PRACTICEexplicitRecur

For each one below find a) explicit formula
b) recursive formula

#1. $3x+3$, $4x+10$, $5x+29$, $6x+66$, $7x+127$

2. Determine a rule for calculating the terms of the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \ldots$. Explain your reasoning.

#3. $\frac{10}{3}, \frac{14}{6}, \frac{18}{12}, \frac{22}{24}, \ldots$

#4. $\frac{3}{1}, \frac{8}{8}, \frac{15}{27}, \frac{24}{64}, \ldots$

5. Determine an expression for the general term of the sequence $x + \frac{1}{y}, 2x + \frac{1}{y^2}, 3x + \frac{1}{y^3}, \ldots$

6. Determine a rule for calculating the terms of the sequence $\frac{3}{5}, \frac{21}{55}, \frac{147}{555}, \frac{1029}{5555}, \frac{7203}{55\,555}, \frac{50\,421}{555\,555}, \ldots$. Explain your reasoning.

ANSWERS:

should know key # patterns $\to n^2: 1, 4, 9, 16, 25, \ldots$
$n^3: 1, 8, 27, 64, 125 \ldots$

#1. $3x+3, \quad 4x+10, \quad 5x+29, \quad 6x+66, \quad 7x+127$

ⓐ $\boxed{t_n = a_n + b_n}$ where $a_n: 3x, 4x, 5x \ldots$

$b_n: 3, 10, 29, 66, 127 \ldots$

differences: $7, 19, 37, 61$

$12, 18, 24$

$6, 6$ cubic, compare to $n^3: 1, 8, 27, \ldots$ always 2 more

$\boxed{\begin{aligned} a_n &= (n+2)x \\ b_n &= n^3+2 \end{aligned}}$ explicit

ⓑ recursive $\boxed{\begin{aligned} &a_n = a_{n-1} + x, \quad a_1 = 3x \\ &b_n = \left(\sqrt[3]{b_{n-1} - 2} + 1\right)^3 + 2, \quad b_1 = 3 \end{aligned}}$

2. Determine a rule for calculating the terms of the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \ldots$. Explain your reasoning.

ⓐ $\boxed{t_n = \frac{N_n}{D_n}}$ where $N_n: 1, 2, 3, \ldots$

$D_n: 2, 3, 4, \ldots$

$\boxed{\begin{aligned} N_n &= n \\ D_n &= n+1 \end{aligned}}$ or $\boxed{t_n = \frac{n}{n+1}}$

explicit

ⓑ recursive $\boxed{\begin{aligned} &N_n = N_{n-1} + 1, \quad N_1 = 1 \\ &D_n = D_{n-1} + 1, \quad D_1 = 2 \end{aligned}}$

#3. $\frac{10}{3}, \frac{14}{6}, \frac{18}{12}, \frac{22}{24}, \ldots$

ⓐ $\boxed{t_n = \frac{N_n}{D_n}}$ where $N_n: 10, 14, 18, \ldots$

$D_n: 3, 6, 12, \ldots$

$\boxed{\begin{aligned} N_n &= 10 + 4(n-1) = 4n+6 \\ D_n &= 3(2)^{n-1} \end{aligned}}$ OR $\boxed{t_n = \frac{4n+6}{3(2)^{n-1}}}$

explicit.

(b) recursive $\boxed{\begin{array}{l} N_n = N_{n-1} + 4, \ N_1 = 10 \\ D_n = D_{n-1} \times 2, \ D_1 = 3 \end{array}}$

#4. $\frac{3}{1}, \frac{8}{8}, \frac{15}{27}, \frac{24}{64}, \ldots$

(a) $t_n = \frac{N_n}{D_n}$ where $N_n: 3, 8, 15, 24$

$+5 \ +7 \ +9$

$+2 \ +2$ quad. compare with n^2 #'s $1, 4, 9, 16, \ldots$ always one less, and offset by 2

$\therefore N_n = (n+1)^2 - 1$

$\boxed{t_n = \frac{(n+1)^2 - 1}{n^3}}$ explicit

$D_n: 1, 8, 27, 64 \ldots$ these are cubes $\therefore D_n = n^3$

(b) recursive $\boxed{\begin{array}{ll} N_n = \left(\sqrt{(N_{n-1} + 1)} + 1\right)^2 - 1 & , N_1 = 3 \\ D_n = \left(\sqrt[3]{D_{n-1}} + 1\right)^3 & , D_1 = 1 \end{array}}$

5. Determine an expression for the general term of the sequence $x + \frac{1}{y}$, $2x + \frac{1}{y^2}$, $3x + \frac{1}{y^3}$, ...

(a) $t_n = nx + \frac{1}{y^n}$ explicit

(b) Recursive $t_n = a_n + b_n$ where $a_n = a_{n-1} + x$, $a_1 = x$

$b_n = b_{n-1} \div y$, $b_1 = \frac{1}{y}$

6. Determine a rule for calculating the terms of the sequence $\frac{3}{5}, \frac{21}{55}, \frac{147}{555}, \frac{1029}{5555}, \frac{7203}{55\,555}, \frac{50\,421}{555\,555}$, ... Explain your reasoning.

a) explicit $\boxed{t_n = \frac{a_n}{b_n}}$ where $a_n: 3, 27, 147, \ldots$ $\boxed{a_n = 3(7)^{n-1}}$

$b_n: 5, 55, 555, \ldots$

seems exponential.

notice that: $\frac{5}{9} = 0.\overline{55}$

$$\boxed{b_n = \frac{5}{9}(10^n) - \frac{5}{9}}$$ Hard :(

b) recursive

$$\boxed{\begin{array}{l} a_n = a_{n-1} \times 7, \quad a_1 = 3 \\ b_n = (b_{n-1} \times 10) + 5, \quad b_1 = 5 \end{array}}$$ easy :)

this is a reason
to know both explicit
and recursive!