S14RationalsNOTES

July 8, 2014 2:05 PM

RationalsNOTES

see below

Rational Expressions Unit 2

Tentative TEST date_______________

Big idea/Learning Goals

The difference between expression and an equation is equations have equals sign.
Because of this difference, it matters where you place the equals sign when simplifying an expression or when solving an equation. Show examples:

ex. $(x+3)(x-2)$ expression
$= x^2+1x-6$
no other side, don't assume it's zero!

ex. $2x = x(x-5)$
$2x = x^2-5x$
$0 = x^2-7x$ .... solve by factoring

In this unit you will learn how to work with algebra of more complex nature than you've seen so far. The expressions that you will be asked to simplify will have variables in the numerators and the denominators. These expressions are called rational expressions since they are just fractions. There will be certain rules that you will have to follow in order to keep the expressions equivalent or the same.

Corrections for the textbook answers:

Sec 2.2 #9 a) $\pi(16x^2+8x)$

Sec 2.3 #7 f) $2[m(m+5)-n(n-5)]$

Sec 2.3 #9 f) $(\sqrt{2}m+\sqrt{5})(\sqrt{2}m-\sqrt{5})(6m-7)$

Sec 2.7 #11 $\frac{t^2-t-s^2-s}{st(s+1)(t-1)}$

Success Criteria

☐ I understand the new topics for this unit if I can do the practice questions in the textbook/handouts

Date	pg	Topics	# of quest. done? You may be asked to show them	Questions I had difficulty with ask teacher before test!
	2-3	Operations with Polynomials Section 2.1 & 2.2 — 2.1 # 8, 12	2.2 # 5, 9	
Day (3)	4-6	Factoring Polynomials & Solving by Factoring Section 2.3 & Two Handout — #4, 5, 6		
	7-9	Working with Complicated Fractions – if there is time Handout — all		
		Simplifying Rational Functions Section 2.4 & Handout — #4, 5, 8, 11		
Day (4)	10-12	Multiplying and Dividing Rational Expressions Section 2.6 & two Handouts — # 4, 5, 6, 9		
	13-14	Adding and Subtracting Rational Expressions Section 2.7 & two Handouts — # 5, 6, 8, 12		
+ Start Quad		REVIEW		

Reflect – previous TEST mark __________, Overall mark now__________.

$2x^{-1} = \frac{2}{x}$ Rational

$2x^{1/2} = 2\sqrt{x}$ ← Root

Operations with Polynomials

Before you begin analyzing rational expressions (fractions with polynomials in the numerator and denominator), you must be comfortable with polynomial operations.

1. What is a polynomial?

- Many terms with variable(s) on which the powers are only whole #'s (no negatives, no fraction powers)

2. How can you simplify a polynomial?

- Collect like terms (add/subtract coeffient)
- When mult/dividing terms do not have to be like, exponents of like bases add/subtract.

3. Simplify, then identify the name of the polynomial.

a. $2a\left(-3a^2b^4\right)^3$ $(-3)^3$

$= 2a(-3a^2b^4)(-3a^2b^4)(-3a^2b^4)$

skip $= -54a^7b^{12}$

b. $-2xy+5x^3+2x(4xy)-xy+x(10x^2)-6y(-3x^2)$

$= -2xy+5x^3+8x^2y-xy+10x^3+18x^2y$

$= -3xy+15x^3+26x^2y$

c. $1(\frac{8}{9}a^2+b-6)-1(\frac{5}{4}b+3c-2a^2-9)$

$= \frac{8}{9}a^2+1b-6-\frac{5}{4}b-3c+2a^2+9$ ($\times 4$, 1×4; $9\times$, 9×1)

$= \frac{26}{9}a^2-\frac{1}{4}b-3c+3$

d. $3x(x+2)(x-1)-6(-x^2+x-4)^2$ $(-x^2+x-4)$

$= 3x(x^2+1x-2)-6(x^4-x^3+4x^2$
$-x^3+x^2-4x$
$+4x^2-4x+16)$

$= 3x^3+3x^2-6x-6(x^4-2x^3+9x^2-8x+16)$

$-6x^4+12x^3-54x^2+48x-96$

$= -6x^4+15x^3-51x^2+42x-96$

e. $(1+y)^4$
you will learn how to do this faster later on in the course

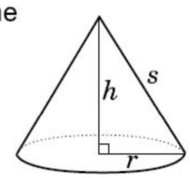
LONG FOIL 4 times

⋮

$= 1+4y+6y^2+4y^3+y^4$

4. Simplify the expression for the volume of the cone if $V=\frac{\pi r^2 h}{3}$, $r=2+x$, and $h=2x-3$

Cone



$V=\frac{\pi}{3}(2+x)^2(2x-3)$

$V=\frac{\pi}{3}(4+4x+x^2)(2x-3)$

$\frac{\pi}{3}($

$)$

$\therefore V=\frac{\pi}{3}(2x^3+5x^2-4x-12)$

5. There are certain rules that are different for monomials in comparison to polynomials. Complete the following questions and then summarize the rules.

MONOMIALS (1 term)

a. $(5x^2y^3)^2$

can distribute exponent over mult./div.

$= 5^2x^4y^6$

$= 25x^4y^6$

c. $\sqrt{25x^2y^4}$

can distribute root

$\sqrt{25}\sqrt{x^2}\sqrt{y^4}$

$= 5xy^2$

Technically $5|x|y^2$

e. $2x(3x)(4x^2)$

multiply all at once

$= 24x^4$

g. $\dfrac{\cancel{2xy}}{(\cancel{2}x^2\cancel{y})(4\cancel{x}y^3)}$

can cancel identical "factors" (things are multiplied)

$= \dfrac{1}{4x^2y^3}$

POLYNOMIALS → Have +/− HARDER some rules don't apply.

b. $(5x^2+y^3)^2$

FOIL can't distribute exp. in!

$= 25x^4 + 10x^2y^3 + y^6$

d. $\sqrt{25x^2-y^4}$

can't even simplify

f. $(2x)(3x+4x^2)$

do distribute

$= 6x^2 + 8x^3$

h. $\dfrac{2xy}{2x^2y-4xy^3}$ ← factor 1st

$= \dfrac{\cancel{2xy}}{\cancel{2xy}(x-2y^2)}$

$= \dfrac{1}{x-2y^2}$

This unit

6. Clarify the terms:

'inverse operations'

EXPAND
↳ multiply + Remove brackets

FACTOR
↳ divide into factors + get brackets in your answer

SIMPLIFY
- collect like terms
- multiply/divide like bases
⋮

EVALUATE
Sub in the given #

SOLVE
↳ isolate for a variable
"SAMDEB" if x appears once.
"Factor" if x appears more than once.

3

Factoring Polynomials & Solving by Factoring

Part of simplifying rational expressions will involve factoring the expressions in the numerator and also in the denominator to see if there are any cancellations. It is time to review factoring methods from grade 10.

1. Show how difference of square method can be used even if **numbers** are not perfect squares. → not factorable over integers

ex. $x^2 - 17 = (x+\sqrt{17})(x-\sqrt{17})$ ← factor over real #'s or irrational #'s.

2. Factor completely. NOTE: if you see brackets, and it asks to factor, don't expand!

a. $16c^4 - 81d^8$

$= (4c^2 + 9d^4)(4c^2 - 9d^4)$

can't

$= (4c^2+9d^4)(2c+3d^2)(2c-3d^2)$

can't

c is not squared ∴ stop.

b. $(2-x)^2 - 4(x+1)^2$

$= [(2-x)+2(x+1)][(2-x)-2(x+1)]$

$= (2-x+2x+2)(2-x-2x-2)$

$= (4+x)(-3x)$

c. $-24a^3b^2 + 48a^2b^2 - 4ab$

$= -4ab(6a^2b - 12ab + 1)$

d. $2\pi h(r+h)^2 + 4\pi(r+h)h^2$

$2\pi h(r+h)$ $\quad 2\pi \cancel{h}(\cancel{r+h})$

$= 2\pi h(r+h)(r+3h)$

$= \underbrace{2\pi h(r+h)}_{\text{G.C.F}}[\underbrace{(r+h)+2h}_{\text{"Leftovers"}}]$

e. $16x^4 - 8x^2 + 1 - 9x^6$

$4 \quad -1$
$4 \quad -1$

$= (4x^2-1)^2 - 9x^6$

$= [4x^2-1+3x^3][4x^2-1-3x^3]$

f. $56x^3y^2 + 18x^2y^2 - 8xy^2$

$2xy^2(28x^2 + 9x - 4)$

$2xy^2(4x-1)(7x+4)$

g. $27(g-h)^3 - 12(h-g)$

$27(g-h)^3 - 12(-g+h)$

$\dfrac{27(g-h)^{2}}{3(g-h)} + \dfrac{12(\cancel{g-h})}{3(\cancel{g-h})}$

$= \underbrace{3(g-h)}_{\text{GCF}}[\underbrace{9(g-h)^2 + 4}_{\text{Leftovers}}]$

can't do more.

h. $6(x+y)^3 + 4(x+y)^{-2}$ CALC

$2(x+y)^{-2}$ $\quad 2\cancel{(x+y)^{-2}}$

$= \underbrace{2(x+y)^{-2}}_{\text{G.C.F.}}[\underbrace{3(x+y)^5 + 2}_{\text{Leftovers}}]$

pull out smallest exp.

$= \dfrac{2[3(x+y)^5+2]}{(x+y)^2}$

i. $2x^4y^2 - 13x^2y + 20$

both neg.

$$= (1x^2y - 4)(2x^2y - 5)$$

j. $15k^2 - 7km - 2m^2$

$$(3k - 2m)(5k + 1m)$$

k. $6d^4 - 29d^2 + 35$

$$(2d^2 - 5)(3d^2 - 7) \leftarrow \text{factored over integers.}$$

$$= (\sqrt{2}d + \sqrt{5})(\sqrt{2}d - \sqrt{5})(\sqrt{3}d + \sqrt{7})(\sqrt{3}d - \sqrt{7}) \leftarrow \text{over real \#'s.}$$

l. $3y^2 + 5y - 10$

one neg.

no combination that works.

3. The last question above cannot be done with rational numbers. BUT there IS a solution! The answers are irrational as can be seen from the quadratic formula. Show how you can still record the factored version by using the qudratic formula.

$3y^2 + 5y - 10$ (a, b, c)

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{145}}{6}$$

$y \doteq 1.17$ and $y \doteq -2.84$

$a(x-s)(x-t)$

Record in factored form. $\boxed{y = a(x-r)(x-t)}$

approx $\doteq 3(y - 1.17)(y + 2.84)$

zeros

exact $=$

$$3\left(y - \frac{-5+\sqrt{145}}{6}\right)\left(y - \frac{-5-\sqrt{145}}{6}\right)$$

4. Please don't use the quadratic formula all the time. Can you think of reasons why I'm asking you not to resort to this method unless absolutely necessary?

- Takes longer
- Confusing if more than one variable is involved.
- may look different

ex. $6(x - \frac{1}{2})(x - \frac{1}{3})$

instead $(2x-1)(3x-1)$

$a(x-h)^2 + k$

$ax^2 + bx + c$

Solve by factoring.

5. $3x^2 - 14x + 8 = 0$

$(3x-2)(x-4)=0$

$3x-2=0$ OR $x-4=0$ — do in your head

$x=\frac{2}{3}$ $x=4$

6. $p^4 + 21p^2 = 100$

$p^4+21p^2-100=0$

$(p^2-4)(p^2+25)=0$

$(p+2)(p-2)(p^2+25)=0$

$p=-2$ $p=2$ $p=$ N/A.

7. $24x^2y - 16x^2y^2 + 8x^3y^2 = 0$

$(8)(x^2)(y)(3-2y+xy)=0$

$x=0$ $y=0$ $3-2y+xy=0$

CAN'T solve 1 equation but 2 unknowns.

8. $3\sin^2 A - 14\sin A = -8$

let $x=\sin A$

$3x^2-14x+8=0$

$(3x-2)(x-4)=0$

$x=\frac{2}{3}$ $x=4$

$\sin A=\frac{2}{3}$ $\sin A=4$

$A=\sin^{-1}\left(\frac{2}{3}\right)\doteq 42^\circ$ $A=\sin^{-1}(4)=$ N/A.

9. $256a^4 = 625$

$256a^4-625=0$

$(16a^2+25)(16a^2-25)=0$

$(16a^2+25)(4a+5)(4a-5)=0$

$a=$ N/A $a=-\frac{5}{4}$ $a=\frac{5}{4}$

10. $121-(3+b)^2 = 0$

$[11+(3+b)][11-(3+b)]=0$

$(14+b)(8-b)=0$

$b=-14$ $b=8$

11. The last two questions can be solved by isolating, why can't the other questions be isolated but the last two can be?

SAMDEB → variable appears once!

9. $\frac{256a^4}{256}=\frac{625}{256}$

$\sqrt[4]{a^4}=\pm\sqrt[4]{\frac{625}{256}}$

$a=\pm\frac{\sqrt[4]{625}}{\sqrt[4]{256}}=\pm\frac{5}{4}$

10. $121-(3+b)^2=0$

$\pm\sqrt{121}=\sqrt{(3+b)^2}$

$\pm 11 = 3+b$

6

Working with Complicated Fractions – if there is time

1. Recall the rules of dividing and adding fractions.

ex. $\frac{2}{5} \div \frac{1}{5}$

$= \frac{2}{5} \times \frac{5}{1} = \frac{10}{5} = 2$

ex. $\frac{5\times 2}{5\times 3} + \frac{1\times 3}{5\times 3}$

$= \frac{10+3}{15} = \frac{13}{15}$

$\frac{10}{15} + \frac{3}{15}$

2. Explain each mistake you should avoid and give a correct version of the result.

ex. $\frac{360}{2100} = \frac{10\times 36}{21\times 100}$

$= \frac{\cancel{(2)}\cancel{(5)}(2)\cancel{(3)}\cancel{(2)}(3)}{\cancel{(3)}(7)\cancel{(2)}\cancel{(2)}\cancel{(5)}(5)}$

$= \frac{6}{35}$

Errors Involving Fractions

Expression	Does NOT Equal
$\frac{1}{a}+\frac{1}{b}$	$\frac{1}{a+b}$
$\frac{a}{x+b}$	$\frac{a}{x}+\frac{a}{b}$
$\frac{\left(\frac{x}{a}\right)}{\left(\frac{b}{1}\right)}$	$\frac{bx}{a}$
$\frac{1}{3}x$	$\frac{1}{3x}$
$\left(\frac{1}{x}\right)+2$	$\frac{1}{x+2}$ or $\frac{3}{x}$

do LCD

can't split up denominator (can with numerators)

multiply by reciprocal

x should be in numerator

do LCD

Errors Involving Cancellation

Expression	Does NOT Equal
$\frac{a+bx}{a}$	$1+bx$
$\frac{a+ax}{a}$	$a+x$
$1+\frac{x}{2x}$	$1+\frac{1}{x}$

can't cancel unless everything is factored (multiplied)

not x^2

3. Simplify the following.

Not proper to have fraction within fraction.

a. $\frac{5}{\left(\frac{2\times 4}{3\times 4}-\frac{1\times 3}{4\times 3}\right)}$

$= \frac{5}{\left(\frac{5}{12}\right)}$

$= 5 \div \frac{5}{12}$

$= \frac{\cancel{5} \times 12}{1 \times \cancel{5}}$

$= 12$

b. $\frac{\left(\frac{x+2}{x}\right)}{\left(\frac{1}{x}-\frac{2}{1}\cdot\frac{x}{x}\right)}$ ← LCD

$= \left(\frac{x+2}{x}\right) \div \left(\frac{1-2x}{x}\right)$

$= \left(\frac{x+2}{\cancel{x}}\right)\cdot\left(\frac{\cancel{x}}{1-2x}\right) = \frac{\cancel{x}(x+2)}{\cancel{x}(1-2x)}$

$= \frac{(x+2)}{(1-2x)}$

← dividing by ZERO $\ddot\frown$
$a-b=0$
since $a=b$

Restrictions are values that are not part of the domain
ie. values that make the expression undefined dividing by zero

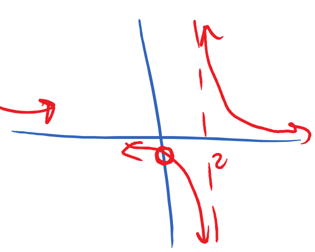
It's important to state them since these restrictions will be vertical asymptotes of rational graphs (adv. functions)

1. Factor both numerator + denominator
2. Cancel like factors (must be multiplied to do so)
3. State restrictions (look at denominators of every line in your solution, not just the end)

* The factored line before cancel is best.

$= \dfrac{\cancel{3x}}{\cancel{3x}(x-2)}$

$= \dfrac{1}{x-2}, \quad x \neq 2, x \neq 0$



$x^2-36 \neq 0$
$\sqrt{x^2} \neq \sqrt{36}$
$x \neq \pm 6$

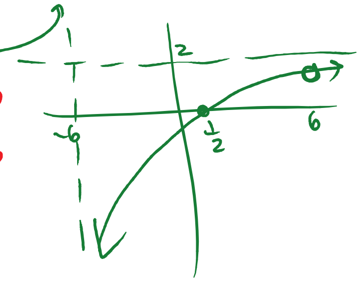
$= \dfrac{(2x-1)\cancel{(x-6)}}{(x+6)\cancel{(x-6)}}$

$= \dfrac{2x-1}{(x+6)}, \quad x \neq 6, \; x \neq -6$

$\binom{2}{1}$ $\quad$ $6\ \ \text{-}1\ \ 2\ \ 3$
$\quad$ $1\ \ \text{-}6\ \ 3\ \ 2$

$(x+6)(x-6) \neq 0$

$x+6 \neq 0 \quad x-6 \neq 0$
$x=-6 \quad x \neq 6$



c. $\frac{42x^3y^5}{-12x^4y^2}$

$\frac{xxx}{xxxx}$

$= \frac{7y^3}{-2x}$

$\frac{-12}{-12}(x^4)(y^2) \neq \frac{0}{-12}$

$x^4 \neq 0$ or $y^2 \neq 0$

$x \neq 0$ $y \neq 0$

d. $\frac{18-2m^2}{m^2-6m+9}$

$= \frac{-2m^2+18}{m^2-6m+9}$

$= \frac{-2(m^2-9)}{(m-3)(m-3)}$

$= \frac{-2(m+3)\cancel{(m-3)}}{(m-3)\cancel{(m-3)}}$

$= \frac{-2(m+3)}{m-3}$, $m \neq 3$

e. $\frac{2a-a^2}{(3a+4)(a-2)-2(a-2)}$

$= \frac{-a^2+2a}{(a-2)((3a+4)-2)}$

G.C.F Leftovers

$= \frac{-a\cancel{(a-2)}}{\cancel{(a-2)}(3a+2)}$

$= \frac{-a}{(3a+2)}$, $a \neq 2$, $a \neq -\frac{2}{3}$

f. $\frac{2x^2+1xy-3y^2}{2y^2-xy-x^2}$ $-3y^2$ $+2x^2$ * fix order before start.

$= \frac{(2x+3y)(x-y)}{(2y+x)(y-x)}$

$(-x+y)$

$\frac{\ }{-1}\ \frac{\ }{-1}$

$-1(x-y)$

$2y+x \neq 0$

$x \neq -2y$

$y-x \neq 0$

$y \neq x$

$\begin{pmatrix}2\\1\end{pmatrix}$ $\begin{matrix}1\\3\end{matrix}\begin{pmatrix}3\\-1\end{pmatrix}$

$\begin{pmatrix}2\\1\end{pmatrix}$ $\begin{pmatrix}1\\-1\end{pmatrix}$

$= \frac{(2x+3y)\cancel{(x-y)}}{-1(2y+x)\cancel{(x-y)}}$

$= \frac{2x+3y}{-1(2y+x)}$

Multiplying and Dividing Rational Expressions

1. Working with rational expressions is similar to working with rational numbers. Remind yourself of the rules of multiplying and dividing fractions.

fractions

ex. $\frac{2}{3} \cdot \frac{5}{4} = \frac{10}{12} = \frac{5}{6}$

ex. $5 \div \frac{2}{3} = \frac{5}{1} \times \frac{3}{2} = \frac{15}{2}$

2. What are the steps of multiplying rational expressions?

1.) Factor everything
2.) Multiply $\frac{\text{top} \times \text{top}}{\text{bottom} \times \text{bottom}}$
3.) Cancel like factors
4.) State restrictions (best to see right before you cancel)

3. What are the steps of dividing rational expressions?

1.) Factor everything before flip!!
2.) Flip the 2nd fraction and multiply
3.) Cancel like factors
4.) State restrictions of ALL denom (before flip + after flip.)

4. Simplify each of the following and state restrictions.

a. $\frac{k^2+k}{k^2-k} \times \frac{3k-21}{2k^2-11k-21}$

$= \frac{k(k+1)}{k(k-1)} \cdot \frac{3(k-7)}{(2k+3)(k-7)}$

$= \frac{\cancel{k}(k+1)(3)\cancel{(k-7)}}{\cancel{k}(k-1)(2k+3)\cancel{(k-7)}}$

$= \frac{3(k+1)}{(k-1)(2k+3)}$, $k \neq 0$, $k \neq 1$, $k \neq -\frac{3}{2}$, $k \neq 7$

b. $\frac{6x^2-15x}{3x^2+5x-12} \div \frac{4x^2-25}{3x+9}$

$= \frac{3x(2x-5)}{(3x-4)(x+3)} \div \frac{(2x-5)(2x+5)}{3(x+3)}$

$= \frac{3x\cancel{(2x-5)}}{(3x-4)\cancel{(x+3)}} \cdot \frac{(3)\cancel{(x+3)}}{\cancel{(2x-5)}(2x+5)}$

$= \frac{9x}{(3x-4)(2x+5)}$ $x \neq \frac{4}{3}$, $x \neq -3$, $x \neq \pm\frac{5}{2}$

c. $\dfrac{3a^2(b+2)}{b(3-a)^2} \times \dfrac{10b(a-3)}{a^3b^4}$

$-1\ \dfrac{-a+3}{-1\ \ -1}$

$= \dfrac{3a^2(b+2)(10)(b)(a-3)}{(b)(-1)(a-3)a^3b^4}$

$= \dfrac{-30(b+2)}{ab^4}$

$a \neq 3, 0$

$b \neq 0$

d. $\dfrac{6(x-1)}{x^2} \div \dfrac{3(x-1)}{x(x+2)}$

$= \dfrac{6(x-1)}{x^2} \cdot \dfrac{x(x+2)}{3(x-1)}$

$= \dfrac{6(x+2)}{3x}$

$= \dfrac{2(x+2)}{x}$, $x \neq 0, -2, 1$

e. $\dfrac{4x^2-25y^2}{(5y-2x)^2} \times \dfrac{4}{4x+10y}$

$= \dfrac{(2x+5y)(2x-5y)(4)}{(5y-2x)(5y-2x)(2)(2x+5y)}$ -1

$5y-2x \neq 0$

$2x+5y \neq 0$

$\dfrac{-4}{2(5y-2x)}$

$= \dfrac{2}{2x-5y}$ OR $= \dfrac{-2}{5y-2x}$

$\dfrac{2}{-1(5y-2x)}$

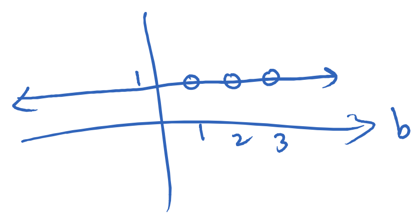
f. $\dfrac{3b^2-9b+6}{2b^2-10b+12} \div \dfrac{3-3b}{6-2b}$

$= \dfrac{3(b^2-3b+2)}{2(b^2-5b+6)} \div \dfrac{-3(b-1)}{-2(b-3)}$

$= \dfrac{3(b-2)(b-1)}{2(b-2)(b-3)} \cdot \dfrac{-2(b-3)}{-3(b-1)}$

$= \dfrac{-6}{-6}$

$= 1$, $b \neq 2, 3, 1$



g. $\dfrac{3y^2-7y+2}{y^2+3y+2}\times\dfrac{8y+8}{4y-8}$

$= \dfrac{2(3y-1)}{(y+2)}$ $y \neq 2, -1, -2$

h. $\dfrac{\dfrac{(m+n)}{5m^4n}}{\dfrac{(m^2-n^2)}{15n^3m}}$

$= \dfrac{(m+n)}{5m^4n} \div \dfrac{(m^2-n^2)}{15n^3m}$

$= \dfrac{(m+n)}{5m^4n} \div \dfrac{(m+n)(m-n)}{15n^3m}$

$= \dfrac{\cancel{(m+n)}(15n^3m)}{5m^4n\cancel{(m+n)}(m-n)}$

$= \dfrac{3n^2}{m^3(m-n)}$

$m \neq 0$
$n \neq 0$
$m+n \neq 0$
$m-n \neq 0$

5. Create an expression that has the following conditions.

Zeros at $x=-2$ and $x=\frac{1}{3}$ and restrictions at $x \neq -5$ and $x \neq \frac{2}{7}$

x-int

numerator = 0 gives x-int
denominator = 0 gives restrictions

$\dfrac{(x+2)(3x-1)}{(x+5)(7x-2)}$

6. For the cylinder,
   a. simplify the ratio of its surface area to volume. fraction
   b. if height is 3 times as long as radius, simplify the ratio again.

$V = \pi r^2 h$
$SA = 2\pi r^2 + 2\pi r h$

ⓐ $\dfrac{SA}{V} = \dfrac{(2\pi r^2 + 2\pi r h)}{\pi r^2 h}$

$= \dfrac{2\cancel{\pi}\cancel{r}(r+h)}{\cancel{\pi} r^{\cancel{2}} h}$

$\dfrac{SA}{V} = \dfrac{2(r+h)}{rh}$, $r \neq 0$, $h \neq 0$

ⓑ $h = 3r$

$\dfrac{SA}{V} = \dfrac{2(r+3r)}{r(3r)}$

$= \dfrac{2(4r)}{3r^2}$

$= \dfrac{8r}{3r^2} = \dfrac{8}{3r}$

$\dfrac{8\cancel{r}}{3\cancel{r}r}$

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$24 = (4)(6) = (2)(2)(2)(3)$

$90 = (45)(2) = (9)(5)(2) = (3)(3)(5)(2)$

Adding and Subtracting Rational Expressions

1. Working with rational expressions is similar to working with rational numbers. Remind yourself of the rules of adding and subtracting fractions.

ex. $\frac{1\cdot 5\cdot 3}{24\cdot 5\cdot 3} + \frac{7\cdot 2\cdot 2}{90\cdot 2\cdot 2} = \frac{15 + 28}{(2)(2)(2)(3)(5)(3)} = \frac{43}{360}$

* LCD = record all factors of 1st denominator then include any missing parts from 2nd denom.

2. What are the steps of adding/subtracting rational expressions?

1. Factor only the denominators to find LCD
2. Create the LCD by multiplying by the missing factors.
3. Simplify the numerator only (don't expand denominator) ← or can't see restrictions or potential cancellations
4. Check for cancellations, state restrictions "RARE"

3. Simplify each of the following and state restrictions.

with MONOMIALS

a. $\frac{3xy(6-x)}{3xy\cdot x^2y} - \frac{3x^2(3+y)}{3x^2\cdot xy^2} + \frac{1}{3x^3y^2}$ LCD $= x^2y\,y(3)x$

$= \frac{3xy(6-x) - 3x^2(3+y) + 1}{3x^3y^2}$ ← simplify top: factor ~~X~~ or expand then factor

$= \frac{18xy - 3x^2y - 9x^2 - 3x^2y + 1}{3x^3y^2}$

$= \frac{18xy - 6x^2y - 9x^2 + 1}{3x^3y^2}$ $x \neq 0$, $y \neq 0$

only POLYNOMIALS

b. $\frac{5x+2}{25x^2-1} - \frac{3x-1}{25x^2+10x+1}$

$= \frac{(5x+2)}{(5x+1)(5x-1)} - \frac{(3x-1)}{(5x+1)(5x+1)}$

$= \frac{(5x+1)(5x+2) - (5x-1)(3x-1)}{(5x+1)(5x-1)(5x+1)}$

$= \frac{25x^2+15x+2 - (15x^2-8x+1)}{(5x+1)(5x-1)(5x+1)}$

$\frac{10x^2+23x+1}{(5x+1)(5x-1)(5x+1)}$ $x \neq \pm\frac{1}{5}$

c. $\frac{2x-1}{12} + \frac{(3x-2)\cdot 4}{3\cdot 4} - \frac{(x+1)\cdot 6}{(2)\cdot 6}$

$= \frac{(2x-1) + 4(3x-2) - 6(x+1)}{12}$

$= \frac{2x-1+12x-8-6x-6}{12}$ ← mistake

$= \frac{8x-15}{12}$, no restrictions

OR $\frac{2}{3}x - \frac{5}{4}$

d. $\frac{x+3}{x^2-4} - \frac{3-x}{4-x^2}$ $\frac{-x^2+4}{-1\quad -1}$ $-1(x^2-4)$

$= \frac{x+3}{x^2-4} + \frac{3-x}{x^2-4}$

$= \frac{x+3+3-x}{x^2-4}$

$= \frac{6}{x^2-4}$, $x \neq \pm 2$ can't see restrictions unless it's factored.

$= \frac{6}{(x+2)(x-2)}$

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e. $\frac{3}{(x+1)} - \frac{4}{(x-2)}$

$= \frac{3(x-2) - 4(x+1)}{(x+1)(x-2)}$

$= \frac{-x-10}{(x+1)(x-2)}$ $x \neq -1, 2$

$\frac{}{1x} - \frac{}{2x}$

$\frac{}{2x}$

f. $\frac{x-6}{x^2-3x+2} - \frac{3x+2}{x^2-x-2}$

$= \frac{(x-6)}{(x-2)(x-1)} - \frac{(3x+2)}{(x-2)(x+1)}$

$= \frac{(x-6)(x+1) - (3x+2)(x-1)}{(x-2)(x-1)(x+1)}$ $x \neq 2, 1, -1$

$= \frac{x^2-5x-6-(3x^2-x-2)}{(x-2)(x-1)(x+1)}$

$= \frac{-2x^2-4x-4}{(x-2)(x-1)(x+1)} = \frac{-2(x^2+2x+2)}{(x-2)(x-1)(x+1)}$

g. $\frac{x}{(x+2)(x-3)} + \frac{4}{-1(-2-x)(x-4)}$

$= \frac{x}{(x+2)(x-3)} - \frac{4}{(x+2)(x-4)}$

$= \frac{x(x-4) - 4(x-3)}{(x+2)(x-3)(x-4)}$

$= \frac{x^2-4x-4x+12}{(\ )(\ )(\ )}$

$= \frac{x^2-8x+12}{(x+2)(x-3)(x-4)}$ OR $\frac{(x-6)(x-2)}{(x+2)(x-3)(x-4)}$ $x \neq -2, 3, 4$

h. $\frac{x^2-x}{2x^2-3x} - \frac{3x^2+1-4x}{3-5x+2x^2}$

$2x^2-5x+3$

$= \frac{(x^2-x)}{x(2x-3)} - \frac{(3x^2-4x+1)}{(2x-3)(x-1)}$

$= \frac{(x^2-x)(x-1) - (3x^2-4x+1)(x)}{x(2x-3)(x-1)}$

$= \frac{x^3-x^2-x^2+x - (3x^3-4x^2+x)}{x(2x-3)(x-1)}$

$= \frac{-2x^3+2x^2}{x(2x-3)(x-1)}$ RARE $= \frac{-2x^2(x-1)}{x(2x-3)(x-1)}$

$= \frac{-2x}{(2x-3)}$, $x \neq \frac{3}{2}, 0, 1$

i. $\frac{2m^3}{12m^2-8m} + \frac{12m}{4m-6m^2}$

$= \frac{\cancel{2m^3}\, m^2}{2\cancel{4m}(3m-2)} + \frac{-6\ \cancel{12m}}{-\cancel{2m}(3m-2)}$

$= \frac{m^2}{2(3m-2)} - \frac{6}{(3m-2)}$

$= \frac{m^2 - 6(2)}{2(3m-2)}$

$= \frac{m^2-12}{2(3m-2)}$, $m \neq \frac{2}{3}, 0$

j. $\frac{x+y}{(x+y)^2-5(x+y)-6} + \frac{y}{3x(x+y)+3x}$

a^2-5a-6

$(a-6)(a+1)$

$(x+y-6)(x+y+1)$

$= \frac{(x+y)(3x)}{(x+y-6)(x+y+1)(3x)} + \frac{y(x+y-6)}{3x(x+y+1)(x+y-6)}$

$= \frac{[(x+y)(3x) + y(x+y-6)]}{3x(x+y-6)(x+y+1)}$

$= \frac{(3x^2+3xy+xy+y^2-6y)}{3x(x+y+6)(x+y+1)}$

$= \frac{(3x^2+4xy+y^2-6y)}{3x(x+y+6)(x+y+1)}$

$3x \neq 0$, $x \neq 0$; $x+y+6 \neq 0$, $x \neq -6-y$; $x+y+1 \neq 0$, $x \neq -1-y$

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Practice: LCD only

c) $\dfrac{1}{42x^3y} + \dfrac{3}{70x^2y} - \dfrac{2}{45xy^4z} = \dfrac{\quad}{?}$

d) $\dfrac{3}{2x^2(x+2)} - \dfrac{1}{6x(x+2)^2(x-3)} = \dfrac{\quad}{?}$

e) $\dfrac{1}{2x^2-8x} - \dfrac{1}{2x^2-7x-4} = \dfrac{\quad}{?}$