one year?"

Number of pairs: 1, 1, 2, 3, 5
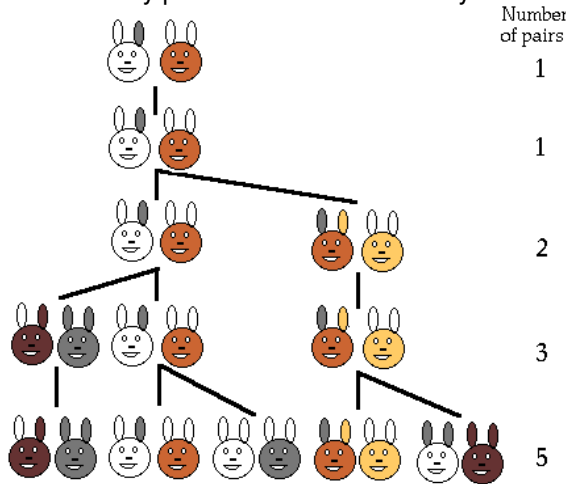
1. What is the **Fibonacci sequence**?

2. Give some reasons why the rabbit puzzle is not realistic?

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Even though the problem is not realistic, Fibonacci does what mathematicians often do at first, simplify the problem and see what happens - and the series bearing his name *does* have *lots* of other interesting and practical applications. For example the honeybee ancestry:

First, some unusual facts about honeybees such as: not all of them have two parents!

- In a colony of honeybees there is one special female called the queen.
- There are many worker bees who are female too but unlike the queen bee, they produce no eggs.
- There are some drone bees who are male and do no work.
- Males are produced by the queen's unfertilised eggs, so male bees only have a mother but no father!
- All the females are produced when the queen has mated with a male and so have two parents.
- Females usually end up as worker bees but some are fed with a special substance called royal jelly which makes them grow into queens ready to go off to start a new colony when the bees form a swarm and leave their home (a hive) in search of a place to build a new nest.

3. Fill in the table below, are the numbers from the Fibonacci sequence, yes or no?

| Number of	parents	grand-parents	great-grand-parents	great, great grand parents	gt, gt, gt, grand parents
for male bee					
for female bee					

Looking at your answers to the previous question, your friend Dee says to you:
- You have 2 parents. They each have two parents, so that's 4 grand-parents you've got.
- They also had two parents each making 8 great-grand-parents in total ... and 16 great-great-grand-parents ... and so on.
- So the farther back you go in your Family Tree the more people there are.
- It is the same for the Family Tree of *everyone* alive in the world today. It shows that the farther back in time we go, the more people there must have been. So it is a logical deduction that the population of the world *must* be getting smaller and smaller as time goes on!

4. Is there an error in Dee's argument? If so, what is it?

5. Show that the ratios $\dfrac{t_n}{t_{n-1}}$ of the successive numbers in the Fibonacci sequence get closer and closer to a number called the **Golden Number**, ϕ , called phi, which is approximately 1.618034…

6. Draw the Fibonacci Rectangles that create a spiral that is seen in nature
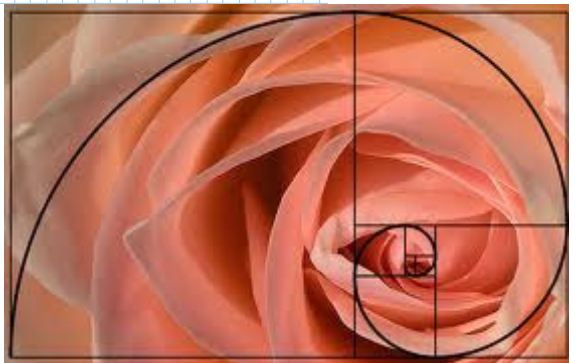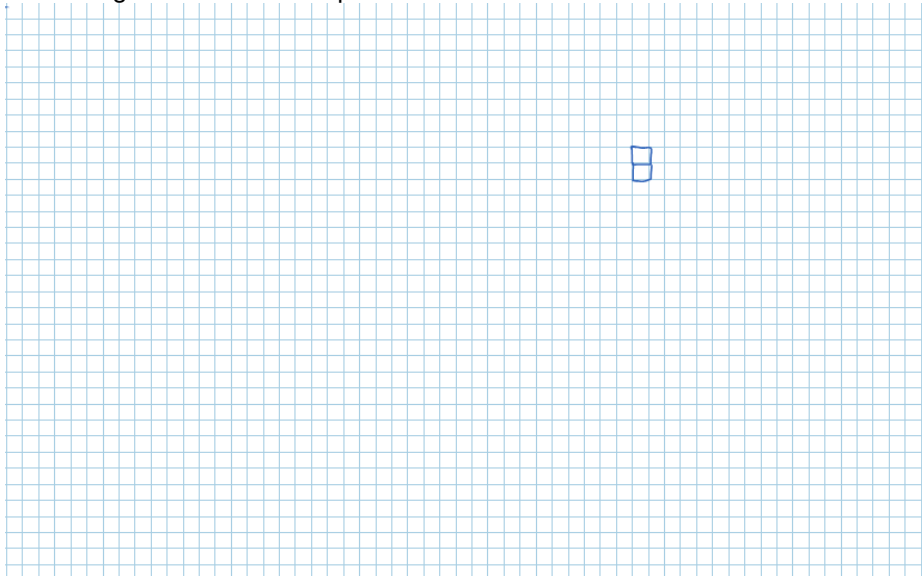
"All life is biology. All biology is physiology. All physiology is chemistry. All chemistry is physics. All physics is math."

Dr. Stephen Marquardt

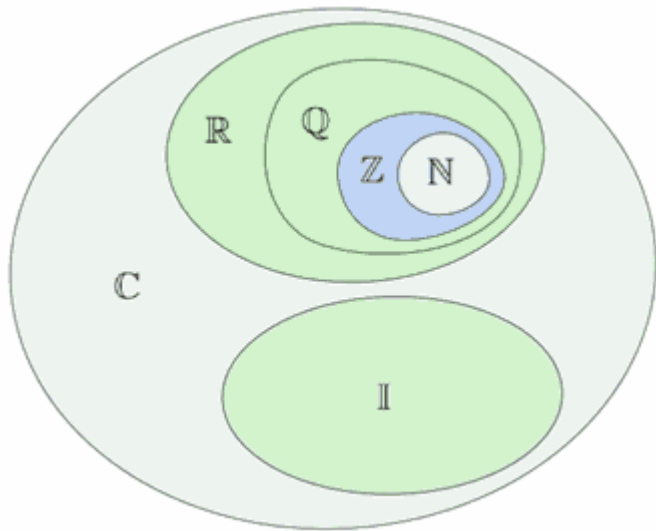The Golden Ratio has a pervasive appearance in real life and the universe.

Please visit
http://www.goldennumber.net/life.htm
(note to teacher: screen shots of this website is in oneNote)

EXTRA

Number Sets

Give an example of a number in each set, record in the corresponding circle.

What type of number is $\sqrt{-4}$?

What type of number is 7?

What type of number is -3.91?

What type of number is $\dfrac{\sqrt{3}}{2}$?

What type of number is 5.67777777…?

Explain:

N

Z

Q

R

I

C

Decimals to Fractions

How to convert terminating decimals to fractions

N=34.765

How to convert repeating decimals to fractions

N=45.232323….

N=2.0168

N=5.12222222…