Rational Expressions Unit 2

Tentative TEST date_____

Big idea/Learning Goals

The difference between expression and an equation is _____

Because of this difference, it matters where you place the equals sign when simplifying an expression or when solving an equation. Show examples:

In this unit you will learn how to work with algebra of more complex nature than you've seen so far. The expressions that you will be asked to simplify will have variables in the numerators and the denominators. These expressions are called rational expressions since they are just fractions. There will be certain rules that you will have to follow in order to keep the expressions equivalent or the same.

Corrections for the textbook answers:

Sec 2.2 #9 a) $\pi(16x^2 + 8x)$

Sec 2.3 #7 f) $2[m(m+5) - n(n-5)]$

Sec 2.3 #9 f) $(\sqrt{2}m + \sqrt{5})(\sqrt{2}m - \sqrt{5})(6m - 7)$

Sec 2.7 #11 $\dfrac{t^2 - t - s^2 - s}{st(s+1)(t-1)}$

Success Criteria

☐   I <u>understand the new topics</u> for this unit if I can do the practice questions in the textbook/handouts

Date	pg	Topics	# of quest. done? You may be asked to show them	Questions I had difficulty with ask teacher before test!
	2-3	Operations with Polynomials Section 2.1 & 2.2		
	4-6	Factoring Polynomials & Solving by Factoring Section 2.3 & Two Handout		
	7-9	Working with Complicated Fractions – if there is time Handout		
		Simplifying Rational Functions Section 2.4 & Handout		
	10-12	Multiplying and Dividing Rational Expressions Section 2.6 & two Handouts		
	13-14	Adding and Subtracting Rational Expressions Section 2.7 & two Handouts		
		REVIEW		

Reflect – previous TEST mark _____, Overall mark now_____.

Operations with Polynomials

Before you begin analyzing rational expressions (fractions with polynomials in the numerator and denominator), you must be comfortable with polynomial operations.

1. What is a polynomial?   2. How can you simplify a polynomial?

3. Simplify, then identify the name of the polynomial.

a. $2a\left(-3a^2b^4\right)^3$

b. $-2xy+5x^3+2x(4xy)-xy+x\left(10x^2\right)-6y\left(-3x^2\right)$

c. $(\dfrac{8}{9}a^2+b-6)-(\dfrac{5}{4}b+3c-2a^2-9)$
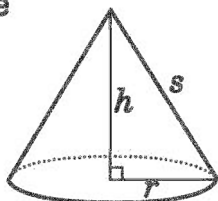
d. $3x(x+2)(x-1)-6(-x^2+x-4)^2$

e. $(1+y)^4$

you will learn how to do this faster later on in the course

4. Simplify the expression for the volume of the cone if $V=\dfrac{\pi r^2h}{3}$, $r=2+x$, and $h=2x-3$

Cone

5. There are certain rules that are different for monomials in comparison to polynomials. Complete the following questions and then summarize the rules.

MONOMIALS	**POLYNOMIALS**

a. $(5x^2y^3)^2$

b. $(5x^2 + y^3)^2$

c. $\sqrt{25x^2y^4}$

d. $\sqrt{25x^2 - y^4}$

e. $2x(3x)(4x^2)$

f. $(2x)(3x + 4x^2)$

g. $\dfrac{2xy}{(2x^2y)(4xy^3)}$

h. $\dfrac{2xy}{2x^2y - 4xy^3}$

6. Clarify the terms:
EXPAND   FACTOR   SIMPLIFY   EVALUATE   SOLVE

Factoring Polynomials & Solving by Factoring

Part of simplifying rational expressions will involve factoring the expressions in the numerator and also in the denominator to see if there are any cancellations. It is time to review factoring methods from grade 10.

1. Show how difference of square method can be used even if **numbers** are not perfect squares.

2. Factor completely. NOTE: if you see brackets, and it asks to factor, don't expand!

a. $16c^4 - 81d^8$

b. $(2-x)^2 - 4(x+1)^2$

c. $-24a^3b^2 + 48a^2b^2 - 4ab$

d. $2\pi h(r+h)^2 + 4\pi(r+h)h^2$

e. $16x^4 - 8x^2 + 1 - 9x^6$

f. $56x^3y^2 + 18x^2y^2 - 8xy^2$

g. $27(g-h)^3 - 12(h-g)$

h. $6(x+y)^3 + 4(x+y)^{-2}$

i. $2x^4y^2 - 13x^2y + 20$

j. $15k^2 - 7km - 2m^2$

k. $6d^4 - 29d^2 + 35$

l. $3y^2 + 5y - 10$

3. The last question above cannot be done with rational numbers. BUT there IS a solution! The answers are irrational as can be seen from the quadratic formula. Show how you can still record the factored version by using the qudratic formula.

4. Please don't use the quadratic formula all the time. Can you think of reasons why I'm asking you not to resort to this method unless absolutely necessary?

Solve by factoring.

5. $3x^2 - 14x + 8 = 0$

6. $p^4 + 21p^2 = 100$

7. $24x^2y - 16x^2y^2 + 8x^3y^2 = 0$

8. $3\sin^2 A - 14\sin A = -8$

9. $256a^4 = 625$

10. $121 - (3+b)^2 = 0$

11. The last two questions can be solved by isolating, why can't the other questions be isolated but the last two can be?

Working with Complicated Fractions – if there is time

1. Recall the rules of dividing and adding fractions.

2. Explain each mistake you should avoid and give a correct version of the result.

Errors Involving Fractions

Expression	Does NOT Equal
$\dfrac{1}{a}+\dfrac{1}{b}$	$\dfrac{1}{a+b}$
$\dfrac{a}{x+b}$	$\dfrac{a}{x}+\dfrac{a}{b}$
$\dfrac{\left(\frac{x}{a}\right)}{b}$	$\dfrac{bx}{a}$
$\dfrac{1}{3}x$	$\dfrac{1}{3x}$
$\left(\dfrac{1}{x}\right)+2$	$\dfrac{1}{x+2}$ or $\dfrac{3}{x}$

Errors Involving Cancellation

Expression	Does NOT Equal
$\dfrac{a+bx}{a}$	$1+bx$
$\dfrac{a+ax}{a}$	$a+x$
$1+\dfrac{x}{2x}$	$1+\dfrac{1}{x}$

3. Simplify the following.

a. $\dfrac{5}{\dfrac{2}{3}-\dfrac{1}{4}}$

b. $\dfrac{\dfrac{x+2}{x}}{\dfrac{1}{x}-2}$

Simplifying Rational Expressions

Rational expressions are fractions composed of polynomials. There are rules to follow. If rules are not followed you can get results like this →

1. Explain what calculations are done in each step:

$$\text{If } a = b$$
$$ab = b^2$$
$$-ab = -b^2$$
$$a^2 - ab = a^2 - b^2$$
$$a(a-b) = (a+b)(a-b)$$
$$\frac{a(a-b)}{(a-b)} = \frac{(a+b)(a-b)}{(a-b)}$$
$$a = (a+b)$$
$$\text{but since } a = b \text{ then}$$
$$a = (a+a)$$
$$a = 2a$$
$$1 = 2$$

2. What are **restrictions**, and why are they important?

3. What are the steps of simplifying rational expressions?

4. Simplify each of the following and state restrictions.

   a. $\dfrac{3x}{3x^2 - 6x}$

   b. $\dfrac{2x^2 - 13x + 6}{x^2 - 36}$

c. $\dfrac{42x^3y^5}{-12x^4y^2}$

d. $\dfrac{18-2m^2}{m^2-6m+9}$

e. $\dfrac{2a-a^2}{(3a+4)(a-2)-2(a-2)}$

f. $\dfrac{2x^2+xy-3y^2}{2y^2-xy-x^2}$

Multiplying and Dividing Rational Expressions

1. Working with rational expressions is similar to working with rational numbers. Remind yourself of the rules of multiplying and dividing fractions.

2. What are the steps of multiplying rational expressions?   3. What are the steps of dividing rational expressions?

4. Simplify each of the following and state restrictions.

a. $\dfrac{k^2+k}{k^2-k} \times \dfrac{3k-21}{2k^2-11k-21}$

b. $\dfrac{6x^2-15x}{3x^2+5x-12} \div \dfrac{4x^2-25}{3x+9}$

c. $\dfrac{3a^2(b+2)}{b(3-a)} \times \dfrac{10b(a-3)}{a^3b^4}$

d. $\dfrac{6(x-1)}{x^2} \div \dfrac{3(x-1)}{x(x+2)}$

e. $\dfrac{4x^2-25y^2}{(5y-2x)^2} \times \dfrac{4}{4x+10y}$

f. $\dfrac{3b^2-9b+6}{2b^2-10b+12} \div \dfrac{3-3b}{6-2b}$

g. $\dfrac{3y^2 - 7y + 2}{y^2 + 3y + 2} \times \dfrac{8y + 8}{4y - 8}$

h. $\dfrac{\dfrac{m+n}{5m^4n}}{\dfrac{m^2 - n^2}{15n^3m}}$

5. Create an expression that has the following conditions.

Zeros at $x = -2$ and $x = \dfrac{1}{3}$ and restrictions at $x = -5$ and $x = \dfrac{2}{7}$

6. For the cylinder,
   a. simplify the ratio of its surface area to volume.
   b. if height is 3 times as long as radius, simplify the ratio again.

Adding and Subtracting Rational Expressions

1. Working with rational expressions is similar to working with rational numbers. Remind yourself of the rules of adding and subtracting fractions.

2. What are the steps of adding/subtracting rational expressions?

3. Simplify each of the following and state restrictions.

with MONOMIALS

a. $\dfrac{6-x}{x^2 y} - \dfrac{3+y}{xy^2} + \dfrac{1}{3x^3 y^2}$

only POLYNOMIALS

b. $\dfrac{5x+2}{25x^2 -1} - \dfrac{3x-1}{25x^2 +10x+1}$

c. $\dfrac{2x-1}{12} + \dfrac{3x-2}{3} - \dfrac{x+1}{2}$

d. $\dfrac{x+3}{x^2 -4} - \dfrac{3-x}{4-x^2}$

e. $\dfrac{3}{x+1} - \dfrac{4}{x-2}$

f. $\dfrac{x-6}{x^2-3x+2} - \dfrac{3x+2}{x^2-x-2}$

g. $\dfrac{x}{(x+2)(x-3)} + \dfrac{4}{(-2-x)(x-4)}$

h. $\dfrac{x^2-x}{2x^2-3x} - \dfrac{3x^2+1-4x}{3-5x+2x^2}$

i. $\dfrac{2m^3}{12m^2-8m} + \dfrac{12m}{4m-6m^2}$

j. $\dfrac{x+y}{(x+y)^2-5(x+y)-6} + \dfrac{y}{3x(x+y)+3x}$