

PRACTICE Series Word Problems

1. A runner begins training by running 5 mi. one week. The second week she runs a total of 6.5 mi. The third week she runs 8 mi. Assume this pattern continues.
- How far will she run in the tenth week?
 - At the end of the tenth week, what will be the total distance she has run since she started training?
 - Express the total distance with summation notation (Σ).

$$5, 6.5, 8, \dots \quad t_n = a + d(n-1)$$

$$\begin{array}{l} n=1 \\ \text{week 1} \end{array} \quad (\text{no offset}) \quad t_n = 5 + 1.5(n-1)$$

$$n=10 \text{ at week 10} \quad t_n = 1.5n + 3.5$$

$$t_{10} = 1.5(10) + 3.5$$

$$\therefore t_{10} = 18.5 \text{ miles for week 10.}$$

Total distance add: $5 + 6.5 + 8 + 9.5 + 11 + 12.5 + 14 + 15.5 + 17 + 18.5$
 or use sum formula $\therefore S_{10} = 117.5$ miles in total

$$\text{Total dist } \sum_{n=1}^{10} 1.5n + 3.5$$

2.



You visit the Grand Canyon and drop a penny off the edge of a cliff. The distance the penny will fall is 16 feet the first second, 48 feet the next second, 80 feet the third second, and so on in an arithmetic sequence. What is the total distance the object will fall in 6 seconds?

Arithmetic sequence: 16, 48, 80, ...

$$a_n = a_1 + (n-1)d$$

$$a_6 = 16 + (6-1)32 = 176$$

The 6th term is 176.

Now, we are ready to find the sum:

$$S_n = \frac{n(a_1 + a_n)}{2}$$

$$S_6 = \frac{6(16 + 176)}{2} = 576 \text{ feet}$$

3. A side of an apartment building is shaped like a steep staircase. The windows are arranged in columns. The first column has 2 windows, the next has 4, then 6, and so on. How many windows are on the side of the apartment building if it has 15 columns?

2, 4, 6, ...

$$S_{15} = \frac{15}{2}(2+30)$$

$$a_{15} = 2 + (15-1)2$$

$$a_{15} = 30$$

$$S_{15} = 240$$

4. Nathan has a collection of barbells for his home gym. He has 2 barbells that weigh 5 pounds each, 10 pounds each, 15 pounds each, and so on, up to 80 pounds. What is the total weight of all his barbells?

10, 20, 30, ... 160

$$S_n = \frac{16}{2}(10+160)$$

$$S_n = 1360 \text{ pounds}$$

5. Amanda wants to host a party. She invites 3 friends and tells them to invite 3 of their friends. The 3 friends do invite 3 others and ask each to invite 3 more people. This invitation process goes on for 5 generations of invitations. Including herself, how many people can Amanda expect at her party?

1, 3, 9, ...

$$S_5 = \frac{1(1-3^5)}{1-3}$$

$$S_5 = 121 \text{ people}$$

6. A health club rolls its towels and stacks them in layers on a shelf. Each layer of towels has one less towel than the layer below it. If there are 20 towels on the bottom layer and one towel on the top layer, how many towels are stacked on the shelf?

20, 19, 18, ... 1

$$S_{20} = \frac{20}{2}(20+1)$$

$$S_{20} = 210 \text{ towels}$$

7. Heavy rain in Brienne's town caused the river to rise. The river rose three inches the first day, and each day twice as much as the previous day. How much did the river rise in five days?

3, 6, 12, ...

$$a_5 = 3(2)^{5-1}$$

$$a_5 = 48 \text{ inches}$$

8. More than 380,000 people run in marathons each year. Matthew is training to run a marathon. He runs 20 miles his first week of training. Each week, he increases the number of miles he runs by 4 miles. How many total miles did he run in 8 weeks of training?

20, 24, 28, ...

$$S_8 = \frac{8}{2}(20+48)$$

$$a_8 = 20 + (8-1)4$$

$$S_8 = 272 \text{ miles}$$

$$a_8 = 48$$

9. A teacher teaches 8 students how to fold an origami model. Each of these students goes on to teach 8 students of their own how to fold the same model. If this teaching process goes on for 6 generations, how many people total will know how to fold the origami model?

8, 64, ...

$$S_6 = \frac{8(1-8^6)}{1-8}$$

$$S_6 = 299,592 \text{ people}$$

10. Daniela borrowed some money from her parents. She agreed to pay \$50 at the end of the first month and \$25 more for each additional month for 12 months. How much does she pay in total after the 12 months?

\$50, \$75, \$100, ...

$$S_{12} = \frac{12}{2}(50+325)$$

$$a_{12} = 50 + (12-1)25$$

$$S_{12} = \$2250$$

$$a_{12} = \$325$$

Name: _____

11. When an object is in free fall and air resistance is ignored, it falls 16 feet in the first second, an additional 48 feet in the next second, and 80 feet during the third second. How many total feet will the object fall in 10 seconds?

16, 48, 80

$$S_{10} = \frac{10}{2}(16 + 304) \quad \Delta_{10} = 16 + (10-1)32$$

$$S_{10} = 1600 \text{ Feet} \quad \Delta_{10} = 304$$

12. A virus goes through a computer, infecting files. If one file was infected initially and the total number of files infected doubles every minute, how many total files will be infected in 20 minutes?

1, 2, 4, 8, ...

$$S_{20} = \frac{1(1-2^{20})}{(1-2)}$$

$$S_{20} = 1048575$$