S14QuadraticsNOTES

QuadraticsNOTES

see below

Quadratics Unit 3

Tentative TEST date_______________

Big idea/Learning Goals

This unit is mostly review from grade 10. However, you will apply function terminology as you describe domain, range, transformations and inverses of quadratics. You will also see more advanced applications of quadratics in real life. Finally, you will make an extension from solving linear systems to solving linear-quadratic systems.

Corrections for the textbook answers:
Sec 3.1 #16 $-1/105(x+12.5)^2+1805/84$

Success Criteria

☐ I understand the new topics for this unit if I can do the practice questions in the textbook/handouts

finish Expressions +

Date	pages	Topics	# of quest. done? You may be asked to show them	Questions I had difficulty with ask teacher before test!
Day 4	2-4	Properties of Quadratics Section 3.1 # 1,5,7,11		
	5-7	Max & Min of Quadratics Section 3.2 & Handout # 3-13		
	8-9	Inverses of Quadratics Section 3.3 # 4,9 10,13		
Day 5	10-12	Operations with Radicals Section 3.4 & three Handouts (text doesn't have division!)	3,4 # 4,5,6,7	Handout 4 # 15-20
	13-15	Solve Quadratics Section 3.5 & Handout all		
		QUIZ no calculators (radicals 3.4 and set up word prob like 3.1 #16, 3.2 #12,15, 3.3 #16, 3.5 #9,10,11,12,14)		
skip p.17	16-17	# of Zeros of Quadratics Section 3.6 # 4,8,9,13		
Day 6	18-19	Systems Section 3.8 # 6,10,12		
	20	Solving Square Root Equations – if there is time Handout # 1-10		
+ start Exp	skip p.22,23 21-23	Families of ~~Quadratics~~ Section 3.7 & Handout all		
		REVIEW		

Reflect – previous TEST mark __________, Overall mark now__________.

Properties of Quadratics

Review from grade 10:

1. What do differences of the dependent variable tell you about the relation given?

next y − prev. y

If 1st differences in y are constant then it's LINEAR
If 2nd diff. " " " " " " QUADRATIC

* double check that x's go up evenly.

2. Determine what type of functions are these?

x	$f(x)$	1st	2nd
0	-90		
2	-30	$-30--90=60$	
4	6	36	-24
6	18	$18-6=12$	-24
8	6	-12	-24
10	-30	-36	-24

vertex

∴ QUAD.

x	$f(x)$	1st
1 x_1	-8 y_1	
7 x_2	2 y_2	10
13	12	10
19	22	10
22	27	10
25	32	10

∴ LINEAR

3. Find the equations for the functions above.

$y=a(x-h)^2+k$

$y=a(x-6)^2+18$ pt. $(4,6)$

$6=a(4-6)^2+18$

$6=a(-2)^2+18$

$6=a(4)+18$

$6=4a+18$

-18

$-12=4a$

$-3=a$

$\boxed{\therefore y=-3(x-6)^2+18}$

$y=mx+b$

$m=\frac{rise}{run}=\frac{\Delta y}{\Delta x}=\frac{y_2-y_1}{x_2-x_1}$

$m=\frac{2+8}{7-1}=\frac{10}{6}=\frac{5}{3}$

$y=\frac{5}{3}x+b$ pt. $(1,-8)$

$-8=\frac{5}{3}(1)+b$

$\frac{3\times}{3\times1}-8-\frac{5}{3}=b$

$-\frac{29}{3}=b$

$\boxed{\therefore y=\frac{5}{3}x-\frac{29}{3}}$

4. Quadratic equations can be written in 3 forms. What are they? What is the key information that can be found from each form?

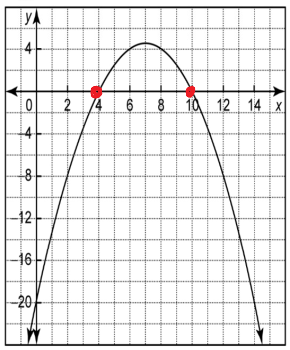
standard:
$y=ax^2+bx+c$
opens up/down
step pattern.
y-int $(0,c)$

factored:
$y=a(x-r)(x-t)$
x-ints / zeros
$(r,0)$ $(t,0)$

vertex:
$y=a(x-h)^2+k$
vertex (h,k)
* see transformations in this form.

5. Find the equation of the following parabolas. Use the most convenient form.

a.



$y=a(x-r)(x-t)$

$y=a(x-4)(x-10)$ pt. $(0,-20)$

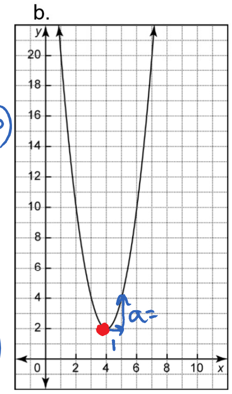
$-20=a(0-4)(0-10)$

$-20=40a$

$-\frac{1}{2}=a$

$\therefore y=-\frac{1}{2}(x-4)(x-10)$

b.



$y=a(x-h)^2+k$

$y=2(x-4)^2+2$

c. The parabola is vertically compressed by 2, opens down, goes through the point (-6, 10) and has a y-intercept of 4.

$a=-\frac{1}{2}$

$y=ax^2+bx+c$

$y=-\frac{1}{2}x^2+bx+4$ pt. $(-6,10)$

$10=-\frac{1}{2}(-6)^2+B(-6)+4$

$10-4+18$

$24=-6B$ $B=-4$ $\boxed{\therefore y=-\frac{1}{2}x^2-4x+4}$

d. The parabola has the axis of symmetry at 3 and x-intercept at 7 and y-intercept at -2.

$y=a(x-r)(x-t)$

$y=a(x+1)(x-7)$ pt. $(0,-2)$

$-2=a(0+1)(0-7)$

$-2=-7a$

$\frac{2}{7}=a$

$\therefore y=\frac{2}{7}(x+1)(x-7)$

6. Sketch the following parabolas

FACTORED form

a. $y=(4-2x)(x+8)$

method1 ① expand to standard

② complete sq. to vertex

③ sketch.

method2 ① zeros $x=-8$ $x=2$

$4-2x=0$ OR $x+8=0$

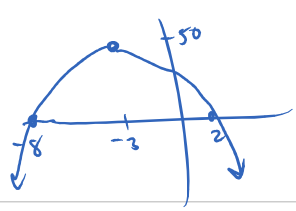
$4=2x$ $x=-8$

$2=x$

② axis of symm $=\frac{\text{add zeros}}{2}=\frac{-8+2}{2}=\frac{-6}{2}=-3$

③ opt. value = sub axis of symm in $=(4-2(-3))(-3+8)=50$

vertex $(-3,50)$



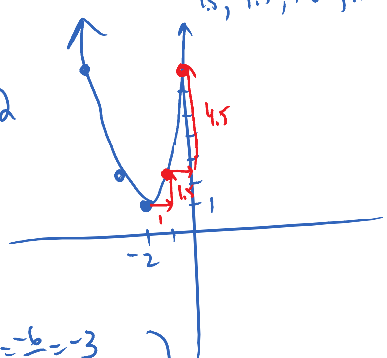
VERTEX form

b. $y=1.5(x+2)^2+1$

vertex $(-2,1)$

step $(1,3,5\ldots)1.5$

$1.5, 4.5, 7.5, \ldots$



STANDARD form

c. $y=x^2+10x-5$

complete square then do it that way

7. A golfer hits a ball into the air. The relationship between the height of the ball, h metres, and the time, t seconds, can be modelled by the equation $h = -4.9t^2 + 19.6t$

   a. Determine the length of time that the ball is in the air.

want zeros
→ factor OR
→ quad. formula.

$h = -4.9(t)(t-4)$

$\therefore t = 0$ and $t = 4$

$\therefore$ The ball was in the air for 4 sec.

   b. Determine the maximum height.

want vertex
→ complete square
→ $\boxed{\text{a.of s} = \frac{\text{add zeros}}{2}}$ then $\boxed{\text{opt. val} = \text{sub a.of s. into eqn.}}$

$\text{a.of s} = \frac{0+4}{2} = 2$

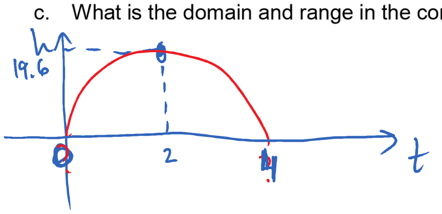
$\text{opt. val} = -4.9(2)^2 + 19.6(2)$
$= 19.6$

$\therefore$ vertex $(2, 19.6)$

$\therefore$ Max height is 19.6m.

   c. What is the domain and range in the context of the real life problem.

$\{t \mid t \in \mathbb{R},$



$D = \{t \in \mathbb{R} \mid 0 \leq t \leq 4\}$

$R = \{h \in \mathbb{R} \mid 0 \leq h \leq 19.6\}$

at vertex.

Maximum and Minimum of Quadratics

1. There are **TWO** ways of finding vertex. One method is faster if you already know zeros. However not all parabolas have zeros so you must review another method of finding the vertex. The two methods of find the vertex are:

(1.) Completing Square (works all the time!)

(2.) $h=$ a.o.f.s = $\frac{\text{add zeros}}{2}$ then $k=$ opt. val = sub in a.o.f.s then vertex = (h, k)

2. Find the vertex in **two** ways for the following parabola. Leave answers as exact values, this is not a word problem so you cannot round.

$$y=\frac{2}{3}x^2-4x-18$$

(1)

$$y=\frac{2}{3}(1x^2-6x+9-9)-18$$

$$y=\frac{2}{3}(x-3)(x-3)\underbrace{-9\left(\frac{2}{3}\right)}_{-6}-18$$

$$y=\frac{2}{3}(x-3)^2-24 \qquad \therefore \text{vertex } (3, -24)$$

(2) factor $y=\frac{2}{3}x^2-4x-18$

$$y=\frac{2}{3}(x^2-6x-27)$$

$$y=\frac{2}{3}(x-9)(x+3) \leftarrow \text{factored form}$$

zeros $x=9$ $x=-3$

$$\text{a.o.f.s} = \frac{9+-3}{2}=\frac{6}{2}=3$$

$$\text{opt. val} = \frac{2}{3}(3)^2-4(3)-18$$

3. How come you are not allowed to just multiply by the LCD in the previous question to get rid of the fraction?

To get rid of fraction (denom. 3) you'd have to multiply BOTH sides by 3 : $3y=2x^2-12x-54$

not proper to have this.

4. What are the steps of completing the square?

(1.) Factor out the "a" from 1st two terms

(2.) Find $\left(\frac{b}{2}\right)^2$, add then subtract in the bracket.

(3.) Pull out the 4th term from the bracket and multiply it by "a"

(4.) Factor the trinomial left over

Half of middle term

5. For word problems sometimes you must find zeros and sometimes the vertex. What are the words you should look out for that would indicate what you must find?

- If you see words "MAX" or "MIN" or "largest/greatest..." Then you must find the vertex
- Otherwise you probably need to solve for $x \to$ zeros

Revenue problem

6. The circus sells 1000 tickets for $6 each. The circus owners want to increase their revenues, so they increase prices. They have noticed that ticket sales decrease by 45 tickets every time the price increases by $0.50.
   a. What is the revenue equation?
   b. What price will maximize the revenue?
   c. What is the quantity at maximum?
   d. What is the maximum revenue?

Profit problem

7. If the total costs are $600 flat and $90 per item, and the revenue is given by $R(x)=150x-x^2$, where x is the number of items sold.
   a. What is the profit equation?
   b. Determine the items sold that would produce the maximum profit.
   c. Find the number of items to sell to have the profit of $100.
   d. What is the initial profit?

$C = 600 + 90x$

(a) $\boxed{\text{Revenue} = (\text{price})(\text{quantity})}$

let x represent the # of times prices changes by $0.50

$R = (6 + 0.50x)(1000 - 45x)$

original

(b) zeros:

$6 + 0.5x = 0$
$0.5x = -6$
$x = -12$

$1000 - 45x = 0$
$1000 = 45x$
$22.\bar{2} = x$
$\left(\frac{200}{9}\right)$

vertex: $a.o.s = \frac{-12 + 22.\bar{2}}{2} = 5.\bar{1} = \frac{46}{9}$

$(5.\bar{1}, 6587.\bar{7})$
x R

opt. val $= (6 + 0.5(5.\bar{1}))(1000 - 45(5.\bar{1}))$
$= (8.55)(770)$
$= 6587.\bar{7}$

price

quantity

(b) price = $8.56

(c) quantity = 770 tickets

(d) Revenue = $6587.78

(a) $\boxed{\text{Profit} = \text{Revenue} - \text{Cost}}$

$P = (150x - x^2) - (600 + 90x)$

$P = -x^2 + 60x - 600$

(b) $P = -(x^2 - 60x + 900 - 900) - 600$

$P = -(\quad)(\quad) - 900(-1) - 600$

$P = -(x-30)^2 + 300$

vertex $(30, 300)$
x P

$\therefore$ for 30 items there's max profit of $300.

(c) $100 = -x^2 + 60x - 600$

$0 = -x^2 + 60x - 700$

$x = \frac{-60 \pm \sqrt{800}}{-2}$

$x \doteq 15.9$ $\quad x \doteq 44.1$

$\therefore$ for 16 or 44 items sold profit is $100.

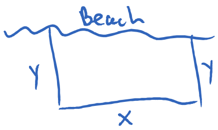
(d) $P(0) = -0^2 + 60(0) - 600$
$= -600$

$\therefore$ loss of $600 initially

Fence (or rope) off an area problem

Rectangular

8. For a park swimming area, 4500 m of line is used to mark off the permissible area. One side not roped off is next to the beach. Find the dimensions of the swimming area that will make it a maximum



Goal: equation that uses 2 variables that is quadratic

$\boxed{4500 = x + 2y}$

$4500 - 2y = x$

$A = lw$

$\boxed{A = xy}$

$A = (4500 - 2y)y$

$A = -2y^2 + 4500y$

$A = -2(\underbrace{y^2 - 2250y + 1265625}) - 1265625$

$A = -2(\quad) - 1265625(-2)$

$A = -2(y - 1125)^2 + 2531250$

max: vertex

(y, A)

vertex$(1125, 2531250)$
$\quad\;\; y \qquad\quad A$

$\therefore$ dimensions are $y = 1125$ m

$x = 4500 - 2y$ OR $\frac{A}{y}$

$x = 2250$ m

Falling object problem

9. A water balloon is catapulted into the air so that its height, in meters after t seconds is given by a quadratic function. The balloon's initial speed is 27 meters/second and it was released from the height of 3 meters. When does the balloon reach the height of 35m?

Key: $h = -4.9t^2 + v_0 t + h_0$

$\frac{-9.8}{2}$ gravity

initial velocity

initial height

$h = -4.9t^2 + 27t + 3$

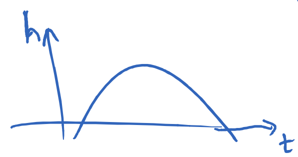
$35 = -4.9t^2 + 27t + 3$

$0 = -4.9t^2 + 27t - 32$

$t = \frac{-27 \pm \sqrt{101.8}}{-9.8}$

$t \doteq 1.7$ or $t \doteq 3.8$

$\therefore$ The balloon reaches 35m at 1.7 sec (at 3.8 sec it's at 35m but on its way down)

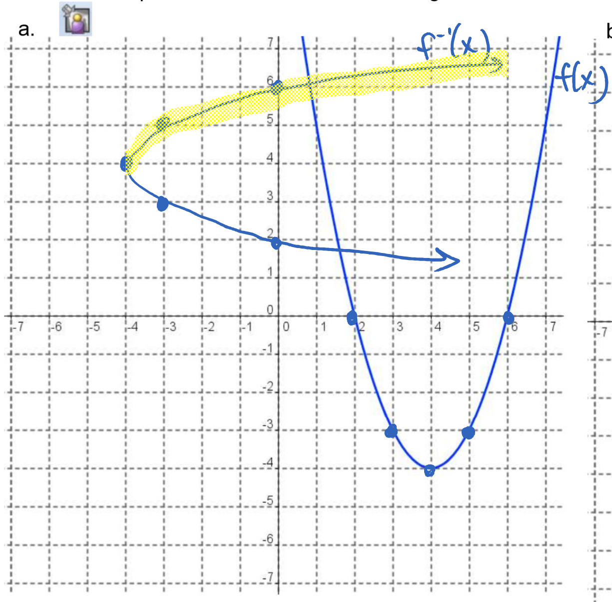


Inverses of Quadratics

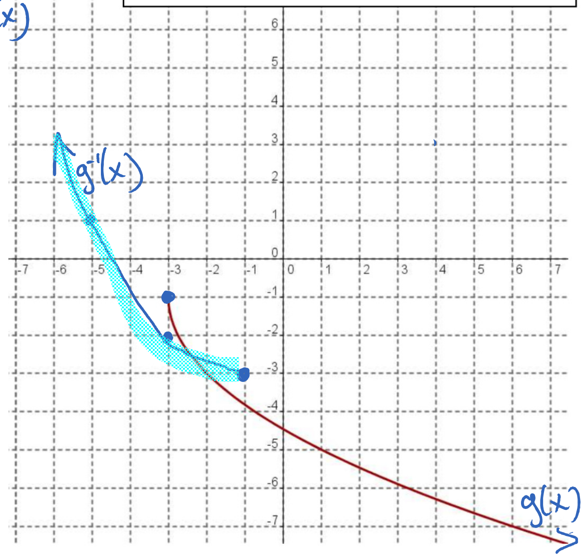
1. Find the equations for each of the following:

For square roots consider if there are any reflections that would make the 'k' negative.
Then use k=1 or k=-1 and solve for 'a' using a point.

a.



b.



$f(x) = 1(x-4)^2-4$

y

$g(x) = a\sqrt{k(x-d)}+c$

$y = -2\sqrt{1(x+3)} - 1$ shifts

step

2. Find the equations for the inverse **functions**.

inverse

$x = (y-4)^2-4$

$\pm\sqrt{x+4} = \sqrt{(y-4)^2}$

$\pm\sqrt{x+4} = y-4$

$\pm\sqrt{x+4}+4 = y$

$f^{-1}(x) = +\sqrt{x+4}+4$

OR

$f^{-1}(x) = -\sqrt{x+4}+4$

inverse

$x = -2\sqrt{y+3} - 1$

$\frac{x+1}{-2} = \frac{-2}{-2}\sqrt{y+3}$

$\left(-\frac{1}{2}(x+1)\right)^2 = \left(\sqrt{y+3}\right)^2$

$\frac{1}{4}(x+1)^2 = y+3$

$\frac{1}{4}(x+1)^2 - 3 = g^{-1}(x)$

3. What would be the domain and range of the inverse **functions**? choose that one

4. Sketch the inverse graphs for the above functions. Do your answers for domain and range make sense?

$D_{f^{-1}} = \{x \in \mathbb{R} \mid x \geq -4\}$

$R_{f^{-1}} = \{y \in \mathbb{R} \mid y \geq 4\}$

$D_{g^{-1}} = \{x \in \mathbb{R} \mid x \leq -1\}$

$R_{g^{-1}} = \{y \in \mathbb{R} \mid y \geq -3\}$

ie. find Range of the inverse $R_{f^{-1}} = \{y \in \mathbb{R}\}$

Cost as a function of items

5. The cost function in thousands of dollars for x thousand of items sold is $C(x) = 0.75x^2 - 1.5x$ $+0$

$\div 0.75 \quad \div 0.75$

$= 0.75(x^2 - 2x + 1 - 1) + 0$

$= 0.75(x-1)^2 - 0.75$

   a. What is the range of this function on the domain of $0 \le x \le 10$?

picture!

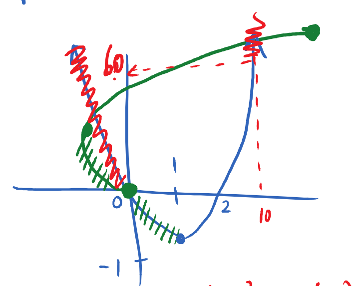
$R: \{y \mid y \in \mathbb{R},\ -0.75 \le y \le 60\}$

   b. How should the domain $0 \le x \le 10$ be further restricted so that the inverse is a function?

$D: \{x \mid x \in \mathbb{R},\ 1 \le x \le 10\}$ then the inverse is a function (passes vertical line test)

   c. Write the model for the items sold as a function of cost.

equation output input.



$0.75(10)^2 - 1.5(10)$
$= 60$

isolate the output x.

$C = 0.75(x-1)^2 - 0.75$

$\dfrac{C + 0.75}{0.75} = \dfrac{\cancel{0.75}(x-1)^2}{\cancel{0.75}}$

$\pm\sqrt{\dfrac{C+0.75}{0.75}} = \sqrt{(x-1)^2}$

items are positive.

$\sqrt{\dfrac{C+0.75}{0.75}} + 1 = x$

items sold

Root $\sqrt[2]{\ }$

Operations with Radicals

1. What are **radical** expressions? What is considered to be a **mixed radical** and what is an **entire radical**?

Like FRACTIONS

Radical expressions have roots in them.

ex. Mixed radical $= 2\sqrt{3}$

ex. Entire Radical $= \sqrt{12} = \sqrt{4 \times 3}$ "Improper"

$\sqrt{4}\sqrt{3}$

$2\sqrt{3}$

2. Why learn about them?

You must not round answers when working with questions in math (except when graphing or word problems.) so you need to know how to reduce/simplify root expressions.

3. When simplifying radicals it is important to know the perfect square numbers as they will help you to ***reduce*** the entire radical into a mixed form. List the first few perfect square numbers.

$\cancel{1}, 4, 9, 25, 36, 49, 64, 81, 100, 121, 144, 169, \ldots \quad x^2, x^4, x^6, \ldots$

4. What are the steps of **simplifying/reducing** radicals?

1.) Find the biggest perfect square that divides into the given radical.

2.) Simplify by writing seperate roots on each factor (must be mult/div.)

- If the number under your radical cannot be divided evenly by any of the perfect squares, your radical is Reduced
- If you do not choose the largest perfect square to start the process, you will have to repeat the process

5. Simplify the following

a. $\sqrt{147}$ $\quad \cancel{\sqrt{7}}\,\cancel{\sqrt{21}}$ not helpful if 7 or 21 is not perfect sq.

$= \sqrt{49}\sqrt{3}$

$= 7\sqrt{3}$

b. $2\sqrt{48p^5}$

$= 2\sqrt{16}\sqrt{3}\sqrt{p^4}\sqrt{p^1}$

$= 2(4)\sqrt{3}\, p^2\sqrt{p} \quad = 8p^2\sqrt{3p}$

c. $3\sqrt{32} = 3\sqrt{4}\sqrt{8}$

$= 3(2)\sqrt{8}$

$= 6\sqrt{8}$ ← reduce!

$6\sqrt{4}\sqrt{2}$

$12\sqrt{2}$

$= 3\sqrt{16}\sqrt{2}$

$= 3(4)\sqrt{2}$

$= 12\sqrt{2}$

d. $\sqrt{200a^4b^3}$

$= \sqrt{100}\sqrt{2}\sqrt{a^4}\sqrt{b^2}\sqrt{b^1}$

$= 10\sqrt{2}\, a^2 b\sqrt{b}$

$= 10a^2b\sqrt{2b}$

6. What are the steps of **adding/subtracting** radicals?

Like Polynomials

1.) Reduce + simplify each term

2.) Add/subtract like radicals ex. $2\sqrt{3}$ and $5\sqrt{3}$ $= 7\sqrt{3}$ not $\cancel{7\sqrt{6}}$

Only coefficient changes. $2x$ and $5x = 7x$

7. Simplify the following

a. $\sqrt{12} + 2\sqrt{48} - 5\sqrt{175}$

$= \sqrt{4}\sqrt{3} + 2\sqrt{16}\sqrt{3} - 5\sqrt{25}\sqrt{7}$

$= 2\sqrt{3} + 2(4)\sqrt{3} - 5(5)\sqrt{7}$

$= 2\sqrt{3} + 8\sqrt{3} - 25\sqrt{7}$

$= 10\sqrt{3} - 25\sqrt{7}$

can't do more.

b. $2\sqrt{18} - 3\sqrt{12} + 4\sqrt{50} + \sqrt{27}$

$= 2\sqrt{9}\sqrt{2} - 3\sqrt{4}\sqrt{3} + 4\sqrt{25}\sqrt{2} + \sqrt{9}\sqrt{3}$

$= 2(3)\sqrt{2} - 3(2)\sqrt{3} + 4(5)\sqrt{2} + 3\sqrt{3}$

$= 6\sqrt{2} - 6\sqrt{3} + 20\sqrt{2} + 3\sqrt{3}$

$= 26\sqrt{2} - 3\sqrt{3}$

8. What are the steps of **multiplying** radicals?

can skip → (1.) Reduce

(2.) Multiply coefficients separately from roots. ex. $2\sqrt{3} \cdot 6\sqrt{2}$

(3.) Reduce

$= (2)(6)\sqrt{(3)(2)}$

$= 12\sqrt{6}$

9. Simplify the following

a. $3\sqrt{12}\times 4\sqrt{6}$

$= (3)(4)\sqrt{(12)(6)}$

$= 12\sqrt{72}$

$= 12\sqrt{36}\sqrt{2} = 12(6)\sqrt{2} = 72\sqrt{2}$

c. $5\sqrt{14x}\left(2\sqrt{x}-4\sqrt{7x}\right)$

$= 10\sqrt{14x^2} - 20\sqrt{98x^2}$

$= 10\sqrt{14}\sqrt{x^2} - 20\sqrt{49}\sqrt{2}\sqrt{x^2}$

$= 10x\sqrt{14} - 20(7)(x)\sqrt{2}$

$= 10x\sqrt{14} - 140x\sqrt{2}$

* b. $\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)$ FOIL

difference of squares

$= \sqrt{25} + \cancel{\sqrt{15}} - \cancel{\sqrt{15}} - \sqrt{9}$

$= 5 \quad - 3$

$= 2$

"conjugate"

ex. $\sqrt{5}+1$

conjugate $\sqrt{5}-1$

d. $\left(-2-3\sqrt{5}\right)\left(5-\sqrt{5}\right)$

$= -10 + \underbrace{2\sqrt{5} - 15\sqrt{5}} + 3\sqrt{25}$

$= -10 - 13\sqrt{5} + 3(5)$

$= 5 - 13\sqrt{5}$

10. Show that the steps of **dividing** radicals are similar to multiplying radicals with the following example $\dfrac{4\sqrt{32}}{12\sqrt{12}}$

$= \dfrac{4}{12}\sqrt{\dfrac{32}{12}}$

$= \dfrac{1}{3}\sqrt{\dfrac{8}{3}}$

$= \dfrac{1\sqrt{8}}{3\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$

$= \dfrac{\sqrt{24}}{3\sqrt{9}}$ ← $\sqrt{4}\sqrt{6}$ ← 3(3)

$= \dfrac{2\sqrt{6}}{9}$

11. It is considered improper form to leave radicals in the denominator. You must rationalize the denominators for your final answers. What does the word rationalize mean to you? Make sure your answer to the previous question is in proper form.

Rationalize means to make the denominator a rational # (fraction or integer)

12. What are the steps of **rationalizing MONOMIAL** denominators? no +/−

Multiply top and bottom by the radical you wish to eliminate

13. What are the steps of **rationalzing BINOMIAL** denominators? have +/−

Multiply top and bottom by the CONJUGATE of the denominator.

14. Simplify the following

a. $\dfrac{2}{\sqrt{6}} \cdot \dfrac{\sqrt{6}}{\sqrt{6}}$ ← improper to have root on bottom.

$= \dfrac{2\sqrt{6}}{\sqrt{36}}$

$= \dfrac{2\sqrt{6}}{6}$

$= \dfrac{1\sqrt{6}}{3}$ or $\dfrac{\sqrt{6}}{3}$

b. $\dfrac{6\sqrt{5}}{7\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{6\sqrt{10}}{7\sqrt{4}}$

$= \dfrac{6\sqrt{10}}{14}$

$= \dfrac{3\sqrt{10}}{7}$

c.) $\dfrac{\sqrt{3x^2y^3}}{4\sqrt{5x^3y}} = \dfrac{1}{4}\sqrt{\dfrac{3\cancel{x^2}\cancel{y^3}\,y^2}{5\cancel{x^3}\cancel{y}\,x}}$

$= \dfrac{\sqrt{3y^2}}{4\sqrt{5x}} \cdot \dfrac{\sqrt{5x}}{\sqrt{5x}}$

no need for conjugate not with +/-

$= \dfrac{y\sqrt{15x}}{4(5x)} = \dfrac{y\sqrt{15x}}{20x}$

e.) $\dfrac{-12\sqrt{24}}{3\sqrt{2}} = -\dfrac{12}{3}\sqrt{\dfrac{24}{2}}$

$= -4\sqrt{12}$

$= -4\sqrt{4}\sqrt{3}$

$= -4(2)\sqrt{3}$

$= -8\sqrt{3}$

d.) $\dfrac{(5-\sqrt{3})}{(2\sqrt{3}-\sqrt{5})} \cdot \dfrac{(2\sqrt{3}+\sqrt{5})}{(2\sqrt{3}+\sqrt{5})}$ ← conjugate of denom!

$= \dfrac{10\sqrt{3}+5\sqrt{5}-2\sqrt{9}-\sqrt{15}}{4(3)-5}$

$= \dfrac{10\sqrt{3}+5\sqrt{5}-6-\sqrt{15}}{7}$

f.) $\dfrac{4}{(\sqrt{2}-5\sqrt{3})} \cdot \dfrac{(\sqrt{2}+5\sqrt{3})}{(\sqrt{2}+5\sqrt{3})}$

$= \dfrac{4\sqrt{2}+20\sqrt{3}}{2-25(3)}$

$= \dfrac{4\sqrt{2}+20\sqrt{3}}{-73}$

15. All of the above rules are for square root questions. How can you reduce cube root or other type of roots?

ex. $\dfrac{2}{\sqrt[3]{3}} \cdot \dfrac{\sqrt[3]{3^2}}{\sqrt[3]{3^2}}$

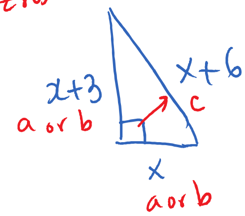
$3^{\frac{1}{3}} \cdot 3^{2/3} = 3^1$

$= \dfrac{9\sqrt[3]{9}}{3}$

Solve Quadratics

the related quadratic that has one side zero!

1. If the quadratic word problem doesn't ask for maximum or minimum then the question often reduces to finding the x-intercepts of the quadratic. Explain why? (Use the following word problem in your explanation.)
"zeros"
"The hypotenuse of a right triangle is 6 cm more than the shorter leg. The longer leg is three more than the shorter leg. Find the lengths of all three sides."



$$a^2+b^2=c^2$$
$$(x+3)^2+x^2=(x+6)^2$$
$$x^2+6x+9+x^2=x^2+12x+36$$
$$\boxed{x^2-6x-27=0}$$

related equation whose zeros provide answers to the Δ question.

2. There are **THREE** ways to find solutions for the independent variable in quadratic type question. What are they?

(1.) Quadratic formula (always works)
(2) Factoring
(3.) SAMDEB (only if variable appears once)

3. Solve the above word problem showing more than one method:

method (2): $(x-9)(x+3)=0$

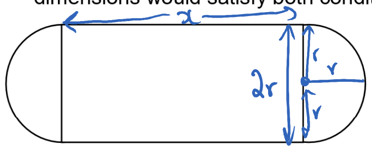
$\therefore x=9$ $\qquad$ ~~$x=-3$~~ makes no sense in Δ.

∴ The lengths of Δ sides are
$x=9$ cm
$x+3=12$ cm
$x+6=15$ cm

method (3) need to complete square then SAMDEB ...

Geometry problem

4. This is a diagram of a practice field. The track and field coach wants two laps around the field to be 1000m. But the Phys. Ed. department wants the rectangular field to be as large as possible. What dimensions would satisfy both conditions?



1 lap = 500m

Goal: - 2 variables
- quadratic
MAX

$500 = 2x + \underbrace{2\pi r}_{\text{circumference}}$

isolate x: $\frac{500 - 2\pi r}{2} = \frac{2x}{2}$

$250 - \pi r = x$

$A = \boxed{lw}$

$A = (x)(2r)$

$A = (250 - \pi r)(2r)$

$A = -2\pi r^2 + 500r$

$A = -2\pi\left(r^2 - \frac{250}{\pi}r + \left(\frac{125}{\pi}\right)^2 - \left(\frac{125}{\pi}\right)^2\right)$

$A = -2\pi\left(r - \frac{125}{\pi}\right)\left(r - \frac{125}{\pi}\right) - \frac{15625}{\pi^2}(-2\pi)$

$A = -2\pi\left(r - \frac{125}{\pi}\right)^2 + \frac{31250}{\pi}$

$A \doteq -2\pi(r - 39.8)^2 + 9947.2$

vertex $(\underset{r}{39.8}, \underset{A}{9947.2})$

$\therefore$ dimensions are → semicircles should have a radius of 39.8 m

→ Length of field $x = 250 - \pi r$

$x = 125$ m

→ width of field $2r = 79.6$ m

Translating English into Math problem

5. Find three consecutive integers such that four times the sum of all three is 2 times the product of the larger two

let x be the 1st integer
let $x+1$ be the 2nd integer
let $x+2$ be the 3rd

$4 \cdot (x + (x+1) + (x+2)) = 2 \cdot (x+2)(x+1)$

$4(3x+3) = 2(x^2 + 3x + 2)$

$12x + 12 - 2x^2 - 6x - 4 = 0$

$-2x^2 + 6x + 8 = 0$

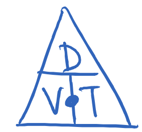
$-2(x^2 - 3x - 4) = 0$

$-2(x-4)(x+1) = 0$

$x = 4$ or $x = -1$

$\therefore$ integers can be 4, 5, 6 OR -1, 0, 1

	D (km)	V (km/h)	T (hrs)
Dan	42	$x+0.4$	stop: 0.5 drive: $\frac{42}{x+0.4}$
Sue	42	x	stop: 0.25 hrs = 15 min drive: $\frac{42}{x}$



$T = \frac{D}{V}$

Dan's Total Time = Sue's Total Time

$D = VT$

$$0.5 + \frac{42}{x+0.4} = 0.25 + \frac{42}{x}$$

$$\frac{(x^2+0.4x)}{(x^2+0.4x)}\frac{0.25}{1} + \frac{42}{x+0.4}\cdot\frac{(x)}{(x)} - \frac{42}{x}\frac{(x+0.4)}{(x+0.4)} = 0$$

LCD

$$\frac{0.25(x^2+0.4x) + 42x - 42(x+0.4)}{1(x+0.4)(x)} = 0$$

$$\frac{0.25x^2 + 0.1x + \cancel{42x} - \cancel{42x} - 16.8}{x(x+0.4)} = \frac{0}{1}$$

$$0.25x^2 + 0.1x - 16.8 = 0$$

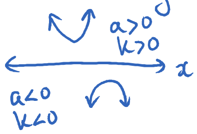
$$x = \frac{-0.1 \pm \sqrt{16.81}}{0.5}$$

$x = 8$

$\cancel{x = -8.4}$

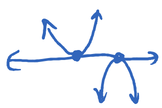
$\therefore$ Sue's speed is 8 km/h
Dan's speed is 8.4 km/h

$y=a(x-h)^2+k$



• NO zeros if "a" and "k" are same sign.

• TWO zeros if "a" and "k" are opposite sign



• ONE zero if $k=0$

$y=ax^2+bx+c$

Discriminant $= b^2-4ac$

↑ under root of quadratic formula

• NO zeros if $b^2-4ac<0$

• TWO zeros if $b^2-4ac>0$

• ONE zero if $b^2-4ac=0$

$y=a(x-r)(x-t)$

TWO zeros

$y=a(x-r)(x-r)$

$y=a(x-r)^2$

ONE zero

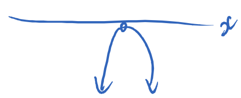
discriminant $=0$?

$b^2-4ac=0$

$b^2-4(-4)(-10)=0$

$b^2-160=0$

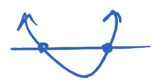
$\sqrt{b^2}=\sqrt{160}$



$b=\pm\sqrt{160}$

$b=\pm\sqrt{16}\sqrt{10}$

$b=\pm 4\sqrt{10}$



✓ reflections?

✗ shifts up/down?

✓ shifts left/right?

✓ Vertical/horiz. stretch/compress?

ⓐ Profit = Rev − Cost

$P=(1600x-x^2)-(1600+1500x)$

$P=-x^2+100x-1600$

ⓑ Break even ⇒ Profit $=0$
(not earn, not lose)

$0=-x^2+100x-1600$

want # of solutions.

discriminant $= b^2-4ac$

$=100^2-4(-1)(-1600)$

$=+3600$ pos #.

∴ Yes, the company will break even for 2 different x, items sold.

5. Show a proof of the quadratic formula. → solves for zeros $(y=0)$

Hint: complete the square using the standard form without actual numbers for a, b, c

$$y = ax^2 + bx + c$$

$$0 = ax^2 + bx + c$$

$$0 = a\left(\underbrace{x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2}_{\div 2 \text{ and square.}} - \left(\frac{b}{2a}\right)^2\right) + c \quad \cdot a$$

$$\frac{\frac{b}{a}}{2} = \frac{b}{a}\cdot\frac{1}{2}$$

$$0 = a\left(x + \frac{b}{2a}\right)\left(x + \frac{b}{2a}\right) - \frac{b^2}{4a^{\cancel{2}}}\cdot\cancel{a} + c$$

$$0 = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$$

* no x appears once. , solve for x

$$\frac{1}{a}\left(\frac{b^2}{4a} - c\right) = \frac{\cancel{a}}{\cancel{a}}\left(x + \frac{b}{2a}\right)^2$$

$$\frac{b^2}{4a^2} - \frac{c\cdot 4a}{a\cdot 4a} = \left(x + \frac{b}{2a}\right)^2$$

LCD

$$\pm\sqrt{\frac{b^2 - 4ac}{4a^2}} = \pm\sqrt{\left(x + \frac{b}{2a}\right)^2}$$

$$\frac{\pm\sqrt{b^2 - 4ac}}{\sqrt{4a^2}} = x + \frac{b}{2a}$$

$$\frac{\pm\sqrt{b^2 - 4ac}}{2a} = x + \frac{b}{2a}$$

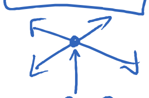
$$\underbrace{\frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}}_{\text{Common denom.}} = x$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = x$$

Systems

1. In grade 10 you have learned how to solve for points of intersection of linear systems. There are three ways of solving linear systems. What are they?

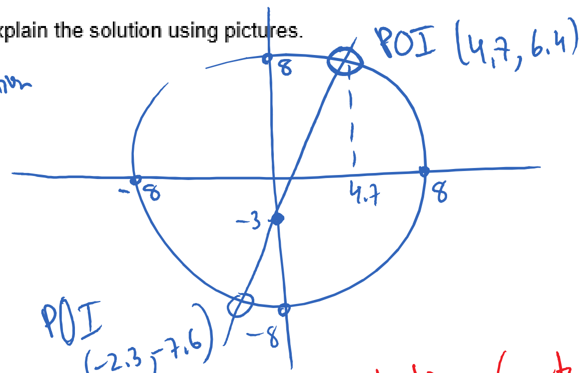
(1.) Substitution Method
(2.) Elimination Method
(3.) Graphing Method ← least accurate.



2. Solve the (1) $y^2 + x^2 = 64$ ← $r = 8$
(2) $y - 2x = -3$ system.

no like terms, so elimination won't work!

Explain the solution using pictures.



isolate y in (2):

$y = 2x - 3$

sub in (1)

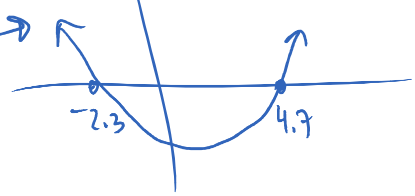
$(2x-3)^2 + x^2 = 64$

FOIL

$4x^2 - 12x + 9 + x^2 = 64$

$\boxed{5x^2 - 12x - 55 = 0}$

←Related equation whose zeros provide solutions (parts of solution) to the original question.



quad formula $x = \frac{+12 \pm \sqrt{1244}}{10}$

$x \doteq 4.7$ or $x \doteq -2.3$

now find y's: $y = 2x - 3$

$y = 2(4.7) - 3$
$y = 6.4$ $\therefore POI(4.7, 6.4)$

$y = 2(-2.3) - 3$
$y = -7.6$ $\therefore POI(-2.3, -7.6)$

check in (1)

	LS	RS
	$y^2 + x^2$	$\neq 64$
	$6.4^2 + 4.7^2$	
	63.05	

rounding error.

$(-7.6)^2 + (-2.3)^2$

3. Find k so that the system has only one solution.

① $y=-3x^2+x-1$

② $y=-x+k$

relation equation to have only one zero.

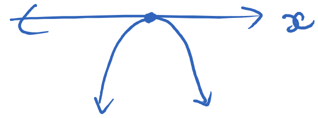
(sub one eqtn into other)

Sub ② into ①

$-x+k=-3x^2+x-1$

$0=-3x^2+2x-1-k$ ← relation equation.

$a=-3$ $b=2$ $c=-1-k$



ONE zero means discriminant = 0

$b^2-4ac=0$

$2^2-4(-3)(-1-k)=0$ $4+12(-1-k)=0$

$4-12-12k=0$

$-8=12k$

$-\frac{2}{3}=k$

4. Predict the number of solutions the following system will have. Explain your reasoning.

① $y=-2x^2+x-3$ # of zeros of Related equation

② $y=-x-2$

sub ② into ①

$-x-2=-2x^2+x-3$

$0=-2x^2+2x-1$

discrim $=b^2-4ac$

$=2^2-4(-2)(-1)$

$=4-8$

neg ∴ related eqtn has no zeros which implies original system has no solutions

5. A toy rocket's flight path is given by ① $h(t)=-5t^2+3t+2$ and a bird's flight path is ② $h(t)=-5t+4$ where t is time in seconds. What time did the bird and the toy rocket collide?

find POI. ② sub in ①

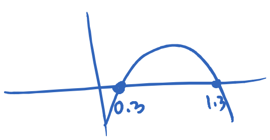
$-5t+4=-5t^2+3t+2$

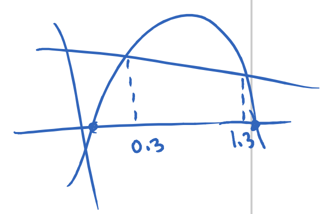
$0=-5t^2+8t-2$

$t=\frac{-8\pm\sqrt{24}}{-10}$

$t=0.3$

$t=1.3$





∴ at time 0.3 sec the bird + toy rocket collide.

Solving Square Root Equations – if there is time

1. The only way to get rid of a square root in an equation is to square both sides of the equation. But before you do that, make sure the square root is isolated. Also make sure you square SIDES not TERMS! Explain why that is so.

Must square whole side, not term by term since terms are separated by +/− and you cannot distribute the square over addition/subtraction ex. $(9+1)^2 \stackrel{?}{=} 9^2+1^2$

2. Solve the following. Check your answers if all of them actually make sense.

a. $\sqrt{x-1}+7=x$

$(\sqrt{x-1})^2=(x-7)^2$ FOIL

$x-1=x^2-14x+49$

$\boxed{0=x^2-15x+50}$ related eqtn.

−10, −5

$0=(x-10)(x-5)$

$x=10$ ✓ OR $x=5$ ✗

LS	RS
$\sqrt{x-1}+7$	x
$\sqrt{9}+7 \stackrel{?}{=}$	10
$\sqrt{4}+7 \stackrel{?}{=}$	5

b. $(\sqrt{9x^2+4})^2=(3x+2)^2$

~~$\sqrt{9x^2}+\sqrt{4}$~~ can't because of addition ex.

$\sqrt{(25+4)} \neq 5+2$

$\sqrt{29} \neq 10$

FOIL

$9x^2+4=9x^2+12x+4$

$0=12x$

$0=x$ ✓ check

LS	RS
$\sqrt{9x^2+4}$	$3x+2$
$\sqrt{0+4}$	$0+2$

c. $(2x^2-7)^{\frac{1}{2}}+x=3$

power $\frac{1}{2}$ $=\sqrt[2]{(\ )}^1$

$\sqrt{2x^2-7}+x=3$

$(\sqrt{2x^2-7})^2=(3-x)^2$

$2x^2-7=9-6x+x^2$

$x^2+6x-16=0$

8, −2

$(x+8)(x-2)=0$

$x=-8$ ✓ OR $x=2$ ✓ Both work :)

d. $\sqrt{x-3}-\sqrt{x}=3$

isolated

$(\sqrt{x-3})^2=(3+\sqrt{x})^2$ FOIL

$x-3=9+6\sqrt{x}+(\sqrt{x})^2$

$x-3=9+6\sqrt{x}+x$

isolate.

$\frac{-12}{6}=\frac{6\sqrt{x}}{6}$

$(-2)^2=(\sqrt{x})^2$

$4=x$ ✗

check: $\sqrt{4-3}-\sqrt{4} \stackrel{?}{=} 3$

$\sqrt{1}-\sqrt{4}$

$1-2$ ∴ no solutions.

20

Families of Quadratics

1. Sometimes the given problems may have many solutions only because not enough information was provided to make the solution unique. The many solutions that satisfy the given conditions are called a **familty** of functions. Give a family of quadratics that share the same vertex, and open down.

vertex $(0,1)$ a neg.

$y=-(x-0)^2+1=-x^2+1$

$y=-0.5(x-0)^2+1=-0.5x^2+1$

$\vdots$ ∞ many of them

To quote all: $\boxed{y=a(x-0)^2+1,\ a<0}$

2. Find a family of functions that have zeros at -9, and 5

$\boxed{y=a(x+9)(x-5),\ a\in\mathbb{R},\ a\neq 0}$ ← family

a. Give another condition that will ensure that this family of many quadratics is one unique parabola.

pt. $(6,3)$ ∴ give another point on a quadratic to get a unique sol.

$3=a(6+9)(6-5)$

$\frac{1}{5}=a$

$\boxed{y=\frac{1}{5}(x+9)(x-5)}$

3. Find a family of functions that have zeros at $2+\sqrt{3}$ and $2-\sqrt{3}$ irrational.

$y=a(x-(2+\sqrt{3}))(x-(2-\sqrt{3}))$

$y=a(x-2-\sqrt{3})(x-2+\sqrt{3})$, $a\neq 0$ ← family

Foil

a. Find the unique equation of the family that passes through the point (1, -5).

make it neater by expanding

$y=a((x-2)^2-\sqrt{9})$

$y=a(x^2-4x+4-3)$

$y=a(x^2-4x+1)$

now unique: pt. $(1,-5)$

$5=a(1^2-4(1)+1)$

$5=-2a$

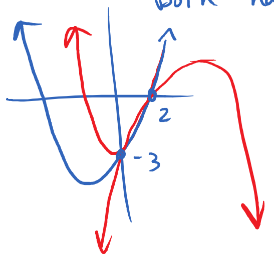
$-\frac{5}{2}=a$

∴ unique: $y=-\frac{5}{2}(x^2-4x+1)$

4. Give two non-equivalent functions that satisfy the following conditions. $f(2)=g(2)=0$ and $f(x)$ and $g(x)$ have the same y-intercept $=-3$

Both have y-int $(0,-3)$

Both have x-int at $(2,0)$



$y=ax^2+bx+c$
? ? ✓

similar steps

$\boxed{y=a(x-r)(x-t)}$
? ? ✓

$y=a(x-r)(x-2)$

sub $(0,-3)$

$-3=a(0-r)(0-2)$

$-3=2ra$

$\frac{-3}{2r}=a$

∴ family

$\boxed{y=-\frac{3}{2r}(x-r)(x-2)\quad r\in\mathbb{R},\ r\neq 0}$

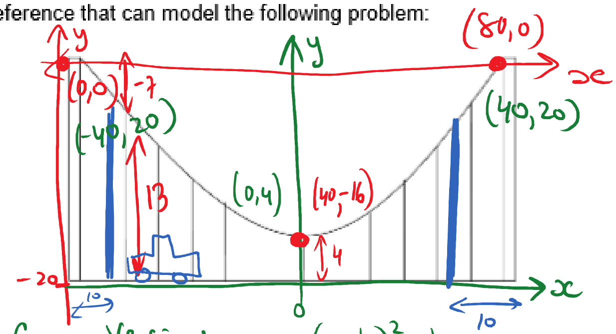
give 2 possibilities: $f(x)=-\frac{3}{2}(x-1)(x-2)$ choose $r=1$

choose $r=2$ $g(x)=-\frac{3}{4}(x-2)(x-2)$

5. Sometimes a given problem can have many different-looking equations that give the same answer. This can happen when many frames of reference can be chosen, i.e. the x and y axes can be placed differently since the question doesn't specify where to place them. (Note: these are NOT considered to be a family of functions since graphing them in the same frame of reference/coordinate system will yield different graphs)

Find TWO different equations using different frames of reference that can model the following problem:

"Many suspension bridges hang from cables that are supported by two towers. The shape of the hanging cables is very close to a parabola. A typical suspension bridge has large cables that are supported by two towers that are 20 m high and 80 m apart. The bridge surface is suspended from the large cables by many smaller vertical cables. The shortest vertical cable is 4 m long."



Red version:

$y = a(x-h)^2 + k$

$0 = a(0-40)^2 - 16$ pt. (0,0)

$0 = 1600a - 16$

$\frac{16}{1600} = a$

$\frac{1}{100} = a$

$\therefore y = \frac{1}{100}(x-40)^2 - 16$

Green Version: $y = a(x-h)^2 + k$

$20 = a(40-0)^2 + 4$ pt. (40,20)

$16 = 1600a$

$\frac{1}{100} = a$

$\therefore y = \frac{1}{100}(x-0)^2 + 4$

a. If another vertical support is needed to be placed 10 m away from the end, and the available support pole is 12 m tall, prove that it will NOT be tall enough to support the cable.

in red system: $x = 10$ or $x = 70$

$y = \frac{1}{100}(10-40)^2 - 16 = -7$

in green: $x = 30$ or $x = -30$

$y = \frac{1}{100}(30-0)^2 + 4 = 13$

same if you compare using the picture.

c. Does it matter which equation is used to answer questions a and b? Why or why not?

c

b. Where should the 12 m support pole be placed?

use only green now:

sub $y = 12$

$12 = \frac{1}{100}(x-0)^2 + 4$

$100 \times \frac{8}{1} = \frac{1}{100}x^2$

$800 = x^2$

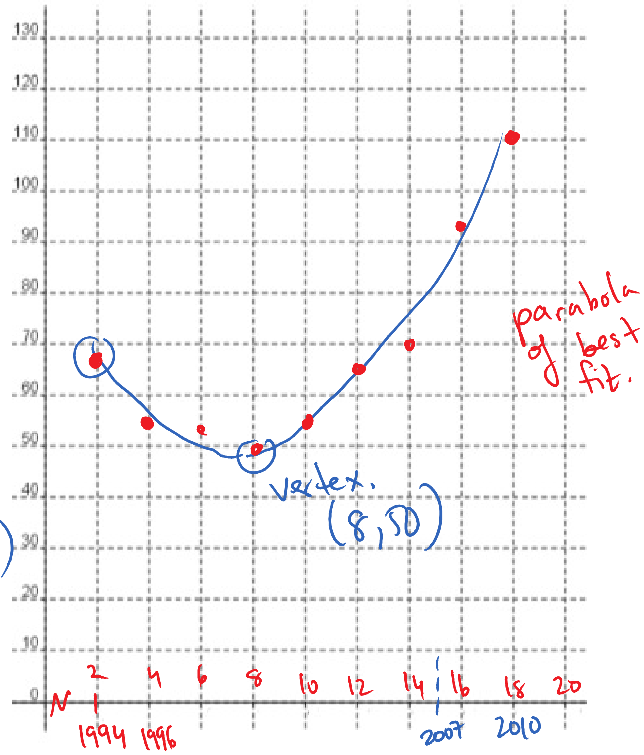
$\pm 28.3 = x$

$\therefore$ place the support poles 28.3 m away from the centre.

6. Sometimes a given problem can have many very close approximate solutions. This occurs when dealing with raw real life data because it is not possible to have perfect measurements and real life situations can fluctuate from many different variables. (Note: again, these are NOT considered to be a family of functions, since the approximate equations will yield slightly different results)

a. Plot the given data as a scatter plot and create a quadratic curve of best fit.

Year	Number (in thousands) of people over 100 years old in a country
$t=0$ 1992	
$t=2$ 1994	68
$t=4$ 1996	55
$t=6$ 1998	54
8 2000	50
10 2002	55
12 2004	65
14 2006	70
16 2008	94
18 2010	110



b. Determine the approximate equation for your graph that will model this senario.

ie. $t=0$ 1992

$y=a(t-8)^2+50$

$68=a(2-8)^2+50$ pt. $(2, 68)$

-50

$18=a(36)$

$\frac{1}{2}=a$ $\therefore y=\frac{1}{2}(t-8)^2+50$

c. Use your equation to **interpolate** (find the value within data points) how many people will be over 100 years old in 2007.

sub $t=15$

$y=\frac{1}{2}(15-8)^2+50=74.5$ $\therefore$ 75 people.

d. Use your equation to **extrapolate** (find the value outside of data points) how many people will be over 100 years old in 2012.

sub $t=20$

$y=\frac{1}{2}(20-8)^2+50=122$ people.

e. Can this model be used to predict number of people very far in the future? Why or why not?

NO. Since parabolas go to ∞ on both sides. Highly unlikely life span would increase that way for more people.

Summary:
Family of functions must ___satisfy___ the given conditions in the ___SAME___ coordinate system(frame of referene) and answers must be ___exact.___.