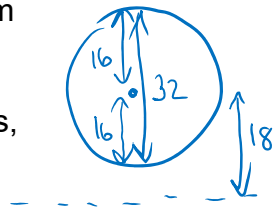
Name: _____

Modeling with Sinusoidal Functions

1. At a country fair, the Ferris wheel has a diameter of 32m, and its centre is 18 m above the ground. The wheel completes one revolution every 30 s.



a) Graph a rider's height above the ground, in metres, versus the time, in seconds, during a 2-min ride. The rider begins at the lowest position on the wheel.
b) Determine the equation of this function. (use cosine)
c) State the domain and range of this function.
d) Where is the rider at 50 s, in to the start of the ride?
e) When will the rider be 10m above the ground?

2. The cycle of ocean tides represents periodic behaviour and can be modeled with a periodic function. Each day, at various locations around the world, the height of the tide above the mean low-water level is recorded. Data for low tide and high tide from one location are shown in the table.

convert to hrs
$10\ \frac{40}{60} = 10.66$

Time	convert time to hours	Tide Height(m)
10:40	10.66	0.3
16:52		2.7
23:05		0.3
05:17 or 29:17		2.7

a) Graph the ocean tide, in metres, versus the time in hours, using the information on the table.
b) What is the equation that models the ocean tides? (use cosine)
c) What is the tide height at noon?
d) When will the height be at 1m?

3. An object suspended from a spring is oscillating up and down. The distance from the high point to the low point is 30 cm, and the object takes 4 s to complete 5 cycles. For the first few cycles, the distance from the mean position, d(t) centimeters, with respect to the time, t seconds, is modeled by a sine function.
a) Sketch a graph of this function for two cycles. → put middle on the x-axis
b) Write an equation that describes the distance of the object from its mean position as a function of time.
c) What position is the object in at 3 s?
d) When will the spring be at 12 cm?

4. A clock pendulum completes a back-and-forth cycle every 2 s. The maximum measure of the angle of the pendulum with the vertical is 12^o.
a) Draw a graph of the function, between 0s and 6s, starting from the vertical position.
b) Express the measure of the angle between the pendulum and the vertical as a function of time. Assume the relationship is a sine function.

5. The volume of air in the lungs during normal breathing is a sinusoidal function of time. Suppose a person's lungs contain from 2200 mL to 2800 mL of air during normal breathing. Suppose a normal breath takes 4 s, and that time t = 0s corresponds to a minimum volume.
a) Let V milliliters represent the volume of air in the person's lungs. Draw a graph of V against t for 20 s.
b) Write an equation for the function.
c) Describe how the graph and the equation would change in each case.
    i) The person breathes more rapidly.
    ii) The person takes bigger breaths.

6. Towers and skyscrapers appear to be rigid, but they are designed to sway with the wind. Suppose you are standing on the glass floor of the CN Tower, which is 342 m above the ground. Starting at t = 0s, the tower is at vertical. It sways 40 cm to the right (positive), returns to vertical, then sways 40 cm to the left (negative). The entire cycle takes 12 s to complete.
a) Draw the graph that represents the CN Tower swaying.
b) Write an equation for the function.
c) At t = 2 s, where is the tower?
d) When will the tower be at 30 cm to the left of the vertical?

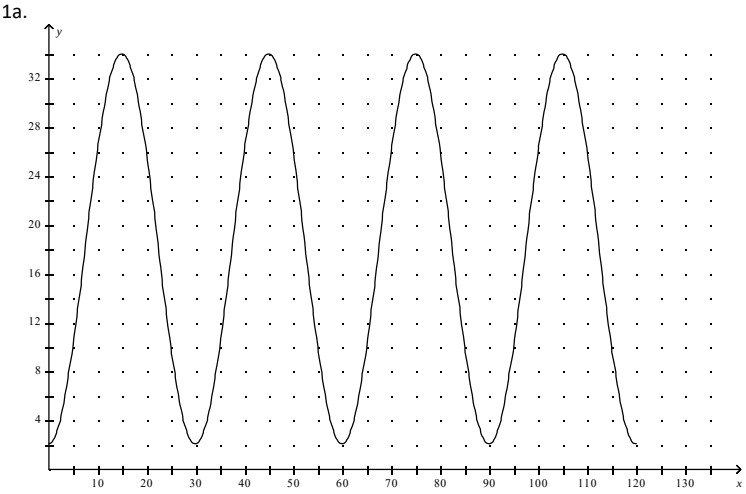
7. The table shows the minimum and maximum number of hours of sunlight in Ajax, Ontario.

Date	Day of Year	Hours of Sunlight
June 21	172	15.35
December 21	355	9.65

a) Graph the information above.
b) Determine an equation for the number of hours of sunlight on the nth day of the year. (use cosine)
c) Predict the number of hours of sunlight on February 10(day 41).
d) Predict a day when there are 14 h of sunlight.

ANSWERS

1a.



1b. $h = 16cos12°(t - 15) + 18$

1c.

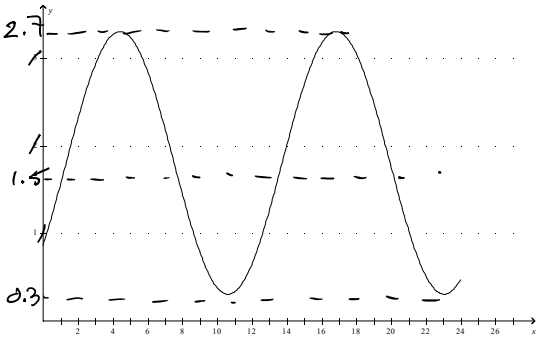
$D = \{t | t \in \mathbf{R}, 0 \leq t \leq 120\}$

$R = \{h | h \in \mathbf{R}, 2 \leq h \leq 34\}$

1d. 26m

1e. 5 sec, 25 sec, 35 sec, 55 sec, 65 sec, 85 sec, 95 sec, 115 sec

2a.



2b.

period = 12:25 = 12.42 hrs

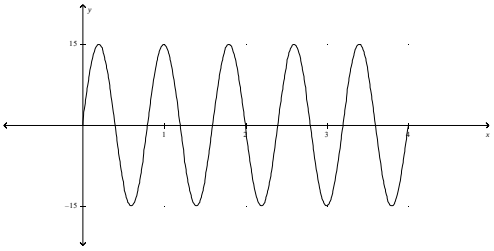
max starts at 16:52 = 16.87 hr

$T = 1.2cos29°(t - 16.87) + 1.5$

2c. 0.6m

2d. 12:55 or 25:21, 8:24 or 20:48

3a

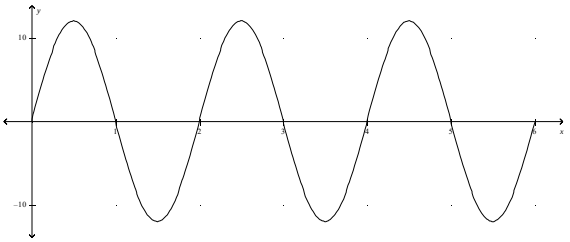


3b. $d = 15sin450°t$

3c. -15cm

3d. 0.12 sec, 0.28 sec, 0.92 sec, 1.08 sec, ...

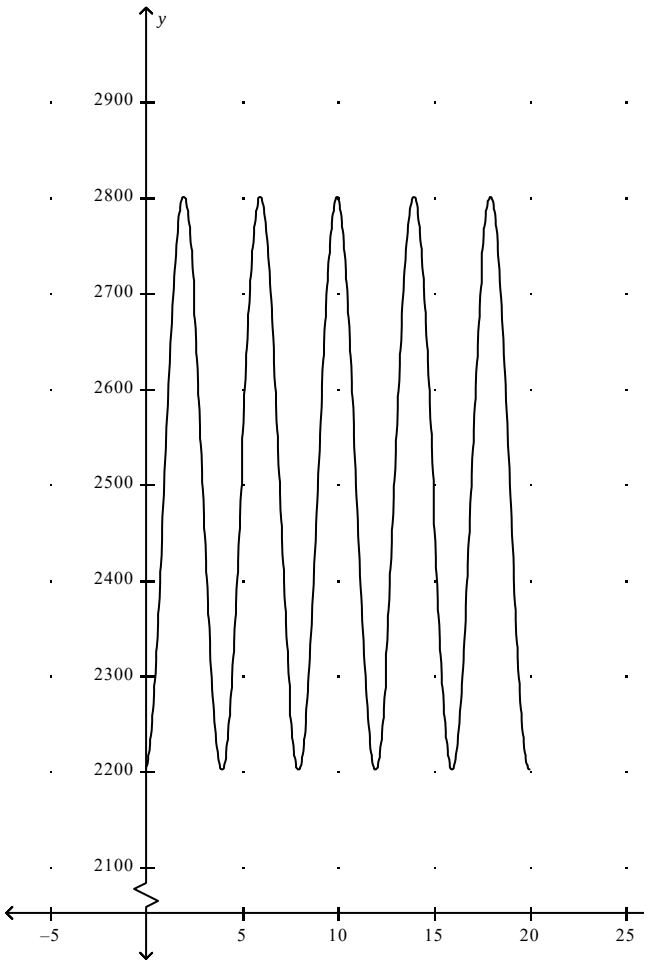
4a.



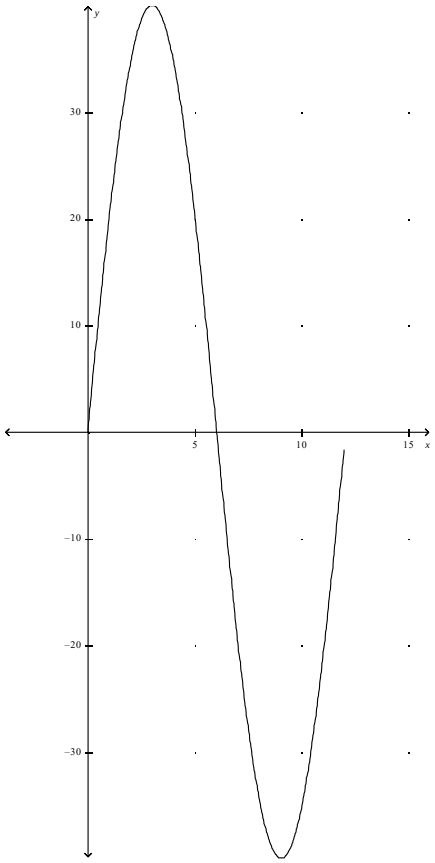
4b.

$A = 12sin180°t$

5a.



6a



5b. $V = 300cos90°(t - 2) + 2500$

5c.

i) rapid breathing – period will get shorter
ie. horizontal compression

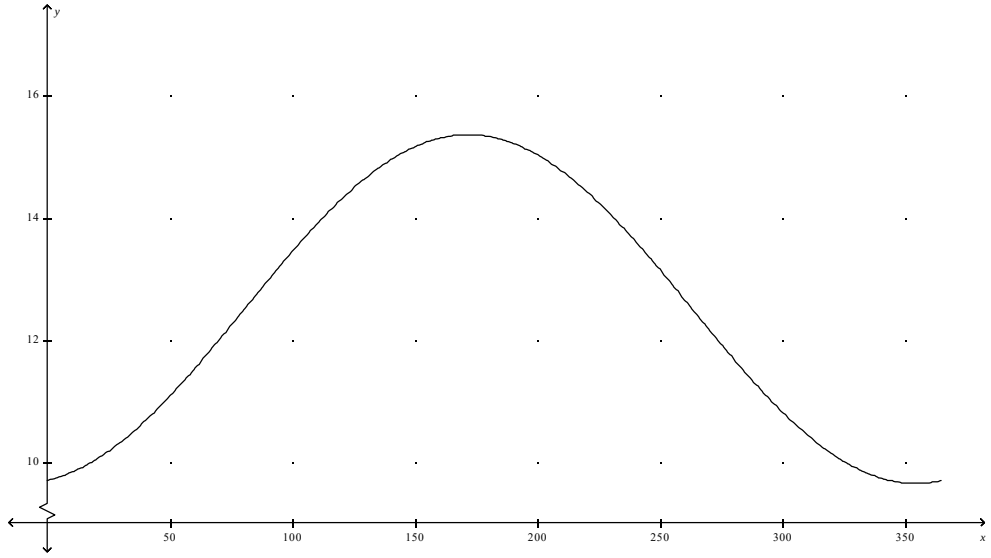
ii) Big breaths – amplitude bigger
ie. vertical stretch

6b. $d = 40sin30°t$

6c.
34.6m right of the vertical

6d.
-1.6 sec, 7.6 sec, 10.4 sec, ...

7a.



7b. $H = 2.85cos0.986°(n - 172) + 12.5$

7c.
10.7 hrs

7d.
day 112.93, day 231.07 ...