Quadratics Unit 3

Tentative TEST date_______________

Big idea/Learning Goals

This unit is mostly review from grade 10. However, you will apply function terminology as you describe domain, range, transformations and inverses of quadratics. You will also see more advanced applications of quadratics in real life. Finally, you will make an extension from solving linear systems to solving linear-quadratic systems.

Corrections for the textbook answers:
Sec 3.1 #16 $-1/105(x+12.5)^2+1805/84$

Success Criteria

- ☐ I understand the new topics for this unit if I can do the practice questions in the textbook/handouts

Date	pages	Topics	# of quest. done? You may be asked to show them	**Questions I had difficulty with** ask teacher before test!
	2-4	Properties of Quadratics Section 3.1		
	5-7	Max & Min of Quadratics Section 3.2 & Handout		
	8-9	Inverses of Quadratics Section 3.3		
	10-12	Operations with Radicals Section 3.4 & three Handouts (text doesn't have division!)		
	13-15	Solve Quadratics Section 3.5 & Handout		
		QUIZ no calculators (radicals 3.4 and set up word prob like 3.1 #16, 3.2 #12,15, 3.3 #16, 3.5 #9,10,11,12,14)		
	16-17	# of Zeros of Quadratics Section 3.6		
	18-19	Systems Section 3.8		
	20	Solving Square Root Equations – if there is time Handout		
	21-23	Families of Quadratics Section 3.7 & Handout		
		REVIEW		

Reflect – previous TEST mark ___________, Overall mark now___________.

Properties of Quadratics

Review from grade 10:

1. What do differences of the dependent variable tell you about the relation given?

2. Determine what type of functions are these?

x	$f(x)$
0	-90
2	-30
4	6
6	18
8	6
10	-30

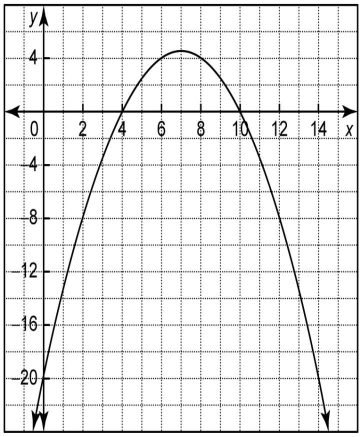
x	$f(x)$
1	-8
7	2
13	12
19	22
22	27
25	32

3. Find the equations for the functions above.

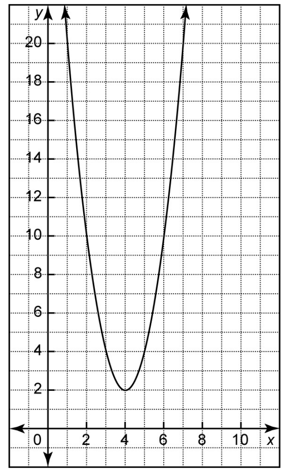
4. Quadratic equations can be written in 3 forms. What are they? What is the key information that can be found from each form?

5. Find the equation of the following parabolas. Use the most convenient form.

a.



b.



c. The parabola is vertically compressed by 2, opens down, goes through the point (-6, 10) and has a y-intercept of 4.

d. The parabola has the axis of symmetry at 3 and x-intercept at 7 and y-intercept at -2.

6. Sketch the following parabolas

FACTORED form

a. $y=(4-2x)(x+8)$

VERTEX form

b. $y=1.5(x+2)^2+1$

STANDARD form

c. $y=x^2+10x-5$

7. A golfer hits a ball into the air. The relationship between the height of the ball, h metres, and the time, t seconds, can be modelled by the equation $h = -4.9t^2 + 19.6t$

   a. Determine the length of time that the ball is in the air.

   b. Determine the maximum height.

   c. What is the domain and range in the context of the real life problem.

Maximum and Minimum of Quadratics

1. There are **TWO** ways of finding vertex. One method is faster if you already know zeros. However not all parabolas have zeros so you must review another method of finding the vertex. The two methods of find the vertex are:

2. Find the vertex in **two** ways for the following parabola. Leave answers as exact values, this is not a word problem so you cannot round.

$$y = \frac{2}{3}x^2 - 4x - 18$$

3. How come you are not allowed to just multiply by the LCD in the previous question to get rid of the fraction?

4. What are the steps of completing the square?

5. For word problems sometimes you must find zeros and sometimes the vertex. What are the words you should look out for that would indicate what you must find?

Revenue problem

6. The circus sells 1000 tickets for $6 each. The circus owners want to increase their revenues, so they increase prices. They have noticed that ticket sales decrease by 45 tickets every time the price increases by $0.50.
   a. What is the revenue equation?
   b. What price will maximize the revenue?
   c. What is the quantity at maximum?
   d. What is the maximum revenue?

Profit problem

7. If the total costs are $600 flat and $90 per item, and the revenue is given by $R(x) = 150x - x^2$, where x is the number of items sold.
   a. What is the profit equation?
   b. Determine the items sold that would produce the maximum profit.
   c. Find the number of items to sell to have the profit of $100.
   d. What is the initial profit?

Fence (or rope) off an area problem

8. For a park swimming area, 4500 m of line is used to mark off the permissible area. One side not roped off is next to the beach. Find the dimensions of the swimming area that will make it a maximum

Falling object problem

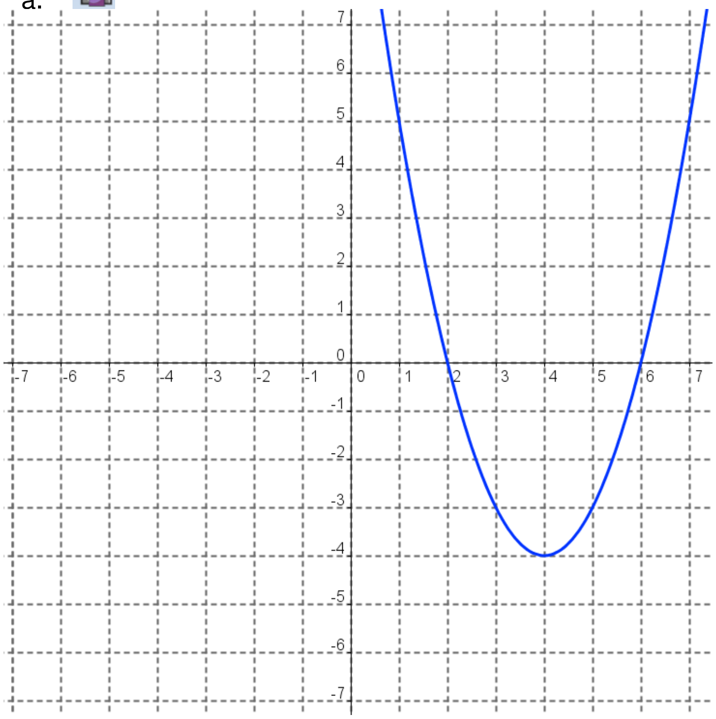
9. A water balloon is catapulted into the air so that its height, in meters after t seconds is given by a quadratic function. The balloon's initial speed is 27 meters/second and it was released from the height of 3 meters. When does the balloon reach the height of 35m?

Inverses of Quadratics

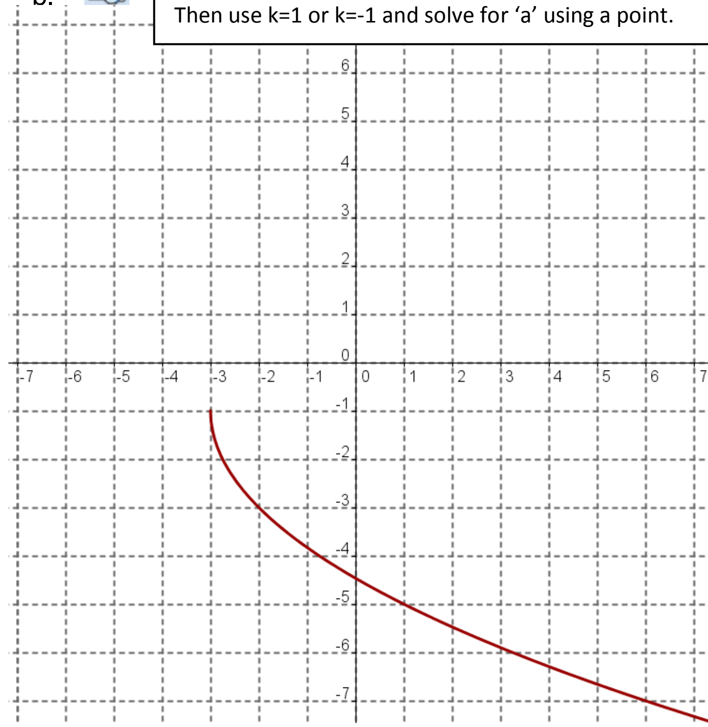
For square roots consider if there are any reflections that would make the 'k' negative.
Then use k=1 or k=-1 and solve for 'a' using a point.

1. Find the equations for each of the following:

a.



b.



2. Find the equations for the inverse **functions**.

3. What would be the domain and range of the inverse **functions**?

4. Sketch the inverse graphs for the above functions. Do your answers for domain and range make sense?


5. The cost function in thousands of dollars for x thousand of items sold is $C(x) = 0.75x^2 - 1.5x$

   a. What is the range of this function on the domain of $0 \le x \le 10$?

   b. How should the domain $0 \le x \le 10$ be further restricted so that the inverse is a function?

   c. Write the model for the items sold as a function of cost.

Operations with Radicals

1. What are **radical** expressions? What is considered to be a **mixed radical** and what is an **entire radical**?

2. Why learn about them?

3. When simplifying radicals it is important to know the perfect square numbers as they will help you to ***reduce*** the entire radical into a mixed form. List the first few perfect square numbers.

4. What are the steps of **simplifying/reducing** radicals?

- If the number under your radical cannot be divided evenly by any of the perfect squares, your radical is ________________________________

- If you do not choose the largest perfect square to start the process, you will have to ________________________________

5. Simplify the following

a. $\sqrt{147}$

b. $2\sqrt{48p^5}$

c. $3\sqrt{32}$

d. $\sqrt{200a^4b^3}$

6. What are the steps of **adding/subtracting** radicals?

7. Simplify the following

a. $\sqrt{12}+2\sqrt{48}-5\sqrt{175}$

b. $2\sqrt{18}-3\sqrt{12}+4\sqrt{50}+\sqrt{27}$

8. What are the steps of **multiplying** radicals?

9. Simplify the following

a. $3\sqrt{12}\times 4\sqrt{6}$

c. $5\sqrt{14x}\left(2\sqrt{x}-4\sqrt{7x}\right)$

b. $\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)$

d. $\left(-2-3\sqrt{5}\right)\left(5-\sqrt{5}\right)$

10. Show that the steps of **dividing** radicals are similar to multiplying radicals with the following example $\dfrac{4\sqrt{32}}{12\sqrt{12}}$

11. It is considered improper form to leave radicals in the denominator. You must rationalize the denominators for your final answers. What does the word rationalize mean to you? Make sure your answer to the previous question is in proper form.

12. What are the steps of **rationalizing MONOMIAL** denominators?

13. What are the steps of **rationalzing BINOMIAL** denominators?

14. Simplify the following

a. $\dfrac{2}{\sqrt{6}}$

b. $\dfrac{6\sqrt{5}}{7\sqrt{2}}$

c. $\dfrac{\sqrt{3x^2y^3}}{4\sqrt{5x^3y}}$

e. $\dfrac{-12\sqrt{24}}{3\sqrt{2}}$

d. $\dfrac{5-\sqrt{3}}{2\sqrt{3}-\sqrt{5}}$

f. $\dfrac{4}{\sqrt{2}-5\sqrt{3}}$

15. All of the above rules are for square root questions. How can you reduce cube root or other type of roots?

Solve Quadratics

1. If the quadratic word problem doesn't ask for maximum or minimum then the question often reduces to finding the x-intercepts of the quadratic. Explain why? (Use the following word problem in your explanation.)

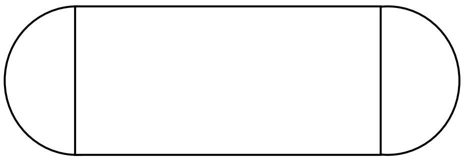
   "The hypotenuse of a right triangle is 6 cm more than the shorter leg. The longer leg is three more than the shorter leg. Find the lengths of all three sides."

2. There are **THREE** ways to find solutions for the independent variable in quadratic type question. What are they?

3. Solve the above word problem showing more than one method:

Geometry problem

4. This is a diagram of a practice field. The track and field coach wants two laps around the field to be 1000m. But the Phys. Ed. department wants the rectangular field to be as large as possible. What dimensions would satisfy both conditions?



Translating English into Math problem

5. Find three consecutive integers such that four times the sum of all three is 2 times the product of the larger two

Motion problem

6. Dan and Sue set off at the same time on a 42 km go-cart race. Dan, drives 0.4 km/min faster than Sue, but has to stop en route and fix his go-cart for one-half hour. This stop costs Dan to arrive 15 min after Sue. How fast was each person driving?

of Zeros of Quadratics

1. Sometimes it is not necessary to actually find a solution but to know if there IS a solution at all, and if there is, to know how many solutions there are. Summarize how you can tell the number of solutions from the following forms of quadratics. Include examples in your explanations.

VERTEX form STANDARD form FACTORED form

2. Find the value of the constant so that there is only one zero $y = -4x^2 + bx - 10$

3. Consider a parabola that has two zeros, state the type(s) transformation(s) that will NOT affect the number of zeros.

4. For producing a certain product, the cost is $C(x) = 1600 + 1500x$ and revenue is $R(x) = 1600x - x^2$, where x is the number of items sold.
   a. What is the profit equation?
   b. Can this company break even? If yes, in how many ways?

 5. Show a proof of the quadratic formula. Hint: complete the square using the standard form without actual numbers for a, b, c

Systems

1. In grade 10 you have learned how to solve for points of intersection of linear systems. There are three ways of solving linear systems. What are they?

2. Solve the $\begin{array}{l} y^2 + x^2 = 64 \\ y - 2x = -3 \end{array}$ system.

Explain the solution using pictures.

3. Find k so that the system has only one solution.

$$y = -3x^2 + x - 1$$
$$y = -x + k$$

4. Predict the number of solutions the following system will have. Explain your reasoning.

$$y = -2x^2 + x - 3$$
$$y = -x - 2$$

5. A toy rocket's flight path is given by $h(t) = -5t^2 + 3t + 2$ and a bird's flight path is $h(t) = -5t + 4$ where t is time in seconds. What time did the bird and the toy rocket collide?

Solving Square Root Equations – if there is time

1. The only way to get rid of a square root in an equation is to square both sides of the equation. But before you do that, make sure the square root is isolated. Also make sure you square SIDES not TERMS! Explain why that is so.

2. Solve the following. Check your answers if all of them actually make sense.

a. $\sqrt{x-1}+7=x$

b. $\sqrt{9x^2+4}=3x+2$

c. $\left(2x^2-7\right)^{\frac{1}{2}}+x=3$

d. $\sqrt{x-3}-\sqrt{x}=3$

Families of Quadratics

1. Sometimes the given problems may have many solutions only because not enough information was provided to make the solution unique. The many solutions that satisfy the given conditions are called a **familty** of functions. Give a family of quadratics that share the same vertex, and open down.

2. Find a family of functions that have zeros at -9, and 5

   a. Give another condition that will ensure that this family of many quadratics is one unique parabola.

3. Find a family of functions that have zeros at, $2+\sqrt{3}$ and $2-\sqrt{3}$

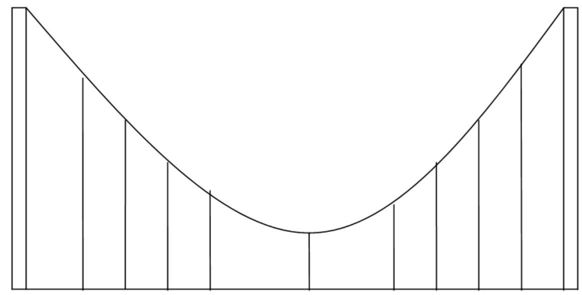
   a. Find the unique equation of the family that passes through the point (1, -5).

4. Give two non-equivalent functions that satisfy the following conditions. $f(2)=g(2)=0$ and $f(x)$ and $g(x)$ have the same y-intercept $=-3$

5. Sometimes a given problem can have many different-looking equations that give the same answer. This can happen when many frames of reference can be chosen, i.e. the x and y axes can be placed differently since the question doesn't specify where to place them. (Note: these are NOT considered to be a family of functions since graphing them in the same frame of reference/coordinate system will yield different graphs)

Find TWO different equations using different frames of reference that can model the following problem:

"Many suspension bridges hang from cables that are supported by two towers. The shape of the hanging cables is very close to a parabola. A typical suspension bridge has large cables that are supported by two towers that are 20 m high and 80 m apart. The bridge surface is suspended from the large cables by many smaller vertical cables. The shortest vertical cable is 4 m long."



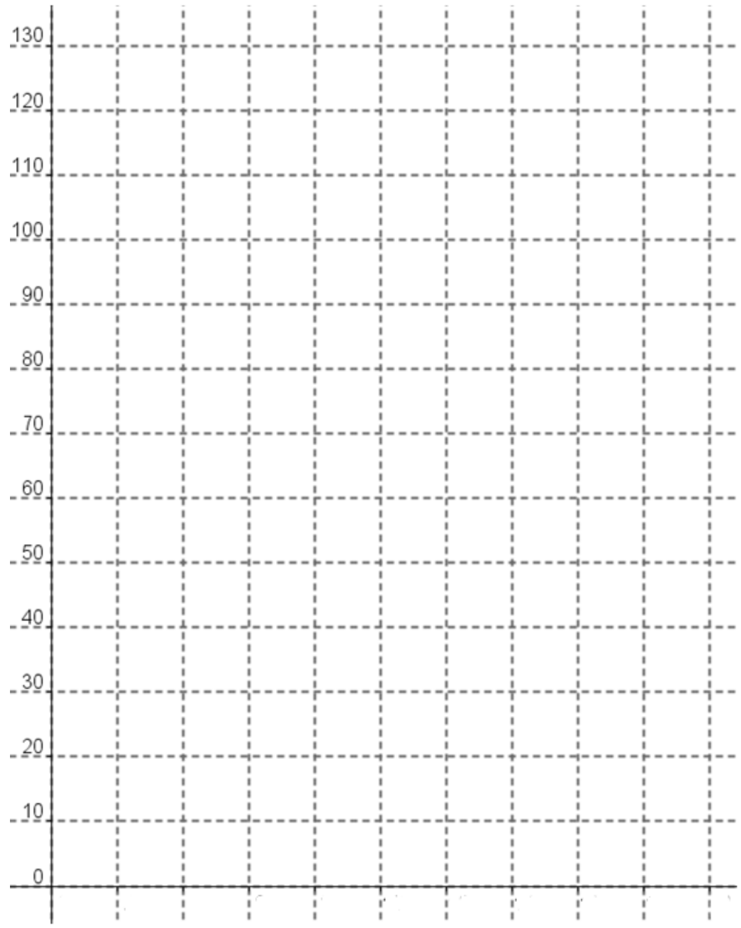
a. If another vertical support is needed to be placed 10 m away from the end, and the available support pole is 12 m tall, prove that it will NOT be tall enough to support the cable.

b. Where should the 12 m support pole be placed?

c. Does it matter which equation is used to answer questions a and b? Why or why not?

6. Sometimes a given problem can have many very close approximate solutions. This occurs when dealing with raw real life data because it is not possible to have perfect measurements and real life situations can fluctuate from many different variables. (Note: again, these are NOT considered to be a family of functions, since the approximate equations will yield slightly different results)
   a. Plot the given data as a scatter plot and create a quadratic curve of best fit.

Year	Number (in thousands) of people over 100 years old in a country
1994	68
1996	55
1998	54
2000	50
2002	55
2004	65
2006	70
2008	94
2010	110



   b. Determine the approximate equation for your graph that will model this senario.

   c. Use your equation to **interpolate** (find the value within data points) how many people will be over 100 years old in 2007.

   d. Use your equation to **extrapolate** (find the value outside of data points) how many people will be over 100 years old in 2012.

   e. Can this model be used to predict number of people very far in the future? Why or why not?

Summary:
Family of functions must __________________ the given conditions in the __________________ coordinate system(frame of referene) and answers must be ________________.