

PRACTICE explicitRecur

For each one below find a) explicit formula
b) recursive formula

#1. $3x + 3$, $4x + 10$, $5x + 29$, $6x + 66$, $7x + 127$

2. Determine a rule for calculating the terms of the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \dots$ Explain your reasoning.

#3. $\frac{10}{3}, \frac{14}{6}, \frac{18}{12}, \frac{22}{24}, \dots$

#4. $\frac{3}{1}, \frac{8}{8}, \frac{15}{27}, \frac{24}{64}, \dots$

5. Determine an expression for the general term of the sequence $x + \frac{1}{y}, 2x + \frac{1}{y^2}, 3x + \frac{1}{y^3}, \dots$

6. Determine a rule for calculating the terms of the sequence $\frac{3}{5}, \frac{21}{55}, \frac{147}{555}, \frac{1029}{5555}, \frac{7203}{55555}, \frac{50421}{555555}, \dots$ Explain your reasoning.

ANSWERS:

should know key # patterns $\rightarrow n^2: 1, 4, 9, 16, 25, \dots$
 $n^3: 1, 8, 27, 64, 125, \dots$

#1. $3x + 3, 4x + 10, 5x + 29, 6x + 66, 7x + 127$

a) $t_n = a_n + b_n$ where $a_n: 3x, 4x, 5x, \dots$ $a_n = (n+2)x$ explicit
 $b_n: 3, 10, 29, 66, 127, \dots$ $b_n = n^3 + 2$
 $\begin{matrix} 7 & 19 & 37 & 61 \\ \underbrace{\quad} & \underbrace{\quad} & \underbrace{\quad} & \underbrace{\quad} \\ 12 & 18 & 24 & \\ \underbrace{\quad} & \underbrace{\quad} & \underbrace{\quad} & \\ 6 & 6 & & \end{matrix}$ cubic, compare to $n^3: 1, 8, 27, \dots$
 always 2 more

b) recursive $a_n = a_{n-1} + x, a_1 = 3x$
 $b_n = \left((b_{n-1} - 2) + 1 \right)^3 + 2, b_1 = 3$

2. Determine a rule for calculating the terms of the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \dots$. Explain your reasoning.

a) $t_n = \frac{N_n}{D_n}$ where $N_n: 1, 2, 3, \dots$ $N_n = n$ or $t_n = \frac{n}{n+1}$
 $D_n: 2, 3, 4, \dots$ $D_n = n+1$ explicit

b) recursive $N_n = N_{n-1} + 1, N_1 = 1$
 $D_n = D_{n-1} + 1, D_1 = 2$

#3. $\frac{10}{3}, \frac{14}{6}, \frac{18}{12}, \frac{22}{24}, \dots$

a) $t_n = \frac{N_n}{D_n}$ where $N_n: 10, 14, 18, \dots$ $N_n = 10 + 4(n-1) = 4n + 6$ explicit.
 $D_n: 3, 6, 12, \dots$ $D_n = 3(2)^{n-1}$ or $t_n = \frac{4n+6}{3(2)^{n-1}}$

b) recursive
$$\begin{aligned} N_n &= N_{n-1} + 4, N_1 = 10 \\ D_n &= D_{n-1} \times 2, D_1 = 3 \end{aligned}$$

#4. $\frac{3}{1}, \frac{8}{8}, \frac{15}{27}, \frac{24}{64}, \dots$

a) $t_n = \frac{N_n}{D_n}$ where $N_n: 3, 8, 15, 24$

$$\begin{array}{ccc} \underbrace{\quad} & \underbrace{\quad} & \underbrace{\quad} \\ +5 & +7 & +9 \\ \underbrace{\quad} & \underbrace{\quad} & \\ +2 & +2 & \end{array}$$

quad. compare with n^2 #'s 1, 4, 9, 16, ...
 always one less, and offset by 2
 $\therefore N_n = (n+1)^2 - 1$

$D_n: 1, 8, 27, 64, \dots$ these are cubes $\therefore D_n = n^3$

$$t_n = \frac{(n+1)^2 - 1}{n^3}$$

explicit

b) recursive
$$\begin{aligned} N_n &= \left(\sqrt{N_{n-1} + 1} + 1 \right)^2 - 1, N_1 = 3 \\ D_n &= \left(\sqrt[3]{D_{n-1} + 1} \right)^3, D_1 = 1 \end{aligned}$$

5. Determine an expression for the general term of the sequence $x + \frac{1}{y}, 2x + \frac{1}{y^2}, 3x + \frac{1}{y^3}, \dots$

a) $t_n = nx + \frac{1}{y^n}$ explicit

b) Recursive $t_n = a_n + b_n$ where $a_n = a_{n-1} + x, a_1 = x$
 $b_n = b_{n-1} \div y, b_1 = \frac{1}{y}$

6. Determine a rule for calculating the terms of the sequence $\frac{3}{5}, \frac{21}{55}, \frac{147}{555}, \frac{1029}{5555}, \frac{7203}{55555}, \frac{50421}{555555}, \dots$ Explain your reasoning.

a) explicit
$$t_n = \frac{a_n}{b_n}$$
 where $a_n: 3, 27, 147, \dots$ $a_n = 3(7)^{n-1}$
 $b_n: 5, 55, 555, \dots$
 seems exponential.

notice that: $\frac{5}{5} = 0.\overline{55}$

$$b_n = \frac{5}{9}(10^n) - \frac{5}{9} \leftarrow \text{Hard!}$$

b) recursive

$$\begin{aligned} a_n &= a_{n-1} \times 7, \quad a_1 = 3 \\ b_n &= (b_{n-1} \times 10) + 5, \quad b_1 = 5 \end{aligned} \leftarrow \text{easy!}$$

this is a reason
to know both explicit
and recursive!