

# Addition and Subtraction of Rational Expressions-Unlike denominators

## I. Model Problems

In this example we will find the least common multiple of rational expressions.

**Example 1: Find the least common multiple of  $3x^3 + 3x^2 - 6x$  and  $4x^3 - 8x^2 + 4x$ .**

Factor each term completely.

$$\begin{array}{ll} 3x^3 + 3x^2 - 6x & 4x^3 - 8x^2 + 4x \\ 3x(x^2 + x - 2) & 4x(x^2 - 2x + 1) \\ 3x(x + 2)(x - 1) & 4x(x - 1)(x - 1) \end{array}$$

List all the factors of the first expression.

$$3x(x + 2)(x - 1)$$

Add any missing factors of the next expression.

$$3x(x + 2)(x - 1) \cdot 4(x - 1)$$

Simplify partially.

$$12x(x + 2)(x - 1)^2$$

**Answer:  $12x(x + 2)(x - 1)^2$**

In this example we will add rational expressions with unlike denominators

**Example 2: Simplify and find the restrictions of  $\frac{4x}{x^2+5x+6} + \frac{5}{x^2-9}$ .**

Factor the denominators.

$$\frac{4x}{(x + 2)(x + 3)} + \frac{5}{(x + 3)(x - 3)}$$

$$x \neq -3, -2, 3$$

$$(x + 2)(x + 3)$$

Find the restrictions.

The least common denominator is the least common multiple. List all the factors of the first expression.

Add any missing factors of the next expression.

$$(x + 2)(x + 3) \cdot (x - 3)$$

Find equivalent expressions by multiplying the numerator of each expression by missing factor(s) in the denominator.

$$\frac{4x}{(x + 2)(x + 3)} = \frac{4x(x - 3)}{(x + 2)(x + 3)(x - 3)}$$

Add the numerators.

$$\frac{5}{(x + 3)(x - 3)} = \frac{5(x + 2)}{(x + 3)(x - 3)(x + 2)}$$

Simplify numerators.

$$\frac{4x^2 - 12x}{(x + 2)(x + 3)(x - 3)} + \frac{5x + 10}{(x + 3)(x - 3)(x + 2)}$$

Add numerators.

$$\frac{4x^2 - 7x + 10}{(x + 2)(x + 3)(x - 3)}$$

**Answer:  $\frac{4x^2 - 7x + 10}{(x + 2)(x + 3)(x - 3)}$   $x \neq -3, -2, 3$**

## II. Practice Problems

Find the LCM.

- $a^3b; ab^2$
- $15xy^2; 6x^2y^2$
- $8xy^4; 10x^2z^3; 4y^5$
- $(x+2); (x+3)$
- $(x-5)(x+2); (x+5)(x-2)$
- $x^2-4; x^2+4x+4$
- $x^2-2x-15; x^2-6x+5$
- $2x^3-14x^2+24x; 3x^4-18x^3+27x^2$

Simplify and find the restrictions

- $\frac{2}{a^3b} + \frac{3}{a^2b^4}$
- $\frac{3z}{x^4y} - \frac{5y}{x^3z}$
- $\frac{6}{4a^3b^2} + \frac{7}{10a^2b^4} - \frac{2}{5a^3b}$
- $\frac{5z}{16xy^2} - \frac{7x^2}{7y^3z} + \frac{3y}{12x^4z}$
- $\frac{3x}{x+1} + \frac{5}{x+2}$
- $\frac{4x}{x+3} - \frac{2}{x-5}$
- $\frac{3}{(x+2)(x+1)} + \frac{5}{(x+1)(x-2)} + \frac{7}{(x+2)(x-2)}$
- $\frac{3x}{x^2-x-6} + \frac{4}{x^2-9}$
- $\frac{7x}{x^2+x-12} - \frac{2x}{x^2+9x+20}$
- $\frac{5x+1}{x^2-10x+25} + \frac{2x}{x^2-x-20}$
- $\frac{x-1}{(x+3)(x-4)} + \frac{3x}{(x-4)(x+2)} + \frac{2}{3x(x-4)}$
- $\frac{1}{x^2+8x+16} + \frac{1}{x^2-16}$

### III. Challenge Problems

1. Find the student's error.

$$\begin{aligned} & \frac{3x}{(x+3)(x-2)} - \frac{2}{(x-2)(x-3)}, x \neq -3, 2, 3 \\ & \frac{3x(x-3)}{(x+3)(x-2)(x-3)} - \frac{2(x+3)}{(x-2)(x-3)(x+3)} \\ & \frac{3x^2 - 9x}{(x+3)(x-2)(x-3)} - \frac{2x+6}{(x-2)(x-3)(x+3)} \end{aligned}$$

$$\frac{3x^2 - 11x + 6}{(x+3)(x-2)(x-3)}, x \neq -3, 2, 3$$

2. Find the student's error.

$$\begin{aligned} & \frac{2x}{(x-5)(x+5)} + \frac{4}{(x+5)(x+5)}, x \neq -5, 5 \\ & \frac{2x}{(x-5)(x+5)} + \frac{4(x-5)}{(x+5)(x-5)} \\ & \frac{6x-20}{(x-5)(x+5)}, x \neq -5, 5 \end{aligned}$$

#### IV. Answer Key

- $a^3b^2$
- $30x^2y^3$
- $40x^2y^5z^2$
- $(x+2)(x+3)$
- $(x+2)(x+5)(x-5)$
- $(x+2)^2(x-2)$
- $(x+3)(x-5)(x-1)$
- $6x^2(x-3)^2(x-4)$
- $\frac{3a+2b^3}{a^3b^4}, a \neq 0; b \neq 0$
- $\frac{3z^2-5xy^2}{x^4yz}, x \neq 0; y \neq 0; z \neq 0$
- $\frac{30b^2+14a-8b^3}{20a^3b^4}, a \neq 0; b \neq 0$
- $\frac{15x^3yz^2-42x^6+12y^4}{48x^4y^3z}, x \neq 0; y \neq 0; z \neq 0$
- $\frac{3x^2+11x+5}{(x+1)(x+2)}, x \neq -1; x \neq -2$
- $\frac{4x^2-22x-6}{(x+3)(x-5)}, x \neq -3; x \neq 5$
- $\frac{15x+11}{(x+2)(x-2)(x+1)}, x \neq -2; x \neq -1; x \neq 2$
- $\frac{7x^2+17x}{(x+2)(x-3)(x+3)}, x \neq -2; x \neq -3; x \neq 3$
- $\frac{5x^2+41x}{(x+4)(x-3)(x+5)}, x \neq -5; x \neq -4; x \neq 3$
- $\frac{7x^2+11x+4}{(x+4)(x-5)^2}, x \neq 5; x \neq -4$
- $\frac{12x^3+32x^2+4x+12}{3x(x+2)(x+3)(x-4)}, x \neq 0; x \neq -2; x \neq -3; x \neq 4$
- $\frac{2x}{(x+4)^2(x-4)}, x \neq -4, 4$

#### Challenge Problems

- Did not distribute negative to all terms of numerator of 2<sup>nd</sup> rational expression.
- Repeated factor needs to be accounted for in LCD. Correct LCD is  $(x-5)(x+5)^2$